The Economics of Sovereign Debt, Bailouts and the Eurozone Crisis[*]

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Abstract

We build a model that analyzes how fiscal transfers and monetary policy are optimally deployed in a monetary union at times of crisis. Because of collateral damage, transfers in a monetary union cannot be ruled out ex-post in order to avoid a costly default. This generates risk shifting with an incentive to overborrow by fiscally fragile countries. However, a more credible no bailout commitment that reduces this incentive, may not be optimal in order to avoid immediate insolvency. Ex-post transfers are such that creditor countries get the whole surplus of avoiding a default and of debt monetization: assistance to a country that is close to default does not improve its fate. Expected debt monetization may reduce the yield because it lowers transfers required to avoid default. When transfers are not possible, the central bank of the monetary union is pushed into inefficient debt monetization.

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1 Introduction.

The markets are deluding themselves when they think at a certain point the other member states will put their hands on their wallets to save Greece. ECB Executive Board member, Júrgen Stark (January 2010):

The euro-region treaties don't foresee any help for insolvent countries, but in reality the other states would have to rescue those running into difficulty. German finance minister Peer Steinbrueck (February 2009)

No, Greece will not default. Please. In the euro area, the default does not exist. Economics Commissioner Joaquin Almunia (January 2010)

These quotes illustrate the uncertainty and the disagreements on sovereign defaults and bailouts in the Eurozone and also the distance between words and deeds. The eurozone crisis has highlighted the unique features of a potential default on government debt in a monetary union comprised of sovereign countries. Compared to the long series of defaults the world has experienced, the costs and benefits that come into play in a decision to default inside a monetary union such as the eurozone are magnified for both debtor and creditor countries. Because a monetary union facilitates financial integration, cross-border holdings of government debts (in particular by banks) inside the monetary union, and therefore potential capital losses in the event of a default, are very large. In addition, a sovereign default inside the eurozone has been interpreted by many policy makers and economists as a first step towards potential exit of the defaulter from the monetary union. Such a dramatic event would in turn impair the credibility of the monetary union as a whole, that may come to be seen as a mere fixed exchange rate regime, leading to a significant re-assessment of risks. The costs of default for the creditor countries inside the eurozone are therefore not only the direct capital losses due to non-repayment but the collateral damage in the form of contagion costs to other member countries as well as the potential disruption of trade and financial flows inside a highly integrated union. For the defaulting party, being part of a monetary union also magnifies the costs of a sovereign default. First, as for creditor countries, the financial and trade disruptions are made worse because of the high level of integration of the eurozone. As illustrated by the Greek case, a sovereign default would endanger the domestic banks which hold large amounts of domestic debt used as collateral to obtain liquidity from the European Central Bank. A potential exit from the eurozone (and even according to several analysts from the European Union) would entail very large economic and political costs with unknown geopolitical consequences. The political dimension of the creation of the euro also transforms a potential default inside the eurozone into a politically charged issue.[1]. These high costs of a default for both the creditors and the debtors and of a potential exit were supposed to be the glue that would make both default and euro exit impossible. They may also have led to excessive debt accumulation.

A distinctive feature of a monetary union comprised of sovereign countries is the way in which debt monetization affects member countries. While benefits and costs of inflation are borne by all members, their distribution is not uniform. Surprise inflation reduces the ex-post real value of debt for all members, benefiting disproportionately highly indebted countries, while the costs of inflation are more uniformly distributed. There is therefore a significant risk that the European Central Bank (ECB) may be pressured to use monetary policy to prevent a default in fiscally weak countries via debt monetization. This was well understood at the time of the creation of the euro and Article 123 of the Treaty on the Functioning of the European Union (TFEU) expressly prohibits the European Central Banks' direct purchase of member countries' public debt.[2]

A final relevant feature is Article 125 of the TFEU which prevents any form of liability of the Union for Member States debt obligations.[3] This clause has been interpreted by some as making bail-outs illegal in case of a sovereign default. For others (see De Grauwe, 2009), the no-bail-out clause only says that the Union shall not be liable for the debt of Member States but does not forbids Member States themselves from providing financial assistance to another member state. [4]. In effect Greece, Ireland, Portugal, Spain and Cyprus lost market access and had to ask for the support of the other eurozone countries in order to avoid a default or to refinance banks. This was mainly done through the creation of the EFSF and ESM that lent these countries at conditions much more favorable than market ones.

[1]The political dimension of the creation of the euro was highlighted by former president of the European Commission Jacques Delors in this declaration of 1997: "people forget too often about the political objectives of European construction. The argument in favor of the single currency should be based on the desire to live together in peace," cited in Prior-Wandersforde and Hacche (2005).

[2]Article 123 stipulates 'Overdraft facilities or any other type of credit facility with the European Central Bank or with the central banks of the Member States (hereinafter referred to as 'national central banks') in favour of Union institutions, bodies, offices or agencies, central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of Member States shall be prohibited, as shall the purchase directly from them by the European Central Bank or national central banks of debt instruments.'

[3]Article 125 stipulates 'The Union shall not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of any Member State, without prejudice to mutual financial guarantees for the joint execution of a specific project. A Member State shall not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of another Member State, without prejudice to mutual financial guarantees for the joint execution of a specific project.'

[4]Article 122 of the TFEU Treaty stipulates "..Where a Member State is in difficulties or is seriously threatened with severe difficulties caused by natural disasters or exceptional occurrences beyond its control, the Council, on a proposal from the Commission, may grant, under certain conditions, Union financial assistance to the Member State concerned.'

In this paper, we present a two-period model of strategic default that integrates these different features unique to the eurozone. The model features two eurozone countries, one fiscally strong and one fiscally fragile, and a third country that represents the rest of the world. Each region issues sovereign debt and private portfolio holdings are determined endogenously. A sovereign default inflicts direct costs on bondholders, but also indirect costs on both the defaulting country and its eurozone partner. The structure of these collateral costs, together with the realization of output and the composition of portfolios determine the conditions under which the fiscally strong country may prefer to bailout its fiscally weak partner. We show that while the bailout allows the union to achieve (ex-post) efficiency, it does so by transferring all the surplus to the fiscally strong country, leaving the debtor country no better off with a bailout (and no default) than with a default (and no bailout). We call this the 'Southern view' of the crisis: financial assistance may come, but it does not help the afflicted country. That financial assistance to a country that is close to default does not improve its fate may seem surprising. However, in absence of political integration, there is no reason creditor countries would offer more than the minimal transfer required that leaves the debtor country indifferent between default and no default. The outcome of the latest negotiations on Greek debt in July 2015 seem to vindicate our analysis.

We then show the presence of such ex-post bailouts distorts the ex-ante incentives of the fiscally weak country and generate excessive borrowing in the first period. We establish this result with a risk neutral borrower, so the incentive to borrow arises exclusively from the expected ex-post transfer. In effect, the likelihood of transfers lowers the cost of borrowing for the weak country below the risk free rate, at the expense of the fiscally strong country. The debtor country then trades off the increased riskiness of debt against the likelihood of a bailout. We call this the 'Northern view' of the crisis: the ability to obtain a bailout weakens fiscal discipline. In the context of the Eurozone crisis, this position has been articulated many times by the German Treasury. Thus our analysis reconciles the 'Northern' and 'Southern' views of the crisis as the two sides of the same coin: risk shifting by the debtor country occurs in the first period because of the transfer, even if ex-post the creditor country captures all the efficiency gains from avoiding a default.

This suggests a simple fix: if the creditor country could credibly commit to a no bail-out clause, this would eliminate ex-ante risk shifting and overborrowing. Yet we show that such commitment may not be optimal, even from the perspective of the creditor country. Instead, we find that, under certain conditions, the creditor country may prefer an imperfect commitment to the no-bailout clause. This is more likely to be the case if the debtor country has an elevated level of debt to rollover. Under a strong no-bailout clause, the debtor country may be immediately insolvent. Instead, if a future bailout is possible, the debtor country might be able to roll-over its debt in the initial period. Of course, this will lead to some risk shifting and excessive borrowing, but the scope for excessive borrowing is less significant the larger is the initial debt to roll-over. Hence the creditor country faces a meaningful trade-off between immediate insolvency and the possibility of a future default. Thus the model provides conditions under which it is optimal for creditor countries to 'gamble for resurrection' or 'kick the can down the road' in official EU parlance and remain evasive about the strength of the no bailout clause. This part of the model captures well what happened between 2000 and 2008 when spreads on sovereign debts were severely compressed.

Finally, we also characterize the impact of a debt monetization through higher inflation in the monetary union. Debt monetization differs from transfers in the sense that the distortion cost is borne by all Member States and that the inflation surprise reduces debt in all countries. As in the case of bailouts, the ECB may prefer, ex-post, to monetize the debt rather than let a default occur. Yet because inflation is more distortionary than a direct bailout, our model implies a pecking order in terms of policies : direct fiscal transfers should be used first before debt monetization. This introduces another complex interaction: if creditor countries adopt credible no-bailout policies in order to reduce ex-ante overborrowing, this may simply push the burden onto the ECB. We show that when debt monetization is the only tool available to avoid defaults, there are more output realizations with default and more output realizations with higher inflation and lower welfare ex-ante. Thus, the no-bailout policy may be counterproductive. Yet in a monetary union with multiple sovereign creditors it might be politically difficult to coordinate a bailout, leaving the European Central Bank as the 'only game in town.'

Our paper relates to several literature The theoretical literature on sovereign debt crisis has focused on the following question: why do countries repay their debt? Two different approaches have emerged (see the recent survey by Bulow and Rogoff (2015)). On the one hand, Eaton and Gersovitz (1981) focus on the reputation cost of default for countries that value access to international capital markets to smooth consumption. On the other hand, Cohen and Sachs (1986), Bulow and Rogoff (1989b), Bulow and Rogoff (1989a) and Fernandez and Rosenthal (1990) focus on the direct costs of default in terms of disruption of trade for example. Our model clearly belongs to this second family of models as we emphasize output loss for the country that defaults which comes from trade and financial disruptions but also which may come from the risk of exit of the eurozone. Empirically, Rose (2005) shows that debt renegotiation entail a decline in bilateral trade of around 8 percent a year which persists for around 15 years.

