

Advertisement and Product Design[☆]

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Abstract

A multi-product firm typically pools different designs of products into groups or tiers and advertises the tiers instead of the individual products. This paper examines an equilibrium foundation of tiers, advertisements, and designs of products. A product is a profile of multiple attributes with different quality levels. We analyze a signaling game in which the firm chooses product designs and advertises. Observing the advertisement, a consumer purchases the good. We show that the firm can pool different products into the same tier and advertise the tier in an equilibrium surviving criterion $D1$ (Cho and Sobel (1990)). The existing marketing literature indicated that the overlap of products in different tiers may undermine firm profitability (Aribarg and Arora (2008)). In contrast, we show that such overlaps may arise in an equilibrium.

Keywords: Product design, Quality, Attributes, Asymmetric information, Signaling, Tiers, Advertisement, Single-crossing property

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1. Introduction

Firms rarely advertise each product they sell. Rather, they organize multiple designs of products into groups or what are called “tiers” and attach a logo or trademark to each tier. The firm then advertises the tiers instead of individual products. Examples abound. The most prominent example is from the decision by Alfred P. Sloan to organize the GM cars into several brands (e.g., Chevrolet, Pontiac, Cadillac), each of which carries its own logo. Casio sells its watches in 11 tiers in the United States, including G-Shock, Baby-G, Edifice, Pro Trek, and the classic Casio. Different tiers may contain products with slightly different properties. However, more often, products share similar features not only within but also across tiers, and consumers rely heavily on logos to distinguish tiers. Within a tier, qualities and prices vary significantly. Figure 1 indicates different tiers of GM’s 4-door sedans and Casio’s men’s solar radio-wave watches and their price ranges.

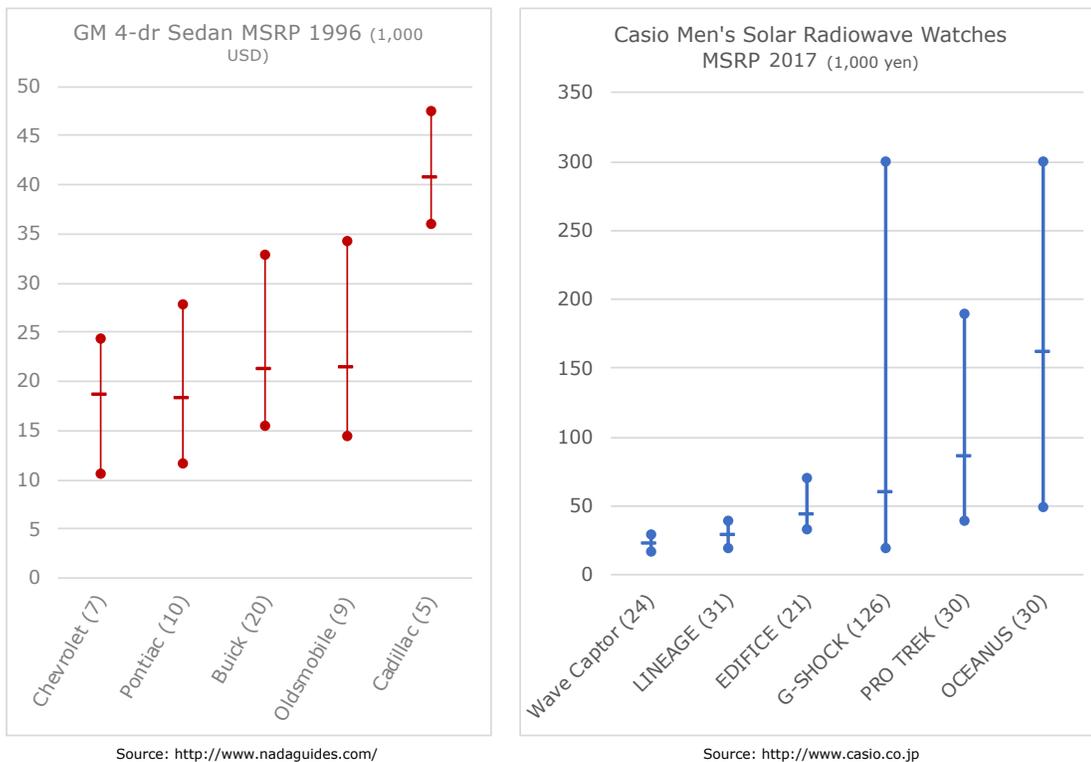


Figure 1: Price ranges of GM’s 4-door sedans (left panel) and Casio’s men’s solar radio-wave watches (right panel) by tiers, interpreted as a proxy for the marginal utility of consuming the first unit. Tick marks indicate the minimum, maximum, and average MSRP (Manufacturer’s Suggested Retail Price). Parentheses contain the number of models in each tier. The horizontal axis lists tiers and does *not* represent the advertisement expense.

Why do firms pool products of different qualities into a tier? This phenomenon is

puzzling in light of the traditional advertising literature, in which firms are considered to advertise to signal product quality (e.g., Milgrom and Roberts (1986), Kihlstrom and Riordan (1984), Bagwell (1989), Bagwell and Riordan (1991), Bagwell (1992), Bagwell and Ramey (1994)). More interestingly, the product quality of a tier often overlaps with those of the others’, as can be seen in Figure 1. Why do firms organize the tiers with overlaps? Overlapping tiers may cannibalize the consumer base, thereby undermining the profitability of the firm (Aribarg and Arora (2008)). By avoiding overlaps, the firm may more efficiently signal the average quality of the tier to a consumer. We address these questions in this paper.

The main objective of this paper is to investigate rigorously the foundation of tiers, advertising, and product design. We choose Milgrom and Roberts (1986) as a baseline model in which the firm decides the advertisement level to signal a product’s utility to a consumer. In the equilibrium of Milgrom and Roberts (1986), the firm advertises more for a higher quality product to separate a high-quality good from the rest in the most efficient manner. We use this separating equilibrium—also known as the Riley outcome—as a benchmark and modify the model in two substantive ways. First, in most advertisement signaling models in the literature, utility is generated by a single attribute and, hence, can be treated as product quality. We allow *multiple* attributes, each having its own scale of quality. A firm designs products by specifying the quality level of each attribute. Some attributes, called “basic attributes”, are easy to communicate to consumers. For example, for a men’s watch, the weight, size, or water-resistance depth can be conveyed to consumers with relative ease, although consumers’ experiences may still vary somewhat. In contrast, the accuracy of radio-wave watches is more difficult to communicate because it depends on the movement, radio-wave reception strength, and the environment in which the watch is used: we call such an attribute more “elaborate.” A more elaborate attribute requires more resources for a firm to explain how it works and how it benefits consumers. But, we do *not* assume that more elaborate attribute generates higher utility.

Second, the firm chooses *endogenously* the attribute level before deciding on the advertisement intensity. We consider the product design as a long-term decision and the advertisement as a short-term decision conditional on the product design. The utility of a product becomes the firm’s type in the signaling stage, which is *endogenous*. The equilibrium structure of an advertisement is critically affected by products’ utility profiles; however at the same time, the firm takes the advertising cost into account when choosing its product design. The interaction between the product design and the advertisement structure is the focus of our investigation.