Collateral damage of a sovereign default plays an important role in our analysis of the euro crisis and the existence of efficient ex post transfers. We are not the first to make this point. A related argument can be found in Bulow and Rogoff (1989a) who show that because protracted debt renegotiation can harm third parties, the debtor country and its lenders can extract side-payments. Mengus (2014) shows that if the creditor's government has limited information on individual domestic portfolios, direct transfers to residents cannot be perfectly targeted so that it may be better off honoring the debtor's liabilities. Tirole (2014) investigates ex ante and ex post forms of solidarity. As in our paper, the impacted countries may stand by the troubled country because they want to avoid the collateral damage inflicted by the latter. A related paper is Farhi and Tirole (2016) which adds a second layer of bailout in the form of domestic bailouts of the banking system by the sovereign to analyze the 'deadly embrace' or two-way link between sovereign and financial balance sheets. The main differences with our paper are that the first paper focuses on the determination of the optimal debt contract, that both rule out strategic default as well as legacy debt and possible debt monetization. Dovis and Kirpalani (2017) also analyze how expected bailouts change the incentives of governments to borrow but concentrate on the conditions under which fiscal rules can correct these incentives in a reputation model. Uhlig (2013) analyzes the interplay between banks holdings of domestic sovereign debt, bank regulation, sovereign default risk and central bank guarantees in a monetary union. Contrary to this paper, we do not model banks explicitly but the home bias in sovereign bonds plays an important role in the incentive to default. A related paper is also Dellas and Niepelt (2016) who show that higher exposure to official lenders improves incentives to repay due to more severe sanctions but that it is also costly because it lowers the value of the sovereign's default option. Our model does not distinguish private and official lenders

Since the seminal paper of Calvo (1988), a large part of the literature on sovereign default has focused on an analysis of crisis as driven by self-fulfilling expectations (see for example Cole and Kehoe (2000)). This view has been very influential to analyze the euro crisis: this is the case for example of de Grauwe (2012), Aguiar, Amador, Farhi and Gopinath (2015) and Corsetti and Dedola (2014)) for whom the crisis can be interpreted as a rollover crisis where some governments (Spain for example) experienced a liquidity crisis. In this framework, the crisis abates once the ECB agrees to backstop the sovereign debt of eurozone members. For example, Corsetti and Dedola (2014)) analyze a model of sovereign default driven by either self-fulfilling expectations, or weak fundamentals, and analyze the mechanisms by which either conventional or unconventional monetary policy can rule out the former. We depart from this literature and do not focus on situations with potential multiple equilibria and on liquidity issues. This is not because we believe that such mechanisms have been absent but in a framework where the crisis is solely driven by self-fulfilling expectations, the bad equilibrium can be eliminated by a credible financial backstop and transfers should remain ”off the equilibrium path”. However, we will show in the next section that transfers (from the EFSF/ESM) to the periphery countries have been substantial and not only to Greece. An important difference between Aguiar et al. (2015) and our work is that they exclude the possibility of transfers and concentrate on the lack of commitment on monetary policy that makes the central bank vulnerable to the temptation to inflate away the real value of its members' nominal debt. We view the lack of commitment on transfers as an distinctive feature of a monetary union and analyze the interaction between the monetary policy and transfers in a situation of possible sovereign default.

The remaining of the paper is organized as follows. In Section 2, we present preliminary stylized facts related to our theoretical analysis. Section 3 introduces the structure of the basic model. Section 4 analyzes the incentives for defaults and bailouts and section 5 studies how these incentives shape optimal debt issuance. Section 6 then introduces monetary policy and debt monetization. Section 7 concludes.

2 Stylized facts (incomplete)

- The size of ex-post transfers

  In our model, we analyze the size and determinants of ex post transfers that are necessary to avoid a default. These transfers have taken several forms in the current eurozone crisis. The most important one for countries under program has been through the EFSF/ESM operations that provide cheaper and longer-term financing than what these countries could get on financial markets. The EFSF/ESM provided loans to Cyprus, Greece, Ireland, Portugal and Spain. A quantitative assessment of the transfer generated by this financial assistance is difficult but was attempted by the European Sability Mechanism (see European Stability Mechanism (2014) and European Stability Mechanism (2015) reports). For the year 2013, the ESM compares the effective interest rate payments on EFSF/ESM loans with the interest rate that these countries would have paid had they been able to cover their financing needs in the market. They use the average theoretical market spread of the 5 and 7-year bond of each country and match it with the EFSF/ESM maturity profile on the three months before and after each country requested support, and compare this with the equivalent EFSF/ESM funding cost. For the year 2013, the transfers range from 0.2 percentage of GDP in Spain to 4.7 percentage of GDP in Greece in 2013. For this country, the ESM also calculated the Net Present Value of the maturity extensions, interest rate reductions and deferrals over the entire debt servicing profile from a net present value (NPV) perspective.[5] They therefore discount the difference between the future cash flows of the loans benefitting from lower financing costs and debt relief measures and the cash flows of the same loans had they not benefited from the relief measures. The implicit transfer from the various relief measures leads to an NPV equivalent transfer of 49 percent of Greece's 2013 GDP. The largest part of this transfer (27 percent of GDP) is due to the low EFSF rates compared to market rates. Official lending at risk-free rate does not constitute a transfer if official lending is indeed risk free. For this reason, estimates of the transfer from ESM official documents may be exaggerated. To do this we need to estimate the difference between the official loan rate on the one hand and the risk free rate and the default risk borne by the ESM. This is work in progress that will follow the methodology of Zettelmeyer and Priyadarshani (2005).

- Notes on collateral rules of the ECB (in relation with liquidity services of sovereign bonds)

  In our model, private portfolios of sovereign bonds are determined by their relative liquidity services. In turn, these liquidity services depend crucially on their use as collateral to obtain liquidity at the ECB. Standard credit operations (or repos) of the ECB indeed involve the provision of liquidity against eligible collateral for a pre-specified period of time. This collateral requirement is also present for less standard open market operations such as Longer Term Refinancing Operations (LTROs). Emergency Liquidity Assistance (ELA) and intraday credit in the payment system TARGET2 are also subject to collateral requirements. Financial assets the ECB considers eligible collateral are only those issued by eurozone based institutions and are the same for all borrowers inside the eurozone. Government bonds have the lowest haircut category. Given that the market price of bonds varies with their perceived riskiness, the collateral and liquidity value is also heterogeneous In addition to this market based differentiation, the ECB exclude some government debt as collateral based on its rating. For example, on February 5, 2015, the ECB decided that debt instruments issued or fully guaranteed by the Greek government would cease to be eligible as collateral. Note also that foreign currency-denominated collateral is accepted but with higher valuation mark-downs than those denominated in euros.

[5] The initial maturity of the loan of May 2010 (€52.9 billion) was 2026 (with a grace period up to 2019 and gradual repayment during 2020-26) and the initial interest rate was linked to the 3-month Euribor with a 300 basis point spread during the first 3 years and 400 basis points afterward. In 2011, the spread was cut to 150 basis points (retroactively) and on 27 November 2012 the spread was cut to 50 basis points. The maturity was extended by 15 years to 2041 with gradual principal amortization between 2020 and 2041. See Bruegel, 2015: http://www.bruegel.org/nc/blog/detail/article/1533-how-to-reduce-the-greek-debt-burden/

3 Model

3.1 Assumptions

The baseline model is similar to Calvo (1988). Consider a world with 2 periods, $t = 0, 1$ and three countries. We label the countries g, i and u. g and i belong to a monetary union, unlike u. g is a fiscally strong country in the sense that its government debt is risk-free. Instead, i is fiscally fragile: the government may be unable or unwilling to repay its debts either in period 0 or period 1. Countries can have different sizes, denoted ω^j with $\sum_j \omega^j = 1$.

Each country/region j receives an exogenous endowment in period t denoted y_t^j. The only source of uncertainty in the model is the realization of the endowment in i in period 1, y_1^i. We assume that $y_1 = \bar{y}_1^i \epsilon_1$ where $E[\epsilon_1] = 1$, so $\bar{y}_1^i$ represents expected total output in i. Lastly, we assume that ϵ_1^i is distributed according to some cdf $G(\epsilon)$ and pdf $g(\epsilon)$, with a bounded support $[\epsilon_{\min}, \epsilon_{\max}]$, with $0 < \epsilon_{\min} < \epsilon_{\max} < \infty$.

In each country j, a representative agent preferences has preferences defined over aggregate consumption c_t^j and government bond-holdings $\{b_t^{k,j}\}_k$ as follows:

$$U^j = c_0^j + \beta E[c_1^i] + \omega^j \lambda^s \ln b_1^{s,j} + \omega^j \lambda^{i,j} \ln b_1^{i,j}$$

The first part of these preferences is straightforward: households are risk neutral over consumption sequences. In addition, we assume that government bonds provide 'money-like' liquidity services that are valued by households (cf. evidence for US Treasuries from Krishnamurthy and Vissing-Jorgensen). We model this in a very simple way, by including bond-holdings in the utility function. Crucially, we consider that bonds from different countries provide different levels of liquidity services, depending on how 'safe' or money-like these bonds are perceived to be for different classes of investors. One potential interpretation is that different government bonds can be used as collateral in various financial transactions and are therefore valued by market participants beyond their financial yield. We don't propose here a theory of what makes some government bonds safe and others not, we simply take as given that:

- u and g bonds are perceived as equally safe and liquid. It follows that they are perfect substitutes and we can consider the total demand for *safe* assets by households in country j, denoted $b_1^{s,j} \equiv b_1^{g,j} + b_1^{u,j}$. Given our assumptions, if aggregate safe bond holdings increase by 1%, aggregate utility in country j increases by $\omega^j \lambda^s / 100$.