After the firm chooses a product design, which becomes its type, we follow Milgrom and Roberts (1986) as closely as possible. We maintain the assumption in the signaling game literature, which is that the quality of each attribute cannot be verified without actual consumption of the product, even if the presence of the attribute may be observed. We assume the single-crossing property with respect to quality for each attribute: signaling higher quality is easier than signaling lower quality. Then, the firm decides on how to advertise the products. The firm chooses a message level conditional on an individual product. Observing the message, consumers purchase the good. We assume that the good

is an experience good so that, after initial consumption, the quality of each attribute is revealed to the consumer. Assuming that consumers are competing for the good in a Bertrand manner, they pay the expected utility conditional on each message and then randomly pick up a good within a tier for actual consumption.¹

Following Nelson (1970), we interpret the message as an advertisement or any instrument used to advertise the product, such as a logo or trademark. If different products are assigned to the same message, we treat the message as a tier, which corresponds to a pooling equilibrium in the signaling (sub-)game. The focus of our analysis is to understand whether a pooling equilibrium can arise even if the single-crossing property holds with respect to quality. We are interested in the tier structure, particularly the distribution of the utility of products within a tier. We investigate whether and how the support of the distribution of tiers may overlap.

The equilibrium outcome is characterized by the profile of the products and the structure of the tiers. We impose criterion *D1* in each signaling game to ensure that the equilibrium is sustained by reasonable beliefs (Cho and Sobel (1990)). Roughly speaking, an equilibrium advertisement satisfies criterion *D1*, if the product with the smallest marginal cost of signaling among the products assigned to the same tier cannot benefit by being separated from the rest by sending a slightly larger message than the equilibrium.

For an arbitrary profile of products, the ensuing signaling game does not have the single-crossing property. The key implication of the single-crossing property is that the type (i.e., the product) that carries the least marginal cost of signaling is the highest quality product among those in the same tier. Deriving the single-crossing property along the equilibrium path is the most critical step of our analysis. We show that the marginal rate of substitution between the cost and the benefit of signaling in a tier must be identical to satisfy criterion *D1*. We prove that the marginal rate of substitution between the cost and the benefit of a tier's signaling decreases as the average quality of the product in a tier increases.

By exploiting the single-crossing property over the tiers (not over products), we show that the seller separates tiers of products instead of individual products in an equilibrium. Because the different tiers separate according to the average product utility in the individual tiers, ranking the tiers accordingly makes sense. Through an example, we show that a tier with multiple products can indeed arise. Moreover, the product in a higher tier with the lowest utility may generate a lower utility than all products in such a lower tier in equilibrium, as depicted in Figure 1, refuting the claim by Aribarg and Arora (2008). The rather surprising overlapping price ranges allow the firm conserve the signaling cost, by making the average product utilities closer across tiers.

We formally describe the model in section 2. In section 3, we analyze the equilibrium. Section 4 concludes the paper.

¹One can also interpret a tier as a convex combination of different products, and a consumer purchases the combination of the product.

2. Formal description

Let us imagine a monopolistic firm that has been producing a good, which is indexed as $k = 1$ and has the profile $\boldsymbol{\pi}_1 = (\pi_{11}, \dots, \pi_{1L})$. The firm plans to develop a new product that generates higher utility than the existing product. In the first round, the firm announces K possible products:

$$\mathcal{K} = \{1, 2, \dots, K\},$$

each of which has L attributes. Let

$$\mathcal{L} = \{1, \dots, L\}$$

be the set of feasible attributes, which is the set of L positive integers.² An attribute comes with different qualities. Let $\pi_{k\ell}$ denote the quality of attribute ℓ of product k .³ We assume

$$0 \leq \pi_{k\ell} \leq 1$$

for convenience. Thus, product k is $\boldsymbol{\pi}_k \in [0, 1]^L \forall k = 1, \dots, K$. Product k is a vector of attributes with different quality:

$$\boldsymbol{\pi}_k = (\pi_{k1}, \dots, \pi_{kL}),$$

where $\pi_{k\ell} \geq 0$ is the quality of attribute k . We normalize the quality so that if $\pi_{k\ell} = 0$, then good k carries no attribute ℓ for $\ell = 1, \dots, L$. The firm designs its products by choosing the quality of each of L attributes

$$\boldsymbol{\pi}_k = (\pi_{k1}, \dots, \pi_{kL}) \quad \forall k \geq 2$$

for the new line of products and announces $\boldsymbol{\pi}_k$'s. If the product development fails, the firm continues to produce $\boldsymbol{\pi}_1$.

To focus on how asymmetric information affects the product design, we suppress the production cost, by assuming that the production cost of different products is normalized to 0 or has already been incurred.⁴

With probability $\mathcal{P}_\pi(k)$, product $k \geq 2$ is successfully developed. We interpret $\mathcal{P}_\pi(1)$ as the probability of failing to develop a new product. The firm observes the outcome of

²We choose L to be finite to simplify analysis. The extension is straightforward.

³Let us use a men's watch as an example. We consider $\ell = 1$ as the attribute of water resistance, $\ell = 2$ as the shock resistance capability, and $\ell = 3$ as the radio-wave reception capability. If $(\pi_{k1}, \pi_{k2}, \pi_{k3}) = (1, 0.5, 0)$, the watch remains waterproof in deep water, is moderately robust against shock, and has no radio-wave reception capability.

⁴We impose a capacity constraint on each attribute. To produce $\boldsymbol{\pi}_k$, we assume that the firm must secure a proper supply of each attribute. We assume that the supply of attributes is finite so that the firm cannot produce an unlimited amount of the identical product. The firm can maximize profits by producing only the product with the highest quality of the most elaborate attribute. This assumption is imposed to rationalize product differentiation.

the product development, in particular the utility that product π_k generates. A consumer does not observe π_k and, hence, all goods are ex-ante identical. Let us assume that without any advertisement, the lemon's problem prevails. For simplicity, we assume that $\mathcal{P}_\pi = (\mathcal{P}_\pi(1), \dots, \mathcal{P}_\pi(K))$ is exogenous and independent of π . Under this assumption, we can write $\mathcal{P}(k)$ in place of $\mathcal{P}_\pi(k)$.

Conditional on the realization of the product and its utility, the firm chooses the level of advertisement m . Let $\mathcal{M} = [0, \infty)$ be the space of messages (or the marketing expenses). Conditional on π , the sender chooses $m \in \mathcal{M}$ according to

$$\sigma_\pi : \{1, \dots, K\} \rightarrow \mathcal{M}.$$

Observing m , a representative consumer pays price $a(m)$ to purchase a good. Following Milgrom and Roberts (1986), we assume that the good is an experience good. After initial consumption, the consumer realizes the actual utility of the product. Assuming a Bertrand-type competition among consumers, they continue paying for the actual utility after the first consumption.