- We denote the demand for i-bonds from investors in country j by $b_1^{i,j}$. i-bonds may offer different degrees of liquidity to u investors, g investors and i investors. A reasonable assumption is that i-bonds provide higher liquidity services to i investors, then g investors, then u investors. That is, we assume that $\lambda^{i,i} > \lambda^{i,g} > \lambda^{i,u}$.

It seems quite natural that i investors perceive i debt as more liquid/safe than other investors. For instance, one could argue that i banks optimally discount the states of the world where their own government defaults because they themselves would have to default. The next section provides a fleshed out model of this risk-shifting. The assumption that g bond holders get more liquidity from i debt holdings than u investors could reflect the fact that g banks can obtain liquidity against i bonds from the common monetary authority at favorable terms. In other words, we view the assumption that $\lambda^{i,g} < \lambda^{i,u}$ as a consequence of the monetary union between i and g.[6] We will consider later how changes in perceptions of the liquidity services provided by i bonds (circa 2008-2009) affects equilibrium debt and bailout dynamics.

In order to simplify a number of expressions, we will often consider the *bondless limit* that obtains when $\lambda^s \to 0$ and $\lambda^{i,j} \to 0$, while keeping the ratios $\omega^j \lambda^{i,j} / \sum_k \omega^k \lambda^{i,k}$ constant.[7] In this limit, as we will see, the bond portfolios remain well defined, but the liquidity services become vanishingly small, so the choice of the level of debt does not directly affect utility.

Countries i and g differ in their fiscal strength. We assume that g is fiscally sound, so that its debt is always safe. Instead, i is fiscally fragile: it needs to refinance some external debt in period $t = 0$, and can decide to default in period $t = 1$. Should a default occurs, we follow the literature and assume that i suffers an output loss equal to Φy_1^i with $0 \leq \Phi \leq 1$. This output loss captures the disruption to the domestic economy from a default. There are many dimensions to the economic cost of a default. In particular, for i, a default may force the country to exit the monetary union, potentially raising default costs substantially. One way to capture this dimension is to assume that $\Phi = \Phi_d + \pi_m \Phi_m$ where Φ_d is the share of lost output if the country defaults but remains in the currency union, π_m is the probability of exit, and Φ_m is the additional share of lost output from an exit. While Φ_d and π_m might be low, Φ_m is likely to be very large.[8] We assume

[6] Note that it is not necessarily the case that a monetary union implies that i debt is more valuable to g investors. In practice, though, this seems to have been the case. See Buiter et al.

[7] The terminology here is by analogy with Woodford's cashless limit where the direct utility gains from money holdings become vanishingly small.

[8] In this paper, we take the exit probability as exogenous.

that the default cost is proportional to output, so that, everything else equal, a default is less likely when the economy is doing well.

In case of a default, we assume that creditors can collectively recover an amount ρy_1^i where $0 \leq \rho < 1$. This assumption captures the fact that i's decision not to repay its debt does not generally result in a full expropriation of outstanding creditor claims. Importantly, the amount recovered is proportional to output, and not to the outstanding debt, capturing the idea that i can only commit to repay a fraction of its output. An alternative interpretation is that ρy_1^i represents the collateral value of the outstanding debt. The recovery payment is distributed *pari passu* among all creditors , domestic and foreign, in proportion to their initial debt holdings. We assume that $\Phi + \rho < 1$ so that the country always has enough resources for the recovery amount in case of default.[9]

In addition, we assume that g also suffers a collateral cost from a default in i, equal to κy_1^g, with $0 \leq \kappa \leq 1$, while u does not suffer any collateral damage. There are two ways to interpret this assumption. First, it captures the idea that the economies of countries g and i are deeply intertwined since they share a currency, so that a default in i would disrupt economic activity in g as well, to a greater extent than u. In addition, we can imagine that the contagion cost would be much higher if, as a consequence of its default, i is forced to exit the common currency. By analogy with the cost of default for i, we could write $\kappa = \kappa_d + \pi_m \kappa_m$, with a low κ_d, a low π_m and a high κ_m. Countries outside the monetary union would not face the higher levels of economic disruption caused by a collapse of the monetary union.

As in Tirole (2015), the contagion cost creates a *soft budget constraint* for country i. Our interpretation is that this 'collateral damage' was at the heart of the discussions regarding bailout decisions in the Eurozone. For instance, the decision to bailout Greece in 2010 and avoid a debt restructuring was directly influenced by the perception that a Greek debt restructuring could have created propagated the fiscal crisis to other economies in the Eurozone. For instance, it was argued that the economies of Spain, Italy, Portugal or Ireland could have suffered an adverse market reaction. It was also argued that a Greek restructuring could hurt France or Germany through the exposure of their banking system to Greek sovereign risk. Implicitly, a common perception at the time was that bailing out Greece -so that the Greek government could in turn repay French and

[9] This condition also ensures that i's consumption is always positive.

German banks– was preferable to a default event where German and French governments would have needed to directly recapitalize the losses of their domestic banks on their Greek portfolio. The term κy_1^g captures the additional cost of a default for g above and beyond the direct portfolio exposure $b_1^{i,g}$. Implicitly, we are assuming that this collateral damage is not borne by u investors. This could reflect the fact that u and i are not strongly integrated economically. In that sense, κ captures the economic ties between members of a monetary union that can be disrupted by a sovereign default.

Finally, we allow for ex-ante and ex-post voluntary transfers τ_t from g to i. Crucially, we consider an environment where g can make ex-post transfers to i conditional on the realization of output, and also on i's default decision. Because these transfers are voluntary, they must satisfy: $\tau_t \geq 0$. Since there is no reason for g to make a transfer to i in case of a default, the optimal transfer in that case is zero.

3.2 Budget Constraints

3.2.1 Households

The budget constraints of the households of the different regions are as follows. First consider i's household in period $t = 0$:

$$c_0^i + b_1^{i,i}/R^i + b_1^{s,i}/R^* \quad = \quad y_0^i - T_0^i + b_0^{i,i} + b_0^{s,i}$$

while in period $t = 1$:

$$\begin{cases} c_1^i = y_1^i - T_1^i + b_1^{i,i} + b_1^{s,i} & \text{if } i \text{ repays} \\ c_1^i = y_1^i(1-\Phi) - T_1^i + \rho y_1^i \dfrac{b_1^{i,i}}{b_1^i} + b_1^{s,i} & \text{if } i \text{ defaults} \end{cases}$$

In period $t = 0$, i's representative household consumes, invests in domestic and safe debt. Its revenues consist of after tax income $y_0^i - T_0^i$ where T_0^i denotes lump-sum taxes levied by i's government. R^i denotes the yield on the Italian debt, while R^* is the yield on safe debt. In period $t = 1$, the household consumes after tax income, and liquidates its bond portfolio. In case of default, it suffers the direct cost Φy_1^i and recovers only $\rho y_1^i / b_1^i$ per unit of domestic bond purchased. Note that period 1 taxes T_1^i are state contingent and can depend on the realization of output and the decision to default.

Now consider g's household (using similar notation), in period $t = 0$:

$$c_0^g + b_1^{i,g}/R^i + b_1^{s,g}/R^* \quad = \quad y_0^g - T_0^g + b_0^{i,g} + b_0^{s,g}$$

and in period $t = 1$:

$$\begin{cases} c_1^g = y_1^g - T_1^g + b_1^{i,g} + b_1^{s,g} & \text{if } i \text{ repays} \\ c_1^g = y_1^g(1-\kappa) - T_1^g + \rho y_1^i \dfrac{b_1^{i,g}}{b_1^i} + b_1^{s,g} & \text{if } i \text{ defaults} \end{cases}$$

As in the case of i, taxes raised in $t = 1$, T_1^g, are state contingent.

A similar set of budget constraints hold for investors from the rest of the world. We omit them from simplicity.

3.2.2 Governments

We now write the budget constraints of the governments in i and g.[10]

The budget constraints for i's government in periods $t = 0$ and $t = 1$ are respectively:

$$T_0^i + b_1^i/R^i + \tau_0 \quad = \quad b_0^i$$

and

$$\begin{cases} T_1^i + \tau_1 = b_1^i & \text{if } i \text{ repays} \\ T_1^i = \rho y_1^i & \text{if } i \text{ defaults} \end{cases}$$

In these expressions, τ_t is the direct unilateral transfer from g's government to i's government in period t. As discussed previously, ex-post transfers τ_1 can be made conditional on the decision to default by i. In principle, g's government can make a transfer to i either *ex-ante*, so as to reduce the debt overhang that i is likely to face, or *ex-post* once i is facing the possibility of default.

[10]There is no role for the government in the rest of the world so we ignore it. One can check that under the assumption that $\alpha_1^{i,g} \geq \alpha_1^{i,u}$ and $\kappa \geq 0$, it is never optimal for u to make a transfer. The proof consists in checking that at $\underline{\epsilon}$ u does not want to step in and make a transfer.

The budget constraints for g's government are:

$$T_0^g + b_1^g/R^* \quad = \quad b_0^g + \tau_0$$

and

$$\begin{cases} T_1^g = b_1^g + \tau_1 & \text{if } i \text{ repays} \\ T_1^g = b_1^g & \text{if } i \text{ defaults} \end{cases}$$

3.3 Market Clearing

The markets for safe bonds and i-bonds clear. The following equilibrium conditions obtain:

$$\sum_j b_1^{i,j} = b_1^i \quad ; \quad \sum_j b_1^{s,j} = b_1^s \tag{1}$$

3.4 Optimal Portfolios without Discrimination

Denote $\mathcal{P}^j \leq 1$ the expected payment per unit of i's sovereign debt for j's household, given the optimal choice of default and recovery rate in period $t = 1$. If i cannot discriminate between different types of bondholders when defaulting, this expected payoff is the same for all investors: $\mathcal{P}^j = \mathcal{P}$. It follows that the first-order conditions for the choice of debt by households are:

$$\begin{aligned} \frac{1}{R^i} - \beta\mathcal{P} \quad &= \quad \frac{\omega^i \lambda^{i,i}}{b_1^{i,i}} = \frac{\omega^g \lambda^{i,g}}{b_1^{i,g}} = \frac{\omega^u \lambda^{i,u}}{b_1^{i,u}} \\ \frac{1}{R^*} - \beta \quad &= \quad \frac{\omega^i \lambda^s}{b_1^{s,i}} = \frac{\omega^g \lambda^s}{b_1^{s,g}} = \frac{\omega^u \lambda^s}{b_1^{s,u}} \end{aligned}$$

Denote $\bar{\lambda}^i \equiv \sum_k \omega^k \lambda^{i,k}$. Using the bond market clearing condition, the aggregate share $\alpha^{i,j}$ of i's debt held by country j satisfies::

$$\alpha^{i,j} \equiv \frac{b_1^{i,j}}{b_1^i} = \frac{\omega^j \lambda^{i,j}}{\bar{\lambda}^i} \tag{2}$$

Similarly derivations for safe bonds yield:

$$\alpha^{s,j} = \omega^j \tag{3}$$

In the absence of selective default, the model implies that equilibrium portfolio shares are proportional to relative liquidity benefits of i debt across investor classes. To understand the intuition for this result, observe that all investors expect the same payment per unit of debt, $\beta\mathcal{P}$, and pay the same price, $1/R^i$. Hence, difference in equilibrium portfolios must arise entirely from differences in the relative liquidity services provided by the bonds, i.e. $\omega^j\lambda^{i,j}/\bar{\lambda}^i$. These shares don't depend on the riskiness of i's debt and remain well defined in the *bondless limit*.

For safe assets, liquidity services are the same, up to size differences. It follows that equilibrium portfolios only reflect size differences with larger countries holding more safe assets.[11]

Finally, we can rewrite the equilibrium conditions as:

$$\frac{1}{R^*} = \beta + \frac{\lambda^s}{b_1^s} \quad ; \quad \frac{1}{R^i} = \beta\mathcal{P} + \frac{\bar{\lambda}^i}{b_1^i} \tag{4}$$

The first expression indicates that the yield on safe debt can be lower than the inverse of the discount rate $1/\beta$ because of a liquidity premium that is a function of λ^s/b_1^s. As the supply of safe debt increases, this liquidity premium decreases, as documented empirically by Krishnamurthy and Vissing-Jorgensen (2012). Similarly, the yield on i's debt decreases with the liquidity services equal to $\bar{\lambda}^i/b_1^i$, but increases as the expected payoff per unit of i's debt $\mathcal{P}$ decreases.

In the *bondless limit* these expressions simplify and we obtain:

$$R^* = \beta^{-1} \quad ; \quad R^i = (\beta\mathcal{P})^{-1}$$

In that limit case, portfolio holdings remain determined by (2) and (3) but the liquidity premium on safe debt disappears and the premium on i's debt reflects entirely default risk ($\mathcal{P} \leq 1$).

4 Defaults and Bailouts in $t = 1$

We solve the model by backward induction, starting at $t = 1$. In the final period, i's government can unilaterally decide to repay its debt or default after observing the realization of the income

[11] Since equilibrium portfolios are constant regardless of the riskiness of the bonds, our benchmark portfolio allocation cannot replicate the large shifts in cross-border bond holdings observed first after the introduction of the Euro (globalization), then following the sovereign debt crisis (re-nationalization). In the benchmark version of the model, this re-nationalization can only occur if the liquidity services provided by i's debt to i's banks ($\lambda^{i,i}$) increases, or if the liquidity services provided by i's debt to foreign banks ($\lambda^{i,g}$ or $\lambda^{i,u}$) decrease. A possible extension, left for future work, would allow for either discrimination in default or differential bailout policies, so that $\mathcal{P}^i \neq \mathcal{P}^j$.

shock ϵ_1^i, taking as given the transfer τ_1 it would receive from g's government if it decides to repay. Consolidating the budget constraint of i's government and households, a government maximizing the welfare of domestic agents will decide to repay its debts when:

$$y_1^i \left[\Phi + \rho(1 - \alpha_1^{i,i})\right] + \tau_1 \geq b_1^i(1 - \alpha_1^{i,i}) \tag{5}$$

This equation has a natural interpretation. The left hand side captures the cost of default for i's government. This cost has three components. First there is the direct disruption to the domestic economy captured by Φy_1^i. Second there is the fact that, even if default occurs, the country will have to repay a fraction ρ of output to foreign investors, holding a fraction $1 - \alpha_1^{i,i}$ of marketable debt. Lastly there is the foregone transfer τ_1. Against these costs, the benefit of default consists in not repaying the outstanding debt to foreign investors, both insider the monetary union and in the rest of the world: $b_1^i(1 - \alpha_1^{i,i})$.Intuitively, default is more likely if the direct cost of default is low, the recovery rate is low, transfers are low, and a larger fraction of the public debt is held abroad.

Condition (5) puts a floor under the promised transfer necessary to avoid a default:

$$\tau_1 \geq b_1^i(1 - \alpha_1^{i,i}) - y_1^i \left[\Phi + \rho(1 - \alpha_1^{i,i})\right] \equiv \underline{\tau}_1$$

Since transfers are voluntary, there is a minimum realization of the shock ϵ_1^i such that repayment is optimal, even in the absence of transfer:

$$\epsilon_1^i \geq \frac{(1 - \alpha_1^{i,i}) b_1^i / \bar{y}_1^i}{\Phi + \rho(1 - \alpha_1^{i,i})} \equiv \bar{\epsilon} \tag{6}$$

Intuitively, $\bar{\epsilon}$ increases with the ratio of debt held by foreigners to expected output, $\left(1 - \alpha_1^{i,i}\right) b_1^i / \bar{y}_1^i$, and decreases with the cost of default Φ or the recovery rate ρ. A larger fraction of i's public debt held by domestic investors makes default less appealing to i's government since a default becomes a zero sum transfer from domestic bondholders and domestic taxpayers. In the limit where i's debt is entirely held domestically, ($\alpha_1^{i,i} = 1$), there is never any incentive to default regardless of the realization of output: $\bar{\epsilon} = 0$.

This result suggests one important implication of the re-nationalization of bond markets: everything else equal, it decreases the ex-post likelihood of default. Hence in our model there is no *deadly embrace* between sovereigns and bondholders. In Farhi and Tirole (2016), the deadly embrace arises from the distorted incentives of domestic banks to hold debt issued by their own sovereign, creating an enhanced contagion channel from banks to sovereigns and vice versa, a channel that is absent in this paper.

Let's now consider the choice of optimal ex-post transfers by g. When $\epsilon_1^i < \bar{\epsilon}$, a transfer becomes necessary to avoid default. Given our assumptions, g makes the minimum transfer required to avoid a default: $\tau_1 = \underline{\tau}_1$.[12] Substituting $\underline{\tau}_1$ into g's consolidated budget constraint, we find that g's government will prefer to make a transfer as long as:

$$\Phi y_1^i + \kappa y_1^g \geq \alpha_1^{i,u} \left(b_1^i - \rho y_1^i \right) \tag{7}$$

The left hand side of (7) measures the overall loss from default for the monetary union. It consists of the sum of the direct cost Φy_1^i for i and the contagion cost κy_1^g for g. The right hand side measures the overall benefit of default: from the point of view of the monetary union, the benefits of default consists in not repaying the rest of the world and economizing $\alpha_1^{i,u}(b_1^i - \rho y_1^i)$.

Equation (7) makes clear that g's transfers are *ex-post efficient* from the joint perspective of g and i. The difference between the left and right hand side of equation (7) represents the surplus from avoiding a default. Under our assumption that g makes a take-it-or-leave-it offer to i, g is able to appropriate the entirety of the ex-post surplus from avoiding default.[13]

We can solve equation (7) for the minimum realization of ϵ_1^i such that a transfer (and no-default) is optimal. This defines a threshold $\underline{\epsilon}$ below which default is jointly optimal:

$$\epsilon_1^i \leq \frac{\alpha_1^{i,u} b_1^i / \bar{y}_1^i - \kappa y_1^g / \bar{y}_1^i}{\Phi + \rho \alpha_1^{i,u}} \equiv \underline{\epsilon} \tag{8}$$

Based on the discussion above, we make the following observations about equation (8):

[12] We assume that if i is indifferent between default and no-default, it chooses not to default.

[13] One could imagine an alternative arrangement where i and g bargain over the surplus from avoiding default. Depending on its bargaining weight, i may be able to extract a share of the surplus, reducing the gain to g. In that case, ex-post efficiency would still obtain, but i's utility would increase relative to default. If output is observable, we believe that it is reasonable to assume that g has the strongest bargaining power. Alternatively, one could consider what happens if ϵ_1^i is not perfectly observable. In that case, i would like to claim a low realization of output in order to claim a higher bailout. It would then be in the interest of g to verify the realization of the state whenever i would request a bailout. In practice, this is often what happens (cf. Greece and the monitors from the 'Troika').

- First, it can be immediately checked that $\underline{\epsilon} \leq \bar{\epsilon}$ as long as $\alpha^{i,g} \geq 0$ or $\kappa \geq 0$. In other words, as long as g is exposed directly (through its portfolio) or indirectly (through contagion) to i's default, it has an incentive to offer ex-post transfers.

- It follows immediately that an ex-ante no-transfer commitment - such as a no-bailout clause- is not *renegotiation proof* and therefore will be difficult to enforce.

- It is also immediate from (7) that g will always be willing to bailout i, regardless of its debt level, if $\alpha_1^{i,u} = 0$, that is if all of i's debt is held within the monetary union, as long as i's default is costly, either for i or g.[14]

- The threat of collateral and direct damage to g from i's default relaxes ex-post i's budget constraint, a point emphasized also by Tirole (2012).