The seller's payoff from selling $\pi_k = (\pi_{k1}, \dots, \pi_{kL})$ is

$$a(m) - mf(\pi_k)$$

where $a(m)$ is the payment from a consumer, and

$$f(\pi_k) = \sum_{\ell=1}^L f_\ell(\pi_{k\ell})$$

is the marginal cost of sending signal m .⁵ We assume that $\forall \ell = 1, \dots, L, \forall \pi_{k\ell} \in [0, 1]$;

$$0 \leq f_\ell(1) \leq f_\ell(\pi_{k\ell}) \leq f_\ell(0), \quad f'_\ell(\pi_{k\ell}) < 0 \quad \text{and} \quad f_\ell(\pi_{k\ell}) < f_{\ell+1}(\pi_{k\ell}).$$

Under our assumption, the marginal cost of signaling a given attribute is positive but decreasing as $\pi_{k\ell}$ is increasing, which is the single-crossing property. Given quality, the marginal cost of signaling is higher as the firm uses a more elaborate attribute in the product, implying that communicating with consumers about a more elaborate attribute is more expensive.

Let $h(\pi_k) \geq 0$ be the utility that a consumer receives from product $\pi_k = (\pi_{k1}, \dots, \pi_{kL})$, where h is quasi-concave so that if $h(\pi_k) \geq h(\pi'_k)$, then $\forall \alpha \in (0, 1)$,

$$h(\alpha\pi_k + (1 - \alpha)\pi'_k) \geq h(\pi'_k).$$

⁵The additively separable marginal cost function is only for the convenience of spelling out the restrictions imposed on the signaling cost, particularly the monotonicity with respect to the attributes. The same analysis continues to apply to the case of a continuum of attributes, as long as we assume that the marginal cost is increasing with respect to the attribute and the single-crossing property holds.

We assume that a consumer prefers higher quality within attributes. Function h is increasing with respect to each component but also along the attributes: $\forall \boldsymbol{\pi}_k$ and any positive vector $\boldsymbol{\epsilon} = (\epsilon_1, \dots, \epsilon_L) > 0$,

$$h(\boldsymbol{\pi}_k + \boldsymbol{\epsilon}) \geq h(\boldsymbol{\pi}_k).$$

With respect to the attributes, note that we assume quasi-concavity, but do not assume monotonicity. The fact that it is costlier to signal an attribute does not necessarily mean that the attribute is more desirable than others. If a consumer pays $a(m)$ to purchase good $\boldsymbol{\pi}_k$, his payoff is

$$h(\boldsymbol{\pi}_k) - a(m).$$

For a fixed $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \dots, \boldsymbol{\pi}_K)$ and probability distribution \mathcal{P} , we can treat the continuation game as a signaling game in which seller type is given by $\boldsymbol{\pi}_k$, with the probability $\mathcal{P}(k)$. Let us refer to the signaling game as $G(\boldsymbol{\pi})$. Since

$$f'_\ell(\pi_{k\ell}) < 0 \quad \forall \ell,$$

the marginal cost of signaling is decreasing in individual qualities of attributes. However, in $G(\boldsymbol{\pi})$, the type is $\boldsymbol{\pi}_k$ but no rank order may exist among $\boldsymbol{\pi}_k$. Thus, we do not have the single-crossing property in $G(\boldsymbol{\pi})$.

To define the equilibrium outcome of the entire game, we first define an equilibrium for $G(\boldsymbol{\pi})$ for fixed $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \dots, \boldsymbol{\pi}_K)$. Observing m , a consumer forms posterior conjecture $\mu(\boldsymbol{\pi}_k|m)$. We assume that a mass of homogeneous consumers exists and they compete as Bertrand competitors (Milgrom and Roberts (1986)).⁶ Suppose that a consumer has a belief about k and $\boldsymbol{\pi}_k$ conditional on m as $\mu(\boldsymbol{\pi}_k|m)$, then the payment from the consumer is the expected utility of the product conditional on m :

$$a(m) = \sum_{k=1}^K h(\boldsymbol{\pi}_k) \mu(\boldsymbol{\pi}_k|m).$$

In any sequential equilibrium, sequential rationality requires that a consumer pays his expected utility from the consumption. Sequential equilibrium in $G(\boldsymbol{\pi})$ can then be written as a pair $(\sigma; \mu)$ of strategy profile $(\boldsymbol{\pi}, \sigma)$ and system of beliefs μ , satisfying consistency and sequential rationality (Kreps and Wilson (1982)).

To ensure that a sequential equilibrium is sustained by a system of reasonable beliefs, we impose restrictions on beliefs off the equilibrium path inspired by criterion $D1$ in each signaling game induced by $\boldsymbol{\pi}$ (Cho and Kreps (1987) and Cho and Sobel (1990)). We focus on a set of sequential equilibria, in which the possibility of signaling the true type by making a small deviation from the equilibrium message is exhausted. Given a particular

⁶To focus on the signaling role of advertising, we abstract from the heterogeneity of consumers who prominently appear in the discrete choice models in the empirical IO or marketing literature, such as Berry (1994) and Berry et al. (1995).

sequential equilibrium in $G(\boldsymbol{\pi})$ and m that is not used with a positive probability in the equilibrium, define

$$\mathcal{D}(m, k) = \{a \mid \mathcal{U}_k(m, a) > \mathcal{U}_k^*\}$$

where \mathcal{U}_k^* represents the equilibrium payoff from $\boldsymbol{\pi}_k$. $\mathcal{D}(m, k)$ is the set of responses by the consumers, which generates strictly higher profit for $\boldsymbol{\pi}_k$ than the equilibrium.

Definition 1. A sequential equilibrium satisfies criterion *D1* if, for any m not used in the equilibrium with a positive probability, belief $\mu(\boldsymbol{\pi}_k | m) = 0$ whenever $\exists k'$ such that $\mathcal{D}(m, k) \subset \mathcal{D}(m, k')$. If a sequential equilibrium $(\sigma_{\boldsymbol{\pi}}, \mu)$ satisfies criterion *D1*, then we call it a *D1* equilibrium.

We define a brand as a pair of profiles of products and messages associated with the product lines.⁷

Definition 2. A brand is $(\boldsymbol{\pi}, \mathbf{m})$ where $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \dots, \boldsymbol{\pi}_K)$ is the profile of products and $\mathbf{m} = (m_1, \dots, m_K)$ is the profile of messages, where m_k is assigned to $\boldsymbol{\pi}_k$.

To be a meaningful brand, the message should be supported by a *D1* equilibrium in $G(\boldsymbol{\pi})$. Otherwise, one product line can make a small deviation from the existing message to signal its type, thus, generating a higher profit.

Definition 3. Given $\boldsymbol{\pi}$ and the signaling game $G(\boldsymbol{\pi})$ induced by $\boldsymbol{\pi} = (\boldsymbol{\pi}_k)_{k=1}^K$, we say that brand $(\boldsymbol{\pi}, m)$ is valid if $\forall k, m_k = \sigma_{\boldsymbol{\pi}}(k)$ where $\sigma_{\boldsymbol{\pi}}$ is a *D1* equilibrium in $G(\boldsymbol{\pi})$.

Define the ex-ante expected profit of a firm from a valid brand induced by $\boldsymbol{\pi}$ as $\mathcal{U}_s(\boldsymbol{\pi})$. The firm chooses the profile of products to maximize its profits, among all valid brands.