- Lastly, because g offers the minimum transfer $\underline{\tau}_1$ to avoid a default, it becomes a residual claimant and captures the entire surplus from avoiding default. When $\underline{\epsilon} \leq \epsilon_1^i < \bar{\epsilon}$, i receives a positive transfer but achieves the same utility as under default. In these states of the world, i's consumption in period $t = 1$ is given by

  $$c_1^i = y_1^i(1 - (\Phi + \rho(1 - \alpha_1^{i,i}))) + b_1^{s,i}$$

  This captures an important effect in our model, which we call the *Southern view* of the crisis: the ex-post support that i receives from g does not make i better off. It avoids the deadweight losses imposed by a default, but g captures *all* the corresponding efficiency gains.

The previous discussion fully characterizes the *optimal* ex-post transfer τ_1, default decisions and consumption patterns in both countries and is summarized in Figure 1.

We already noted that the transfer τ_1 is ex-post optimal from the point of view of g. However, it is important to recognize that it may be difficult for g to implement such transfers. For instance, the institutional framework may prevent direct transfers from one country to another. It may also make be difficult for an institution like the Central Bank to implement such a transfer on behalf

[14] Of course, in anticipation of the next section, in that case i would want to issue so much debt in period $t = 0$ that this would eventually threaten g's fiscal capacity. In what follows we always assume that $\alpha_1^{i,u} > 0$ and that g has sufficient fiscal capacity to make the necessary transfers.

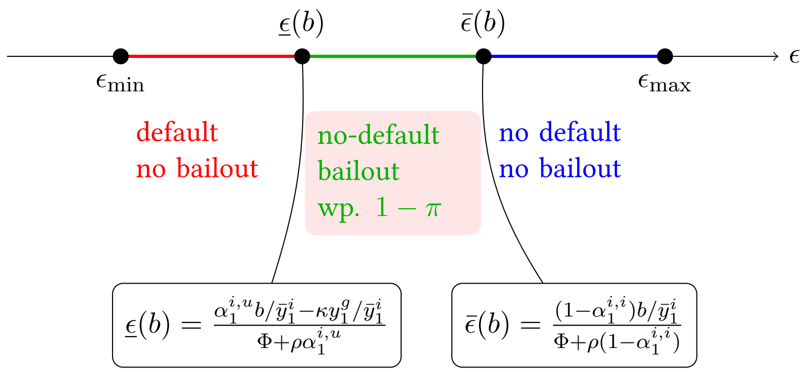


Figure 1: Optimal Ex-Post Bailout Policy.

of g (we explore this possibility in more details in the next section).

These 'no-bailout' clauses have repeatedly been invoked and played an important role in shaping the response to the Eurozone crisis. For instance, the legality of proposed bailout programs has often been questioned and referred to the German constitutional court (the Karlsruhe court), or the European Court of Justice. From pour point of view, the important observation is that the political process contains a certain amount of uncertainty, since it is not known ex-ante how the legal authorities will rule on these matters.

We also note that, even though a bailout from g to i is renegotiation proof in our static model, it may not be optimal from a dynamic perspective. Indeed we will see that in some cases g may prefer ex-ante to commit not to bailout i ex-post.

We capture both the political uncertainty and the attempt to achieve some form of ex-ante commitment with an exogenous parameter π, denoting the probability that ex-post transfers will *not* be implemented, even when they are ex-post in the best interest of both parties. By varying π, we nest the polar cases of full commitment ($\pi = 1$) and full discretion ($\pi = 0$).

The following table summarizes the transfers in period $t = 1$ depending on the realization of the shock ϵ_1.

	ex-post transfer	default	ex-post transfer τ_1
$\epsilon_1 < \underline{\epsilon}$		yes	0
$\underline{\epsilon} \leq \epsilon_1 < \bar{\epsilon}$	ruled out	yes	0
$\underline{\epsilon} \leq \epsilon_1 < \bar{\epsilon}$	authorized	no	$b_1^i(1-\alpha_1^{i,i}) - y_1^i\left[\Phi + \rho(1-\alpha_1^{i,i})\right]$
$\bar{\epsilon} \leq \epsilon_1$		no	0

Observe that the optimal transfer is discontinuous at $\epsilon_1^i = \underline{\epsilon}$. The reason is that a large transfer to i is necessary to avoid a default at that point. A default occurs either if $\epsilon < \underline{\epsilon}$ or when $\underline{\epsilon} < \epsilon_1^i \leq \bar{\epsilon}$ and ex-post transfers are ruled to be illegal. The ex-ante probability of default is then given by:

$$\pi_d = G(\underline{\epsilon}) + \pi(G(\bar{\epsilon}) - G(\underline{\epsilon})) \tag{9}$$

5 Debt Rollover Problem at $t = 0$

5.1 The Debt Laffer Curve.

We now turn to the choice of optimal debt issuance at period $t = 0$, taking the ex-ante transfer τ_0 and initial debt level b_0 as given. If debt with notional value b_1^i has been issued at $t = 0$, then the expected repayment $\mathcal{P}b_1^i$ is given by:

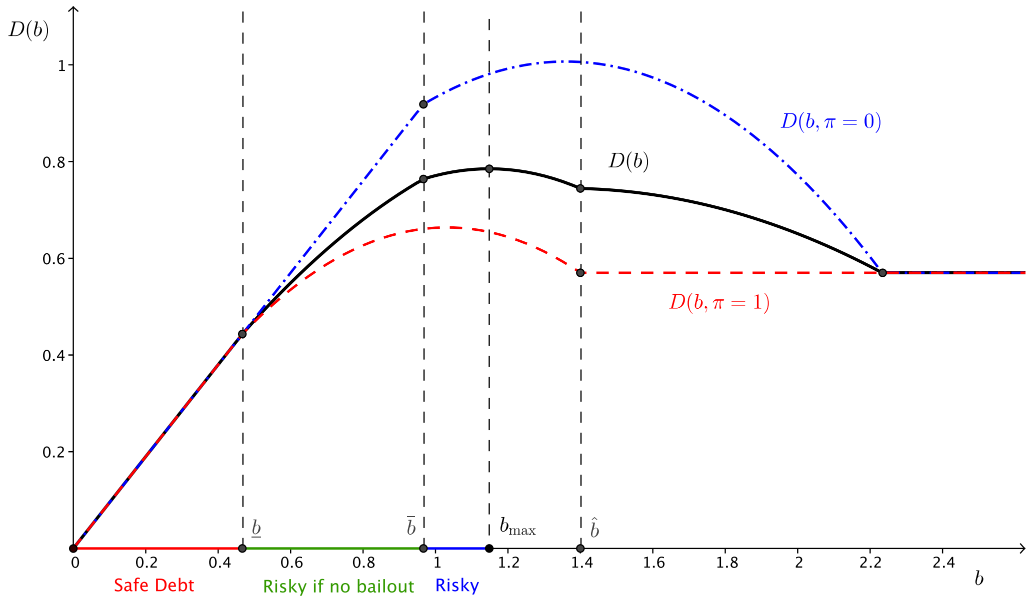
$$\mathcal{P}b_1^i = (1-\pi_d)b_1^i + \rho\bar{y}_1^i\left(\int_{\epsilon_{\min}}^{\underline{\epsilon}} \epsilon dG(\epsilon) + \pi\int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon dG(\epsilon)\right)$$

This expression has three terms. First, if country i does not default (with probability $1-\pi_d$), it repays at face value. If default occurs, investors recover instead ρy_1^i. This can happen either because default is ex-post optimal (when $\epsilon_1^i < \underline{\epsilon}$) or when a transfer is needed but fails to materialize (with probability π when $\underline{\epsilon} \leq \epsilon_1^i < \bar{\epsilon}$).

Substituting this expression into condition (4), we obtain an expression for the fiscal revenues $D(b_1^i) \equiv b_1^i/R^i$ raised by the government of country i in period $t = 0$:

$$\begin{aligned} D(b_1^i) &= \beta\mathcal{P}b_1^i + \overline{\lambda}^i \\ &= \beta b_1^i(1-\pi_d) + \beta\rho\bar{y}_1^i\left(\int_{\epsilon_{\min}}^{\underline{\epsilon}} \epsilon dG(\epsilon) + \pi\int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon dG(\epsilon)\right) + \overline{\lambda}^i \end{aligned} \tag{10}$$

This *Laffer curve* plays an important role in the analysis of the optimal choice of debt. We



$D(b)$ for $\pi = 0$ (max bailout), $\pi = 0.5$ and $\pi = 1$ (no bailout).
[Uniform distribution with $\rho = 0.6$, $\Phi = 0.2$, $\kappa = 0.05$, $\epsilon_{\min} = 0.5$, $\beta = 0.95$, $\bar{y}_1^i = 1$, $y_1^g = 2$, $\alpha_1^{i,i} = 0.4$, $\alpha_1^{i,g} = \alpha_1^{i,u} = 0.3$. $\underline{b} = 0.47$, $\bar{b} = 0.97$ and $\hat{b} = 1.4$]

Figure 2: The Debt-Laffer Curve

report a full characterization in appendix A. Heuristically, we have the following cases, also illustrated on Figure 2:[15]

- When $b_1^i \leq \underline{b} \equiv y_{\min}^i \left(\Phi/(1-\alpha_1^{i,i}) + \rho\right)$). In that case, the debt level is so low that i repays in full without transfers, for all realizations of output. The debt is safe, there is no default risk and no transfers.

- When $\underline{b} < b_1^i \leq \bar{b} \equiv ((\Phi + \rho\alpha_1^{i,u})y_{\min}^i + \kappa y_1^g)/\alpha_1^{i,u}$. In that case, the level of debt is sufficiently low that it is optimal for g to bailout i when output is too low. Default might occur if this bailout is not allowed with probability $\pi > 0$. In that region, the Laffer curve with discretionary bailout ($\pi = 0$, in blue on the figure) lies strictly above the Laffer curve under no bailout ($\pi = 1$, in red on the figure): this is a consequence of the soft budget constraint that is induced by the transfers. Under the assumptions specified in appendix A, the Laffer curve is increasing (at a decreasing rate) over that range.