Definition 4. An optimal brand is $(\boldsymbol{\pi}^*, \mathbf{m}^*)$ where \mathbf{m}^* is a valid brand in $G(\boldsymbol{\pi}^*)$ and $\mathcal{U}_s(\boldsymbol{\pi}^*) \geq \mathcal{U}_s(\boldsymbol{\pi}) \forall \boldsymbol{\pi}$, and $\mathcal{U}_s(\boldsymbol{\pi})$ is the equilibrium payoff of a valid brand in $G(\boldsymbol{\pi})$.

From now on, by an equilibrium, we mean an optimal brand.

Let $D(\boldsymbol{\pi})$ be the set of *D1* equilibria in $G(\boldsymbol{\pi})$. The set of equilibrium payoffs of $D(\boldsymbol{\pi})$ is known to be upper hemi-continuous with respect to $\boldsymbol{\pi}$, from which the existence of a sequential equilibrium of the entire game follows (Cho and Sobel (1990)).

Furthermore, let (m_1, \dots, m_J) be the profile of messages used with a positive probability in an equilibrium: $\forall j, \exists k$ such that $\sigma_{\boldsymbol{\pi}}(k) = m_j$ for $k \in \{1, \dots, K\}$ and $j \in \{1, \dots, J\}$. Without loss of generality, assume that $m_1 < \dots < m_J$. Note that if $J < K$, we have a pooling equilibrium with multiple products in at least in one tier.

Definition 5. Fix $G(\boldsymbol{\pi})$. $\boldsymbol{\pi}_k$ and $\boldsymbol{\pi}_{k'}$ are in the same tier if $\sigma_{\boldsymbol{\pi}}(k) = \sigma_{\boldsymbol{\pi}}(k')$. m is a higher tier than m' if

$$\mathbb{E}[h(\boldsymbol{\pi}_k) \mid m] > \mathbb{E}[h(\boldsymbol{\pi}_k) \mid m']$$

⁷Although there is no consensus in economics, marketing, law, accounting and consumer psychology about what brand is, our use of the brand is consistent with one of the 12 definitions listed in de Chernatony and Riley (1998). We regard a brand as the long term capital for the product and the advertisement.

where m and m' are used with positive probabilities in an equilibrium. We say that a product is in a higher tier than another product if the expected utility conditional on the signal is higher.

3. Properties of optimal brand

3.1. Preliminaries

In our case, the single-crossing property of $G(\boldsymbol{\pi})$ is *not* guaranteed for some $\boldsymbol{\pi}$. Consequently, in a *valid* brand $(\boldsymbol{\pi}, \mathbf{m})$, the tier may not increase with respect to the marketing expense m . We show that for an *optimal* brand, the products are grouped into tiers that are ranked according to the amount of marketing expense, and the marketing expense is strictly increasing with respect to the tier.

We focus on an optimal brand that entails a finite number of messages, for technical reasons.⁸ Let $\mathbf{m} = (m_1, \dots, m_J)$ be the profile of equilibrium messages: $\forall j, \exists k$ such that $\sigma_{\boldsymbol{\pi}}(k) = m_j$ for $k \in \{1, \dots, K\}$ and $j \in \{1, \dots, J\}$ for $J \leq K$. Without loss of generality, assume that

$$m_1 < \dots < m_J.$$

Fix $m \in \{m_1, \dots, m_J\}$. Define m_- and m_+ as the adjacent message immediately below and immediately above m :

$$m_- = \max\{m' < m\} \quad \text{and} \quad m_+ = \min\{m' > m\}$$

If $m = m_1$, let $m_- = m$. If $m = m_J$, then let $m_+ = m$. For each $k \in \sigma_{\boldsymbol{\pi}}^{-1}(m)$, we know the marginal rate of substitution between m and the price,

$$\left. \frac{da}{dm} \right|_k = f(\boldsymbol{\pi}_k).$$

Define \tilde{k} as the type that can signal the true type at the minimal cost among those under the same message m :

$$\left. \frac{da}{dm} \right|_{\tilde{k}} = f(\boldsymbol{\pi}_{\tilde{k}}) \leq f(\boldsymbol{\pi}_k) \quad \forall k \in \sigma_{\boldsymbol{\pi}}^{-1}(m).$$

Similarly, define \hat{k} as the opposite of \tilde{k} , which is the type that bears the highest cost to signal among those under m :

$$\left. \frac{da}{dm} \right|_{\hat{k}} = f(\boldsymbol{\pi}_{\hat{k}}) \geq f(\boldsymbol{\pi}_k) \quad \forall k \in \sigma_{\boldsymbol{\pi}}^{-1}(m).$$

⁸We would like to avoid technical issues arising from mixed strategies over a continuum of pure strategies. As the ensuing analysis reveals, the optimal brand is a pure strategy equilibrium. In fact, we can focus on pure strategies, without loss of generality.

Finally, define $\pi_{\bar{k}}$ as the product with the highest utility under m :

$$h(\pi_{\bar{k}}) = \max\{h(\pi_k) \mid \sigma_{\pi}(k) = m\}.$$

Unless the single-crossing property with respect to the products holds, we should expect $\bar{k} \neq \tilde{k}$ and $\hat{k} \neq \tilde{k}$.

Let \tilde{k}_- , \hat{k}_- and \bar{k}_- be the corresponding elements defined for m_- instead of m . Let p be the unit price of sales of the good under m . Similarly, let p_- be the unit price of sales of the good under m_- .

3.2. Signaling cost

We measure the difficulty of signaling product quality in terms of the marginal rate of substitution between the marketing cost m and the expected return. We show that, for an optimal brand, the marginal rate of substitution under the same tier must be the same for all products in the same tier.

For a valid brand, it is possible that $\tilde{k} \neq \bar{k}$. However, for an optimal brand, $\tilde{k} = \bar{k}$ in any message used in equilibrium with positive probability. Because the proof reveals the critical role of product design, we state the proof in the text.

Lemma 1. *In an optimal brand (π, \mathbf{m}) , $\forall m$ that is used with a positive probability, $\tilde{k} = \bar{k}$.*

Proof. To prove the lemma by contradiction, suppose that

$$h(\pi_{\bar{k}}) > h(\pi_{\tilde{k}}).$$

By the definition of \tilde{k} ,

$$f(\pi_{\tilde{k}}) \leq f(\pi_{\bar{k}}).$$

Since h is quasi concave, $\forall \lambda \in (0, 1)$,

$$\hat{p} = \frac{\sum_{k \neq \bar{k}} h(\pi_k) \mathcal{P}(k) + h((1 - \lambda)\pi_{\bar{k}} + \lambda\pi_{\tilde{k}}) \mathcal{P}(\bar{k})}{\sum_{\sigma_{\pi}(k)=m} \mathcal{P}(k)} > \frac{\sum_{\sigma_{\pi}(k)=m} h(\pi_k) \mathcal{P}(k)}{\sum_{\sigma_{\pi}(k)=m} \mathcal{P}(k)} = p.$$

We cannot yet conclude that the deviation from $\pi_{\bar{k}}$ to $(1 - \lambda)\pi_{\bar{k}} + \lambda\pi_{\tilde{k}}$ by product \bar{k} generates profits higher than what the seller would have received in the candidate equilibrium. We need to construct a profile of products $\tilde{\pi} = (\tilde{\pi}_k)$, signaling game $G(\tilde{\pi})$, and a *valid* brand $(\tilde{\pi}, \tilde{\mathbf{m}})$ so that the seller can generate higher profits.