[15] This figure is drawn under the assumption that the shocks are uniformly distributed.

- When $\bar{b} < b_1^i \leq \hat{b} \equiv y_{\max}^i(\Phi/(1-\alpha^{i,i}) + \rho)$, it becomes optimal for g to let i default when the realizations of output are sufficiently low. This increases default risk and the yield on i's debt. Under the assumptions specified in Appendix A, the Laffer curve is convex in this region and reaches its peak at $b = b_{\max}$ strictly below $\hat{b}$.

- For $\hat{b} < b \leq \tilde{b}$, we enter a region where default would occur with certainty in the absence of transfers. With transfers, it is possible for default to be avoided, if output is sufficiently high. Under the assumptions in the appendix, the Laffer curve slope down over that region.

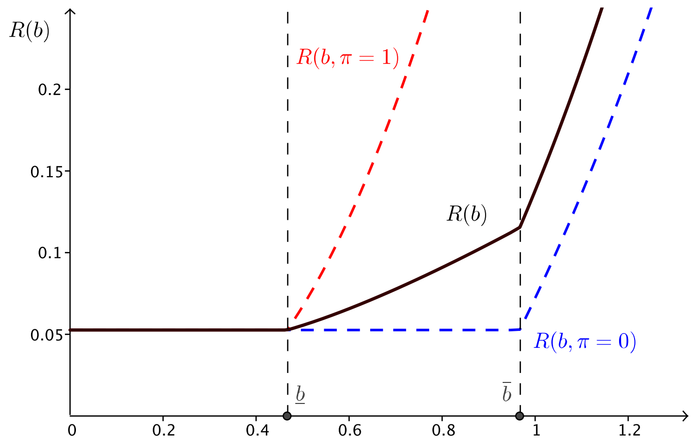
- Finally, for $b > \tilde{b} \equiv ((\Phi + \rho\alpha_1^{i,u})y_{\max}^i + \kappa y_1^g)/\alpha_1^{i,u}$, i always defaults regardless of the realization of output. There are no transfers and investors expected repayment is amount $\rho\bar{y}_1^i$.[16]

Appendix A provides a full characterization of the cut-offs and a set of necessary conditions to ensure that the Laffer curve is convex over the relevant range: $[0, \hat{b})$. The fact that the country can choose its repayment level b_1^i implies that it will never choose to locate itself on the 'wrong side' of the Laffer curve, i.e. it will only consider levels of debt level such that $b \leq b_{\max} < \hat{b}$. This eliminates Calvo (1988)-like rollover crises and multiple equilibria.

Over the relevant range, the Laffer curve is convex, continuous and exhibits two non-differentiable points, at $b = \underline{b}$ and $b = \bar{b}$.

Figure 3 reports the contractual yield R^i on i's debt and shows how it varies with the probability of enforcement of no-bailout clause π. The interesting range is for $\underline{b} < b \leq \bar{b}$ where the yield remains equal to $1/\beta$ if the bailouts are allowed, but increases very rapidly –together with the ex-post probability of default– when bailouts are prohibited. This figure illustrates one possible channel for the rapid surge in yields when the crisis erupted: the perception that implicit bailout guarantees were removed (i.e. a switch from $\pi = 0$ to $\pi = 1$). Similarly, one can interpret the decline in yields following President Draghi's famous pronouncement that the ECB would do 'Whatever it takes' to preserve the Euro, as a sign that bailout guarantees would be reinstated, i.e. a switch from $\pi = 1$ to $\pi = 0$.

[16] There is also another case where $\tilde{b} < \hat{b}$. We view this case as unintuitive: it corresponds to a situation where it would always be ex-post efficient to bail out i. We assume parameter configurations that rule out this case.



Yields for $\pi = 0$, $\pi = 1$ and $\pi = 0.2$.
[Uniform distribution with $\rho = 0.6$, $\Phi = 0.2$, $\kappa = 0.05$, $\epsilon_{\min} = 0.5$, $\beta = 0.95$, $\bar{y}_1^i = 1$, $y_1^g = 2$, $\alpha_1^{i,i} = 0.4$, $\alpha_1^{i,g} = \alpha_1^{i,u} = 0.3$. $\underline{b} = 0.47$ and $\bar{b} = 0.97$]

Figure 3: Yields

5.2 Optimal Debt Issuance

We now consider the optimal choice of debt b_1^i in the *bondless limit* where bond holdings provide infinitesimal liquidity services. This allows us to ignore the direct impact of the debt level on the utility of the agents via liquidity services. Recall that bond portfolios remain pinned down and invariant to the level of debt so we can take the portfolio shares $\alpha_1^{j,k}$ as given.

The consolidated budget constraint for i in period 0 is:

$$c_0^i + \alpha_1^{i,i} b_1^i / R^i + \alpha_1^{s,i} b_1^s / R^* = (y_0^i + \tau_0 + b_0^{i,i} - b_0^i + b_0^{s,i}) + b_1^i / R^i$$

And the consolidated budget constraint for period 1 is:

$$\begin{cases} c_1^i = y_1^i - b_1^i(1 - \alpha_1^{i,i}) + \alpha_1^{s,i} b_1^s & \text{if } \epsilon_1^i \geq \bar{\epsilon} \text{ (} i \text{ repays, no transfer)} \\ c_1^i = y_1^i(1 - \Phi) - \rho y_1^i (1 - \alpha_1^{i,i}) + \alpha_1^{i,s} b_1^s & \text{if } \epsilon_1^i < \bar{\epsilon} \text{ (} i \text{ defaults or receives a transfer)} \end{cases}$$

where we substituted the optimal transfer.

It follows that country i's government solves the following program:[17]

$$\begin{aligned} \max_{b_1^i} \quad & c_0^i + \beta \left(\int_{\epsilon_{\min}}^{\bar{\epsilon}} c_1^i dG(\epsilon) + \int_{\bar{\epsilon}}^{\epsilon_{\max}} c_1^i dG(\epsilon) \right) \\ \text{s.t.} \quad & c_0^i \geq 0 \\ & b_1^i / R^i = D(b_1^i) \\ & 0 \leq b_1^i \leq b_{\max} \end{aligned}$$

where c_0^i and c_1^i are defined above.

Denoting ν_0 the multiplier on period 0 consumption and μ_1 the multiplier on $b_1^i \geq 0$, the first-order condition is:[18]

$$\begin{aligned} 0 \quad \in \quad & \mu_1 + (1 - \alpha_1^{i,i}) \partial D(b_1^i)(1 + \nu_0) - \beta(1 - G(\bar{\epsilon}))(1 - \alpha_1^{i,i}) \\ & \nu_0 c_0^i = 0 \\ & \mu_1 b_1^i = 0 \end{aligned}$$

where $\partial D(b)$ denotes the sub-differential of $D(b)$.[19]

Consider first an interior solution ($c_0^i \geq 0$ and $b_1^i \geq 0$) where the revenue curve is differentiable. The first-order condition becomes:

$$D'(b_1^i) = \beta \left(1 - G(\bar{\epsilon})\right) \tag{11}$$

This first-order condition equates the marginal gain from one additional unit of debt (at face value), $D'(b_1^i)$, with its marginal cost. Equation (11) establishes that this marginal cost is equal to the probability of repayment without transfer $1 - G(\bar{\epsilon})$, discounted back at the risk free rate $1/R^* = \beta$. In other words, i only considers as relevant the states of the world where it is repaying the debt without default or bailout. In case of default, the repayment is proportional to output (and therefore not a function of the debt level). In case of a bailout, the debt is -at the margin- repaid by g. A change in b_1^i also has an effect on the thresholds $\bar{\epsilon}$ and $\underline{\epsilon}$, but since these thresholds are optimally chosen, the Envelope theorem ensures that i does not need to consider their varia-

[17] We do not need to impose the constraint that $c_1^i \geq 0$: it is always satisfied under the assumption that $\Phi + \phi \leq 1$.

[18] The constraint $b \leq b_{\max}$ does not need to be imposed.

[19] The sub-differential is the derivative of $D(b)$ where that derivative exists. It is the convex set $[D(b^-), D(b^+)]$ where that derivative does not exist, at $b = \underline{b}$ and $b = \bar{b}$.

tion.

Substituting the general expression for $D'(b_1^i)$ from equation (10) into equation (11) we obtain:

$$(G(\bar{\epsilon}) - G(\underline{\epsilon}))\,(1-\pi) = (b_1^i - \rho\bar{y}_1^i\underline{\epsilon})(1-\pi)g(\underline{\epsilon})\frac{d\underline{\epsilon}}{db} + (b_1^i - \rho\bar{y}_1^i\bar{\epsilon})\pi g(\bar{\epsilon})\frac{d\bar{\epsilon}}{db} \tag{12}$$

The left hand side of this equation has a very natural interpretation. It represents the probability that i will receive a transfer from g, a benefit for i. Recall that i obtains a bailout from g with probability $1-\pi$ when $\underline{\epsilon} \leq \epsilon < \bar{\epsilon}$. By issuing more or less debt in period 0, i can influence the likelihood of a bailout. The right hand side represents the cost of issuing more debt. It has two components. Let's consider each in turn. The first term captures the cost of an increase in debt due to a change in $\underline{\epsilon}$. Recall that i defaults below $\underline{\epsilon}$, and receives no bailout. An increase in b_1^i increases $\underline{\epsilon}$, making outright default more likely. If $\epsilon = \underline{\epsilon}$, lenders loose b_1^i and receive instead $\rho y_1^i\underline{\epsilon}$, with probability $g(\underline{\epsilon})(1-\pi)$. The second term captures the cost of an increase in debt due to a change in $\bar{\epsilon}$. Recall that, above $\bar{\epsilon}$, i repays its debts and default does not occur. Below $\bar{\epsilon}$, a default can occur when bailouts are not allowed. An increase in debt increases $\bar{\epsilon}$, again making default more likely. At $\epsilon = \bar{\epsilon}$, lenders are now at risk of loosing b_1^i and receiving instead $\rho y_1^i\bar{\epsilon}$, in case a bailout does not materialize, i.e. with probability $g(\bar{\epsilon})\pi$. The increased riskiness of i's debt is reflected into a higher yield, reducing $D'(b)$. Equation (12) makes clear that the possibility of a bailout in period 1 induces i to choose excessively elevated debt levels in period 0. We call this the *Northern view* of the crisis. Note also that a lower collateral cost of default for g, a lower κ, reduces the probability i will receive a transfer from g (the left hand side of (12)) and therefore the incentive to issue debt. Hence, reducing κ has a direct positive impact on g but also serves to discipline i. This resonates with some German proposals to introduce orderly restructuring in case of a default in the eurozone that can be interpreted in the context of our model as lower collateral costs of default.

Equation (12) highlights that i trades off the increased riskiness of debt –and therefore higher yields– against the likelihood of a bailout. In the absence of ex-post transfers (e.g. when $\pi = 1$), the left hand side of (12) is identically zero. The only interior solution is $\bar{\epsilon} \leq \epsilon_{\min}$, so that $g(\bar{\epsilon}) = 0$: i has no incentives to issue risky debt. By contrast, once $\pi > 0$, i may choose to issue risky debt (i.e. $\bar{\epsilon} > \epsilon_{\min}$) in order to maximize the chance of a bailout in period 1. This risk shifting result is a common feature of moral hazard models. Ex-post bailouts partially shield borrowers from the fiscal consequences of excessive borrowing. Not surprisingly, this provides an incentive to borrow excessively.

Appendix B provides a full description of the optimal level of debt issued in period 0. In particular, we show that, under some mild regularity conditions, the optimal choice of debt is either $b \leq \underline{b}$, i.e. a safe level of debt, or $b_{opt} \leq b \leq b_{\max}$, where b_{opt} denotes the unique optimal level of risky debt that obtains when the funding needs are smaller than $D(b_{opt})$.

Define $x_0^i = (b_0^i(1-\alpha_0^{i,i}) + \alpha_1^{s,i} b_1^s/R^* - y_0^i - \tau_0 - b_0^{s,i})/(1-\alpha_1^{i,i})$. x_0^i represents the *funding needs* of country i. It increases with the net amount of debt to be repaid $b_0^i(1-\alpha_0^{i,i})$, and decreases with the amount of resources available in period 0, $y_0^i + \tau_0^i$. The optimal choice of debt as a function of the initial funding needs x_0^i can be summarized as follows:

- For $x_0^i > D(b_{\max})$, i is insolvent in period 0 and must default. No level of debt can ensure solvency.
- For $D(b_{\max}) \geq x_0^i > D(b_{opt})$, i issues a level of debt $b_{\max} \geq b > b_{opt}$ such that $D(b) = x_0^i$ and there is no consumption in period 0. There is no risk shifting in the sense that debt issuance is fully constrained by i's funding in period 0.
- For $D(b_{opt}) \geq x_0^i > \beta \underline{b}$, i chooses to issue b_{opt}. In that range, the possibility of a bailout leads i to issue excessive amounts of debt in the sense that $D(b_{opt}) > x_0^i$ and consequently the probability of default is excessively high.
- Finally, for $x_0^i < \beta \underline{b}$, i can choose to issue either a safe amount debt $x_0^i/\beta \leq b_1^i \leq \underline{b}$ or the risky amount b_{opt}. If i prefers to issue risky debt, then the amount of risk shifting is *maximal*. This will be the case if i achieves a higher level of utility at b_{opt} then by keeping the debt safe. The utility gain from risk shifting is given by $U(b_{opt}) - U_{safe}$, equal to:

$$\begin{aligned} U(b_{opt}) - U_{safe} &= (1-\alpha_1^{i,i})(1-\pi)\beta\left[G(\bar{\epsilon}) - G(\underline{\epsilon})\right]\left(b_{opt} - \rho \bar{y}_1^i E[\epsilon | \underline{\epsilon} \leq \epsilon \leq \bar{\epsilon}]\right) \\ & \quad -\beta \Phi G(\bar{\epsilon}) \bar{y}_1^i E[\epsilon | \epsilon < \bar{\epsilon}] \end{aligned}$$

The first term represents the expected net gain from the bailout (since $b_{opt} > \rho \bar{y}_1^i \bar{\epsilon}$, it follows that $b_{opt} > \rho \bar{y}_1^i E[\epsilon | \underline{\epsilon} < \epsilon < \bar{\epsilon}]$). The second term represents the expected discounted cost of default for i. This cost is borne by i as soon as $\epsilon < \bar{\epsilon}$ since the bailout does not affect i's utility. It follows that i will issue excessively high levels of debt when the following condition holds:

$$(1-\alpha_1^{i,i})(1-\pi)\left[G(\bar{\epsilon})-G(\underline{\epsilon})\right]\left(b_{opt}-\rho\bar{y}_1^i E[\epsilon|\underline{\epsilon}\leq\epsilon\leq\bar{\epsilon}]\right)>\Phi G(\bar{\epsilon})\bar{y}_1^i E[\epsilon|\epsilon<\bar{\epsilon}] \quad (13)$$

Inspecting equation (13), it is immediate that there is no risk shifting when $\pi = 1$ or when i holds most of its own debt ($\alpha_1^{i,i} \approx 1$). Risk shifting is more likely the higher is the optimal debt output ratio $b_{opt}/\bar{y}_1^i$ and the lower the cost of default Φ.

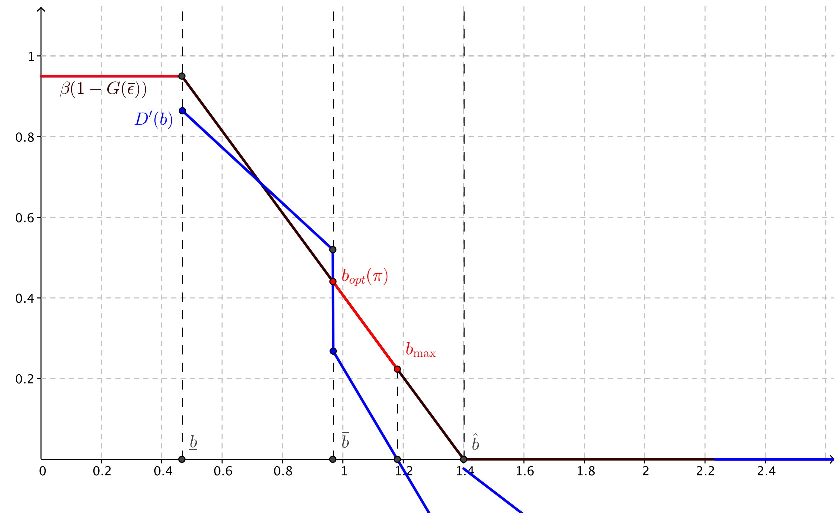
These results are summarized in Figure 4. The figure reports, for the case of a uniform distribution the function $\beta(1-G(\bar{\epsilon}(b)))$ (in black) and the function $D'(b)$ (in blue). There are two discontinuities of the function $D'(b)$ at $b=\underline{b}$ and $b=\bar{b}$. In red, the figure reports the possible optimal equilibrium debt levels. For $b \leq \underline{b}$ the debt is safe and any level -if sufficient to rollover the debt– provides equivalent level of utility; $b_{opt} \geq \bar{b}$ is the optimal level of risky debt when the rollover constraint ($c_0^i \geq 0$) does not bind. Finally, $b_{opt} < b \leq b_{\max}$ obtains when the rollover constraint binds (i.e. $c_0^i = 0$ and $D(b) = x_0^i$.[20]

Figure 5 reports the Laffer curve and the optimal debt levels. It illustrates the extent of risk shifting that occurs when i chooses to issue at b_{opt} instead of a safe level $b < \underline{b}$.

Making i's debt safe: Optimal ex-ante bailout policy for g. The previous analysis makes clear that the extent of risk shifting depends on the likelihood of a bailout, $1-\pi$. When bailouts are very likely ($\pi \approx 0$), and under the regularity conditions described in appendix A and B, b_{opt} is larger than $\bar{b}$. In other words, i chooses a level of risky debt sufficiently high so that there might be a possibility of default, even when ex-posts bailouts are almost guaranteed. In that case, the extent of risk shifting is maximal.

As π increases, this optimal level of risky debt decreases until it reaches $b_{opt} = \bar{b}$. Appendix B shows that there is a critical level of π, denoted π_c such that for $\pi > \pi_c$, the optimal level of debt falls discontinuously from $\bar{b}$ to $b \leq \underline{b}$ and debt becomes safe. This is represented in Figure 6 where we report b_{opt} as a function of π. This analysis indicates that it is not necessary for g to enforce a strict no-bailout policy ($\pi = 1$) to eliminate risk shifting in period 0. Any level π superior to π_c will result either in a safe debt level, or the minimum level of debt necessary to cover funding

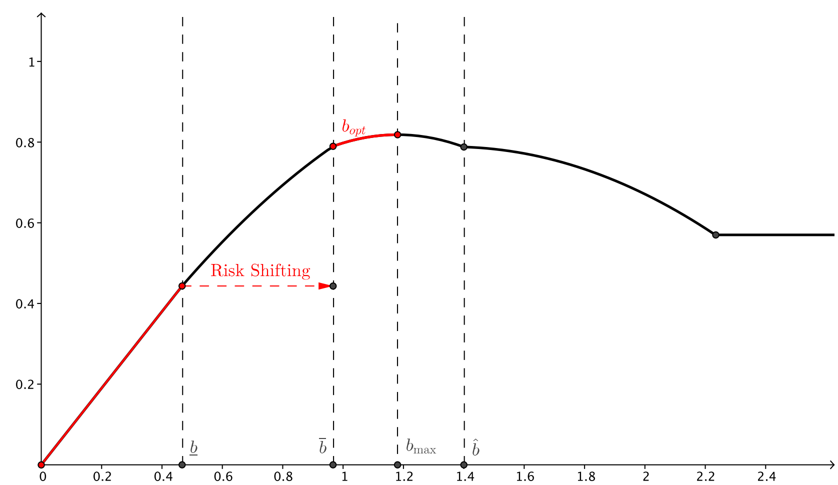
[20] As can be seen on the figure, there is another solution to the first order condition between $\underline{b}$ and $\bar{b}$. However, this solution does not satisfy the second-order conditions.