Notice that $\forall \lambda > 0$, $\hat{p} > p$, and $\lim_{\lambda \rightarrow 0} \hat{p} = p$. First, consider the indifference curves passing through (m, \hat{p}) by type k products where $\sigma_{\pi}(k) = m$ (cf. Figure 2). Locate the indifference curve among those passing through (m, \hat{p}) , which has the steepest slope. By definition, it is the indifference curve of \hat{k} . Second, locate the indifference curve among those passing through (m_-, p_-) that has the “flattest” slope. By definition, it is the indifference curve of \bar{k}_- . Finally, note that to satisfy the incentive constraint, $\forall k \in \sigma_{\pi}^{-1}(m)$,

$$\left. \frac{da}{dm} \right|_{\tilde{k}_-} > \left. \frac{da}{dm} \right|_{\hat{k}} \geq \left. \frac{da}{dm} \right|_k \quad (3.1)$$

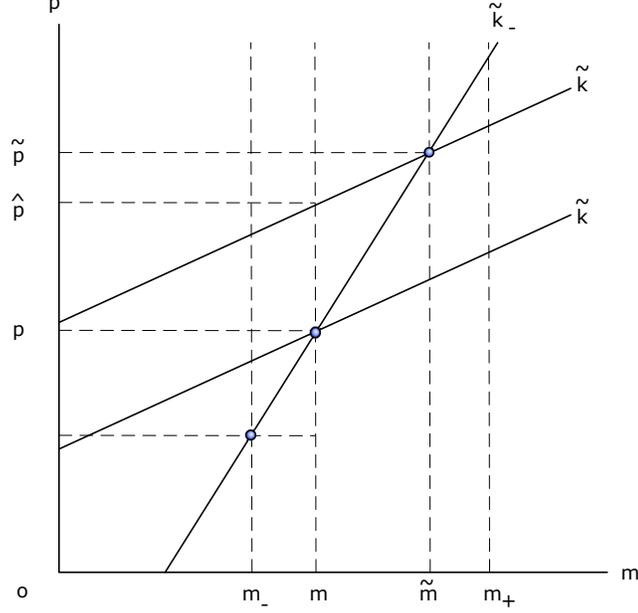


Figure 2: The indifference curves are linear because the marginal cost of sending a message is constant.

because $m_- < m$.

Choose (\tilde{m}, \tilde{p}) with $\tilde{m} > m$, satisfying the two conditions:

$$\hat{p} - mf(\pi_{\hat{k}}) = \tilde{p} - \tilde{m}f(\pi_{\hat{k}}) \quad (3.2)$$

$$p_- - m_-f(\pi_{\tilde{k}_-}) \geq \tilde{p} - \tilde{m}f(\pi_{\tilde{k}_-}). \quad (3.3)$$

(3.2) is the indifference condition for \hat{k} , whereas (3.3) is the indifference condition for \tilde{k}_- . By (3.1), (\tilde{m}, \tilde{p}) exists and is unique. By the definition of \hat{k} , $\forall k \in \sigma_{\pi}^{-1}(m)$,

$$\hat{p} - mf(\pi_k) \leq \tilde{p} - \tilde{m}f(\pi_k).$$

Thus, every type in $\sigma_{\pi}^{-1}(m)$ weakly prefers (\tilde{m}, \tilde{p}) to (m, p) . Recall that $m_- < m < \tilde{m}$. By (3.3),

$$\tilde{p} - \tilde{m}f(\pi_{k_-}) \leq p_- - m_-f(\pi_{k_-})$$

$\forall k_- \in \sigma_{\pi}^{-1}(m_-)$, which ensures the incentive constraint of k_- type.

We can choose $\lambda > 0$ sufficiently small so that $\tilde{m} < m_+$. Define

$$\tilde{\pi}_k = \begin{cases} \pi_k & \text{if } k \neq \bar{k} \\ \tilde{\pi} & \text{if } k = \bar{k} \end{cases}$$

and

$$\tilde{m}_k = \begin{cases} m_k & \text{if } k \neq \bar{k} \\ \tilde{m} & \text{if } k = \bar{k}. \end{cases}$$

By construction, it is straightforward to assign beliefs off the equilibrium path satisfying criterion *D1*. By construction, $\tilde{\mathbf{m}}$ is valid in the induced signaling game $G(\tilde{\pi})$. Since the seller generates a higher payoff in the valid brand $(\tilde{\pi}, \tilde{\mathbf{m}})$ in $G(\tilde{\pi})$ than the candidate equilibrium, we have a contradiction. \square

We can show that every type that sends the same message must have the same marginal rate of substitution between marketing expense m and expected payment a by the consumer.

Proposition 1. *Suppose that π is the products and $m_1 < \dots < m_J$ are the messages that are sent with a positive probability in the optimal brand. Then, if $\sigma_\pi(k) = \sigma_\pi(k')$, then*

$$\left. \frac{da}{dm} \right|_k = \left. \frac{da}{dm} \right|_{k'}.$$

Proof. See Appendix A. \square

Proposition 1 captures the critical feature of the long-term decision of product designs. To maximize profits, the firm designs individual products so that each product under the same message (or logo) m has the same marginal cost of the advertisement.

3.3. Recursive structure

By Proposition 1, it makes sense to write the marginal rate of substitution as a function of m_i ($i = 1, \dots, J$) instead of as a function of π_k under m_i :

$$\gamma_i = \left. \frac{da}{dm} \right|_k \quad \forall k \in \sigma_\pi^{-1}(m_i).$$

In an optimal brand, a weak form of the single-crossing property holds.

Lemma 2. *Suppose that (π, \mathbf{m}) is an optimal brand,*

$$\gamma_i \geq \gamma_{i+1} \text{ if } m_i < m_{i+1}.$$

Proof. See Appendix B. \square

Lemma 2 is weaker than the single-crossing property in the sense that γ_i may not be *strictly* increasing with respect to m_i . However, this property is sufficient to ensure that, in equilibrium, we only have to check the incentive of m_i to imitate m_{i+1} , not vice versa. Thus, Lemma 2 implies the recursive optimization characterization of the optimal brand.

However, the same result is not sufficient to prove that the expected utility conditional on m_i is increasing with respect to m_i . To this end, we need more results. Recall that we rank the equilibrium messages;

$$m_1 < \dots < m_J.$$

Lemma 3. *Suppose that (π, \mathbf{m}) is an optimal brand. $\forall i < J, \exists j > i$ so that*

$$-\gamma_i m_i + a_i = -\gamma_i m_j + a_j. \tag{3.4}$$

Proof. See Appendix C. □

Lemma 3 says that if $\sigma_{\boldsymbol{\pi}}(k) = m_i$ and $i < J$, then at least one message m_j higher than m_i must satisfy the incentive constraint with equality. To satisfy the incentive constraint of $\boldsymbol{\pi}_k$ where $\sigma_{\boldsymbol{\pi}}(k) = m_i$,

$$-\gamma_i m_i + a_i \leq -\gamma_i m_j + a_j$$

must hold for all $j > i$. If the incentive constraint holds with a strict inequality for every $j > i$, then the firm can save the advertisement expense by reducing m_j slightly while satisfying the incentive constraint $\forall j > i$, which contradicts the hypothesis that $(\boldsymbol{\pi}, \mathbf{m})$ is an optimal brand.

Lemma 4 strengthens Lemma 3 and states that if the incentive constraint of $\boldsymbol{\pi}_k$ with $\sigma_{\boldsymbol{\pi}}(k) = m_i$ is binding for m_j as in (3.4), then m_j must be the adjacent message above m_i : $m_j = m_{i+1}$.

Lemma 4. *Suppose that $(\boldsymbol{\pi}, \mathbf{m})$ is an optimal brand. If $\exists j > i$ and*

$$-\gamma_i m_i + a_i = -\gamma_i m_j + a_j,$$

then

$$-\gamma_i m_i + a_i = -\gamma_i m_{i+1} + a_{i+1},$$

Proof. See Appendix D. □

We show that in an optimal brand, the ranking of the advertisement reveals the rank of the tiers according to the average quality of the products conditional on the tier.

Proposition 2. *Let $(\boldsymbol{\pi}, \mathbf{m})$ be an optimal brand where $\mathbf{m} = (m_1, \dots, m_K)$. Given $\boldsymbol{\pi} = (\boldsymbol{\pi}_k)_{k=1}^K$, \mathbf{m} is constructed recursively: m_1 solves*

$$\max_{m_1} \mathbf{E} [h(\boldsymbol{\pi}_k) - m_1 f(\boldsymbol{\pi}_k) \mid].$$

$\forall i \geq 2$, m_i solves

$$\max_{m_i} \mathbf{E} [h(\boldsymbol{\pi}_k) - m_i f(\boldsymbol{\pi}_k) \mid \sigma_{\boldsymbol{\pi}}(k) = m_i]. \quad (3.5)$$

subject to

$$\mathbf{E}[h(\boldsymbol{\pi}_{k'}) \mid \sigma_{\boldsymbol{\pi}}(k') = m_{i-1}] - m_{i-1} f(\boldsymbol{\pi}_{k'}) \geq \mathbf{E}[h(\boldsymbol{\pi}_k) \mid \sigma_{\boldsymbol{\pi}}(k) = m_i] - m_i f(\boldsymbol{\pi}_{k'})$$

$\forall k' \in \sigma_{\boldsymbol{\pi}}^{-1}(m_{i-1})$. *Moreover, $\mathbf{E}[h(\boldsymbol{\pi}_k) \mid \cdot]$ is strictly increasing with respect to m in an optimal brand.*

Proof. Combining the preliminary results, we can characterize the optimal brand recursively. We also know that if $m_i < m_{i+1}$, then

$$-\gamma_i m_i + a_i = -\gamma_i m_{i+1} + a_{i+1}.$$

Thus,

$$a_{i+1} > a_i.$$

Since $a_i = \mathbf{E}[h(\boldsymbol{\pi}_k) \mid \sigma_{\boldsymbol{\pi}}(k) = m_i]$, we conclude that the expected quality of m_i is a strictly increasing function of m_i . □

Since the level of advertisement is ranked according to the expected quality, we can regard each m_j as a tier of products.

3.4. Example

Proposition 2 states that the seller *can* differentiate the products through tiers, instead of fully separating individual products (Milgrom and Roberts (1986)). Through an example, we show that Proposition 2 is not vacuous. We construct an example in which multiple products must be pooled in the same tier, and the quality of the products in different tiers “overlap” in an optimal brand. This example shows that the brand structure depicted in Figure 1 can indeed be optimal, in contrast to the claims by some marketing studies (Aribarg and Arora (2008)).

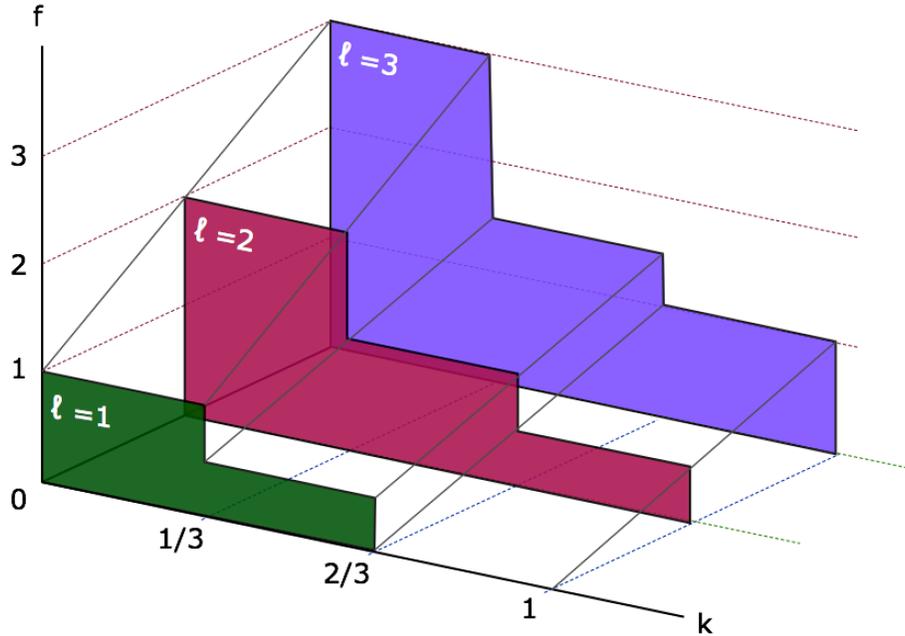


Figure 3: Colored planes are f defined for each $\ell = 1, 2, 3$.

A product is a profile of three attributes with different quality levels: $L = 3$. The firm has been producing the baseline product

$$\pi_1 = (0, 1, 1)$$

but plans to develop two new products π_2 and π_3 . Let us assume that $\mathcal{P}(k) = 1/3$.

Suppose that $f : [0, 1]^3 \rightarrow \mathbb{R}$ is given as in Figure 3;

$$f(\pi_{k1}, \pi_{k2}, \pi_{k3}) = \begin{cases} \ell & \text{if } 0 \leq \pi_{k1} < \frac{1}{3} \\ \frac{1}{3}\ell + \frac{1}{6} & \text{if } \frac{1}{3} \leq \pi_{k1} < \frac{2}{3} \\ \frac{1}{3}\ell - \frac{1}{3} & \text{if } \frac{2}{3} \leq \pi_{k1} < 1 \\ 0 & \text{if } \pi_{k1} = 1. \end{cases}$$

To simplify the example, we choose the first coordinate (quality) to have a step function structure, and the second coordinate (attributes) to have three elements ($L = 3$). We intentionally construct the example to be non-generic to highlight the key idea. We can easily turn the example into a generic one with continuous $f : [0, 1]^3 \rightarrow \mathbb{R}$, admitting a continuum of quality and attributes.

The utility of a product with π_k is

$$h(\pi_{k1}, \pi_{k2}, \pi_{k3}) = \begin{cases} 0 & \text{if } \pi_{k1}\pi_{k2} < 1 \text{ or } \pi_{k3} \leq 1/3 \\ 4 & \text{if } \pi_{k1}\pi_{k2} = 1 \text{ and } 1/3 < \pi_{k3} \leq 2/3 \\ 10 & \text{if } \pi_{k1}\pi_{k2} = 1 \text{ and } 2/3 < \pi_{k3} \leq 1. \end{cases}$$

Observe that h is weakly, if not strictly, increasing with respect to quality and attribute. We can easily modify h to be continuous and strictly quasi-concave over $[0, 1]^3$.

Given the step function structure of f and h with respect to the quality, we can treat quality as having three representative elements $\pi_{k1} \in \{\frac{1}{4}, \frac{1}{2}, 1\}$ for each $\ell \in \{1, 2, 3\}$. The status quo of the firm is to produce a product with $\pi_1 = (0, 1, 1)$.

Under the assumption over the production capacity of the attributes, the firm must select two different products from π_1 . In particular, the first component of π_2 and π_3 must differ: one product must use the quality of the first attribute in $[1/3, 2/3)$, whereas the other must use the quality of the first attribute in $[2/3, 1]$.

Without loss of generality, let $\pi_{21} = \frac{1}{2} \in [1/3, 2/3]$ and $\pi_{31} = 1 \in [2/3, 1]$. The firm decides to choose two other products with a specific combination of quality and attribute, and then to chooses the advertisement level to construct a tier.

Proposition 1 imposes a restriction over π_k with $\sigma_\pi(k) = m$. To be a product in the same tier, the product design must induce the same marginal cost of signaling.

To earn a positive profit, the firm has to produce product k with $\pi_{k3} > 1/3$. A consumer does not observe the outcome of the development and is aware that, with probability $1/3$, the development will fail and the product will remain $\pi_1 = (0, 1, 1)$, which generates 0 utility. The core of the firm's product design problem is to choose the right level of an attribute for each quality level and then package them into a tier to reduce the signaling cost. Pooling different products into the same tier economizes the signaling costs. However, to be valid, the products in the same tier must have the same marginal cost of signaling which may force the firm to choose a less elaborate attribute. Thus, profits may decline, as the ensuing analysis reveals.

First, let us consider a complete pooling equilibrium, in which all three products are pooled into a single tier. Thus, the equilibrium advertisement is $m = 0$. However, to place three different products into the same tier, the marginal cost of signaling of each product must be identical, which forces the seller to lower the level of elaboration.

Since $\pi_1 = (0, 1, 1)$ is realized with probability $1/3$,

$$f(\pi_1) = f(\pi_2) = f(\pi_3)$$

in the pooling equilibrium. The only way to satisfy the constraint is to choose the attribute so that

$$\pi_2 = (1, 1/2, 1) \text{ and } \pi_3 = (1, 1, 1).$$

That is, the mid-quality product must have the attribute of medium elaboration and the highest quality product must have the most elaborate attribute. The (ex-ante) expected profit is then

$$\frac{1}{3}(h(0, 1, 1) + h(1, 1/2, 1) + h(1, 1, 1)) = \frac{10}{3}.$$

An important observation is that to make the tier valid, the firm must design the second line of the product to be less elaborate than the third line of the product, which reduces the profit from the pooling equilibrium.

Second, we constructed the example in such a way that, in any equilibrium in which π_2 is pooled with π_1 or π_3 , $\pi_1 = (0, 1, 1)$ or $\pi_1 = (1, 1, 1)$ must hold to generate any positive profits and satisfy Proposition 1. Thus, any semi-pooling equilibrium in which π_2 is pooled cannot generate higher profits than the pooling equilibrium, because the utility of each product is the same as in the pooling equilibrium, but any separation incurs a positive signaling cost.

Third, we now conclude that to generate higher profits than in the pooling equilibrium, π_2 must be separated. We consider a completely separating equilibrium. To generate higher profits than in the pooling equilibrium, the design of the products should be

$$\pi_1 = (0, 1, 1), \quad \pi_2 = (1, 1, 1/2), \quad \text{and} \quad \pi_3 = (1, 1, 1),$$

such that the firm can generate positive profits from π_2 as well as from π_1 . However, the incentive constraint must be satisfied to reveal the true utility of the product. Let m_1 and m_2 be the amount of advertisement by the π_2 and π_3 products, which satisfies

$$h(\pi_1) - 0 \times f(\pi_1) \geq h(\pi_2) - m_1 f(\pi_1) \quad \text{and} \quad h(\pi_2) - m_1 f(\pi_2) \geq h(\pi_3) - m_2 f(\pi_3).$$

A simple calculation shows that

$$m_1 = 4 \quad \text{and} \quad m_2 = 8.$$

The profit from a separating equilibrium is

$$\frac{1}{3}(h(\pi_1) + (h(\pi_2) - 4f(\pi_2)) + (h(\pi_3) - 8f(\pi_3))) = \frac{1}{3}(0 + (4 - 4 \times 1.5) + (10 - 8 \times 1)) = 0,$$

which is less than what the firm makes from the pooling equilibrium. The signaling cost completely wipes out the profit.

Finally, let us consider a semi-pooling equilibrium in which the firm pools π_1 and π_3 into a higher tier, placing π_2 into a lower tier. Recall that by Proposition 1, if π_1 and π_3 are pooled, then $f(\pi_1) = f(\pi_3)$. Because the average utility of π_1 and π_3 is higher than the utility of π_2 , π_2 forms a lower tier and $f(\pi_2) > f(\pi_1) = f(\pi_3)$.

To separate, the higher tier needs to spend m_3 amount of advertisement to satisfy

$$h(\pi_2) - 0 \times f(\pi_2) \geq \frac{1}{2}(h(\pi_1) + h(\pi_3)) - m_3 \times f(\pi_3)$$

or

$$4 - 0 \times \frac{3}{2} \geq \frac{1}{2}(0 + 10) - m_3 \frac{3}{2}$$

which implies

$$m_3 = \frac{2}{3}.$$

Thus, the expected profit is

$$\begin{aligned} & \frac{1}{3} \left((h(\boldsymbol{\pi}_1) - \frac{2}{3}f(\boldsymbol{\pi}_1)) + (h(\boldsymbol{\pi}_2) - 0 \times f(\boldsymbol{\pi}_2)) + (h(\boldsymbol{\pi}_3) - \frac{2}{3}f(\boldsymbol{\pi}_3)) \right) \\ &= \frac{1}{3} \left((0 - \frac{2}{3} \times 1) + (4 - 0) + (10 - \frac{2}{3} \times 1) \right) = 4\frac{2}{9} \end{aligned}$$

which is larger than the profit from a pooling equilibrium, $10/3$.

Hence, the optimal brand must be $(\boldsymbol{\pi}, \mathbf{m})$ where

$$\boldsymbol{\pi} = (\boldsymbol{\pi}_1; \boldsymbol{\pi}_2; \boldsymbol{\pi}_3) = (0, 1, 1; 1, 1, 1/2; 1, 1, 1)$$

and $\mathbf{m} = (0, \frac{2}{3})$. The firm can produce the most elaborate products for each segment of the first attribute, thus generating the largest profit for each quality level. By pooling $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_3$ and relegating $\boldsymbol{\pi}_2$ to a lower tier, the firm can economize the signaling cost. Thus, the equilibrium must entail an “overlap” of qualities between the two tiers, because the lower tier consists of $\boldsymbol{\pi}_2$ which generates a higher utility than $\boldsymbol{\pi}_1$.

4. Concluding remarks

One might claim that, in our model, each product in the same tier is purchased at the same price, which appears inconsistent with actual practice. Even with the same tiers, different products carry different prices, depending on options. Our theory does not claim that every product in the same tier must receive the same price. As in Milgrom and Roberts (1986), we assumed that the products are experience goods. Thus, after the initial consumption of the product, the actual utility of the good is revealed and carries the price accordingly. The advertisement is a tool to entice a consumer to purchase the good when a new experience good is introduced.⁹ Even if a consumer does not observe the actual utility, different products in the same tier can be sold at different prices according to verifiable options. If the seller offers the same set of options for every product in the same tier, providing an option does not reveal the quality of the underlying product. Because a consumer understands exactly the utility from the optional attributes, the price that he or she pays is the sum of the expected utility for the quality of different attributes, which a consumer cannot verify. One example is a hotel chain. Different hotels under the same

⁹Our analysis applies to credence goods that a consumer does not find the actual utility even after the initial consumption. Life insurance is an example whose utility can be realized only after the policyholder dies.

trademark, such as Holiday Inn, differ in quality depending on location. Still, a consumer pays the same price to stay at Holiday Inn but pays different prices for options such as a large room, and size can be verified.

We believe that such a theory can be useful in providing a theoretical signaling interpretation for the brand fixed effects. In marketing and empirical industrial organizations, brand fixed effects are estimated as a significant component of product sales and prices, and are higher in value for higher tier products.

Appendices

Appendix A. Proof of Proposition 1

Since m is valid, criterion $D1$ requires that the belief conditional on $m' \in (m, m_+)$ should be concentrated at

$$\{k \mid f(\boldsymbol{\pi}_k) = f(\boldsymbol{\pi}_{\bar{k}})\}.$$

Thus, the expected payoff from sending m' should be

$$\mathbb{E}[h(\boldsymbol{\pi}_k) \mid f(\boldsymbol{\pi}_k) = f(\boldsymbol{\pi}_{\bar{k}})] - m' f(\boldsymbol{\pi}_{\bar{k}})$$

since a consumer is paying $\mathbb{E}[h(\boldsymbol{\pi}_k) \mid f(\boldsymbol{\pi}_k) = f(\boldsymbol{\pi}_{\bar{k}})]$. To be an equilibrium

$$\mathbb{E}[h(\boldsymbol{\pi}_k) \mid f(\boldsymbol{\pi}_k) = f(\boldsymbol{\pi}_{\bar{k}})] - m' f(\boldsymbol{\pi}_{\bar{k}}) \leq \mathbb{E}[h(\boldsymbol{\pi}_k) \mid \sigma^{-1}(m)] - m f(\boldsymbol{\pi}_{\bar{k}}).$$

Since $m' - m > 0$ can be arbitrarily small, the equilibrium condition implies

$$\mathbb{E}[h(\boldsymbol{\pi}_k) \mid f(\boldsymbol{\pi}_k) = f(\boldsymbol{\pi}_{\bar{k}})] \leq \mathbb{E}[h(\boldsymbol{\pi}_k) \mid \sigma_{\boldsymbol{\pi}}^{-1}(m)].$$

Recall that \bar{k} is the highest (i.e., most desirable) product type among those k satisfying $\sigma(k) = m$. Thus, the only possibility that the inequality holds is when

$$\{f(\boldsymbol{\pi}_k) = f(\boldsymbol{\pi}_{\bar{k}})\} = \sigma_{\boldsymbol{\pi}}^{-1}(m)$$

which implies that $\forall k \in \sigma_{\boldsymbol{\pi}}^{-1}(m)$

$$f(\boldsymbol{\pi}_k) = f(\boldsymbol{\pi}_{\bar{k}}).$$

Appendix B. Proof of Lemma 2

Suppose that $\exists i$ such that $\gamma_i < \gamma_{i+1}$. Since the incentive constraint of m_i must hold,

$$-\gamma_i m_i + a_i \geq -\gamma_i m_{i+1} + a_{i+1}$$

which is equivalent to

$$\gamma_i(m_{i+1} - m_i) \geq a_{i+1} - a_i.$$

Under the hypothesis of the proof, $\gamma_{i+1} > \gamma_i$,

$$\gamma_{i+1}(m_{i+1} - m_i) \geq a_{i+1} - a_i$$

which then implies

$$-\gamma_{i+1} m_i + a_i \geq -\gamma_{i+1} m_{i+1} + a_{i+1}. \tag{B.1}$$

If so, m_{i+1} has incentive to imitate m_i .

Appendix C. Proof of Lemma 3

Suppose $\exists i, \forall j > i$,

$$-\gamma_i m_i + a_i > -\gamma_i m_j + a_j.$$

$\exists \epsilon > 0$ so that $\forall j > i$,

$$-\gamma_i m_i + a_i > -\gamma_i(m_j - \epsilon) + a_j.$$

That is, we decrease m_j by a small amount, thus saving the marketing cost. By the construction, the incentive constraint for $j > i$ continues to hold, since we simply shift all messages to the left by an equal amount for $j > i$. Also, the incentive constraint of m_i continues to hold, as we choose $\epsilon > 0$ sufficiently small. Thus, the new profile of logos

$$(m_1, \dots, m_i, m_{i+1} - \epsilon, \dots, m_K - \epsilon)$$

is valid. Since the seller saves the marketing cost, the profit increases, which contradicts the hypothesis that m is a part of an optimal brand.

Appendix D. Proof of Lemma 4

Suppose otherwise: $\exists \ell \ i < \ell < j$ so that

$$-\gamma_i m_i + a_i = -\gamma_i m_j + a_j > -\gamma_i m_\ell + a_\ell. \quad (\text{D.1})$$

Incentive compatibility of ℓ implies that

$$-\gamma_\ell m_\ell + a_\ell \geq -\gamma_\ell m_j + a_j. \quad (\text{D.2})$$

By Lemma 2,

$$\gamma_i \leq \gamma_\ell \leq \gamma_j. \quad (\text{D.3})$$

Combining (D.1), (D.2) and (D.3), we have

$$-\gamma_j m_j + a_j > -\gamma_i m_\ell + a_\ell \geq -\gamma_\ell m_\ell + a_\ell > -\gamma_\ell m_j + a_j$$

Since $m_j > m_i \geq 0$, we have

$$\gamma_\ell > \gamma_j$$

which contradicts Lemma 2.

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