$D'(b)$ and $\beta(1-G(\bar{\epsilon}))$ for $\pi = 0.5$.

[Uniform distribution with $\rho = 0.6$, $\Phi = 0.2$, $\kappa = 0.05$, $\epsilon_{\min} = 0.5$, $\beta = 0.95$, $\bar{y}_1^i = 1$, $y_1^g = 2$, $\alpha_1^{i,i} = 0.4$, $\alpha_1^{i,g} = \alpha_1^{i,u} = 0.3$. $\underline{b} = 0.47$, $\bar{b} = 0.97$ and $\hat{b} = 1.4$]

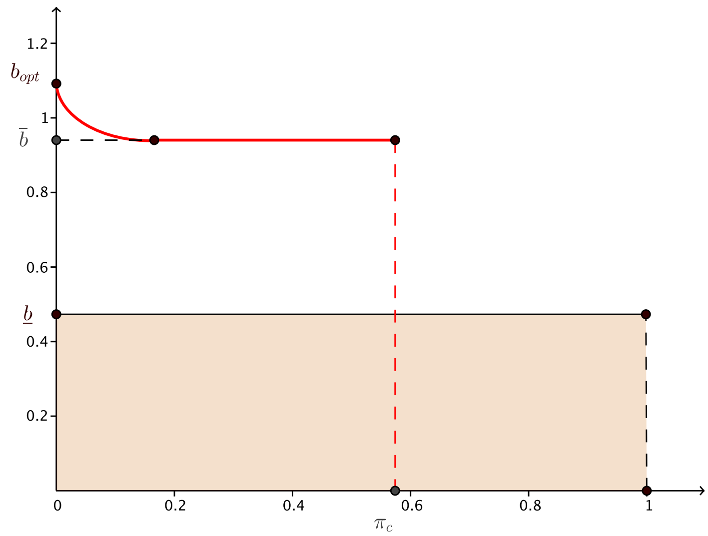
Figure 4: Optimal Debt Issuance



Optimal Debt Issuance for $\pi = 0.5$.

Uniform distribution with $\rho = 0.6$, $\Phi = 0.2$, $\kappa = 0.05$, $\epsilon_{\min} = 0.5$, $\beta = 0.95$, $\bar{y}_1^i = 1$, $y_1^g = 2$, $\alpha_1^{i,i} = 0.4$, $\alpha_1^{i,g} = \alpha_1^{i,u} = 0.3$. $\underline{b} = 0.47$, $\bar{b} = 0.97$ and $\hat{b} = 1.4$

Figure 5: Optimal Debt Issuance: Risk Shifting



Plot of the set of unconstrained solutions $0 \leq b \leq \underline{b}$ and b_{opt} as a function of π. There is a critical value π_c above which risk shifting disappears.

Figure 6: The Effect of No-Bailout Clauses

needs, i.e. $D(b_1^i) = x_0^i$.

It does not necessarily follow that g is indifferent between any bailout policy with $\pi \geq \pi_c$, since higher levels of π reduce ex-post efficiency. Suppose g can choose a commitment technology π in period 0. A higher π reduces the amount of risk shifting. For $\pi > \pi_c$ risk shifting is eliminated entirely. However, this also reduces resources available to i in the ex-post stage and makes a default more likely. It also makes i less solvent, so that, depending on the initial funding needs x_0^i, it could also force i to default in period 0. In other words, there is an option value to wait and see if i's output level will be sufficiently high to allow repayment without transfer and it can be in the interest of g to allow for a possible bailout, even as of $t = 0$.

In the bondless limit, g's utility can be expressed as a function of the optimal debt $b(\pi)$ issued by i and no-bailout probability π (after substitution of the optimal transfer when $\underline{\epsilon} \leq \epsilon < \bar{\epsilon}$):

$$\begin{aligned}
U_g(b(\pi), \pi) &= c_0^g + \beta E[c_1^g] \\
&= y_0^g - b_0^g + b_0^{i,g} + b_0^{s,g} + \beta y_1^g - \alpha_1^{i,g} D(b(\pi); \pi) \\
&\quad + \beta \int_{\epsilon_{\min}}^{\underline{\epsilon}} (\alpha_1^{i,g} \rho \bar{y}_1^i \epsilon - y_1^g \kappa) dG + \beta \int_{\underline{\epsilon}}^{\bar{\epsilon}} (\bar{y}_1^i \epsilon (\Phi + \rho(1 - \alpha_1^{i,i})) - b(\pi) \alpha_1^{i,u}) dG \\
&\quad + \beta \alpha_1^{i,g} b(\pi)(1 - G(\bar{\epsilon})) \\
&= y_0^g - b_0^g + b_0^{i,g} + b_0^{s,g} + \beta y_1^g + \Psi(b(\pi); \pi)
\end{aligned}$$

where

$$\begin{aligned}
\Psi(b; \pi) &= -\alpha_1^{i,g} D(b; \pi) + \beta \int_{\epsilon_{\min}}^{\underline{\epsilon}} (\alpha_1^{i,g} \rho \bar{y}_1^i \epsilon - y_1^g \kappa) dG + \beta \int_{\underline{\epsilon}}^{\bar{\epsilon}} (\bar{y}_1^i \epsilon (\Phi + \rho(1 - \alpha_1^{i,i})) - b \alpha_1^{i,u}) dG \\
&\quad + \beta \alpha_1^{i,g} b(1 - G(\bar{\epsilon}))
\end{aligned}$$

denotes the net gain to g from holding risky debt from i. g's government is not indifferent as to the level of i's debt, despite risk neutral preferences because it internalizes that it will have to provide a bailout τ_1. If the debt is safe (i.e. $\bar{\epsilon} \leq \epsilon_{\min}$), then $\Psi(\pi) = 0$.

The optimal choice of commitment technology satisfies $d\Psi(b(\pi); \pi)/d\pi = 0$. Taking a full derivative of the expression above yields:

$$\begin{aligned}
&-\alpha_1^{i,g} \left(\frac{\partial D(b; \pi)}{\partial \pi} + \frac{\partial D(b; \pi)}{\partial b} \frac{db}{d\pi} \right) + \beta (\bar{y}_1^i \bar{\epsilon} (\Phi + \rho(1 - \alpha_1^{i,i})) - b(1 - \alpha_1^{i,i})) g(\bar{\epsilon}) \frac{d\bar{\epsilon}}{db} \frac{db}{d\pi} \\
&- \beta \alpha_1^{i,u} \frac{db}{d\pi} (G(\bar{\epsilon}) - G(\underline{\epsilon})) + \beta \alpha_1^{i,g} \frac{db}{d\pi} (1 - G(\bar{\epsilon})) = 0
\end{aligned}$$

Suppose that i chooses $b = b_{opt}$. This satisfies $\partial D(b; \pi)/\partial b = \beta(1 - G(\bar{\epsilon}))$. Substituting, and simplifying one obtains:

$$-\alpha_1^{i,g} \frac{\partial D(b; \pi)}{\partial \pi} + \beta (\bar{y}_1^i \bar{\epsilon} (\Phi + \rho(1 - \alpha_1^{i,i})) - b(1 - \alpha_1^{i,i})) g(\bar{\epsilon}) \frac{d\bar{\epsilon}}{db} \frac{db}{d\pi} - \beta \alpha_1^{i,u} \frac{db}{d\pi} (G(\bar{\epsilon}) - G(\underline{\epsilon})) = 0$$

It is easy to check that if risk shifting is optimal for i (i.e. condition (13) holds), all three terms on the left are positive since we have established that $db_{opt}/d\pi \leq 0$ and $\partial D/\partial \pi < 0$: g will choose the highest possible level of ex-ante commitment to eliminate risk-shifting.

This analysis is valid as long as i remains solvent. Denote $b_{\max}(\pi)$ the level of debt that maximizes revenues for i as a function of the commitment level. It is immediate that $dD(b_{\max};\pi)/d\pi \leq 0$. Once $D(b_{\max}(\pi);\pi) < x_0^i$, i cannot honor its debts and is forced to default in the initial period. By analogy with the analysis of period 1, suppose that a default in period 0 has a direct contagion cost κy_0^g on g. In addition, i's bondholders recover a fraction ρ of i's output. Assume also that i is unable to borrow, so $b_1^i = 0$. It follows that g will choose $\pi(x_0^i)$ defined implicitly such that $D(b_{opt};\pi(x_0^i)) = x_0^i$, and will prefer to let i default if the following condition is satisfied:

$$\kappa y_0^g + \alpha_0^{i,g}(b_0^i - \rho y_0^i) + \Psi(b_{opt}, \pi(x_0^i)) \geq 0 \tag{14}$$

Condition (14) states that it can be optimal ex-ante for g to allow ex-post bailouts if these allow i to avoid an immediate default. The logic is quite intuitive: by allowing the possibility of a future bailout, g allows the monetary union to gamble for resurrection: in the event that i's output is sufficiently hight in period 1, debts will be repaid and a default will be avoided in both periods. Even if a bailout is required, the cost to g as of period 0 is less than one for one.

This discussion highlights that g is more likely to adopt an ex-ante lenient position on future bailouts (i.e. a low π) when i has initially a high debt level or a low output level. This provides an interpretation of the early years following the creation of the Eurozone. Countries were allowed to join the Eurozone with vastly different levels of initial public debt. The strict imposition of a no bailout guarantee could have pushed these countries towards an immediate default and debt restructuring. Instead, it may have been optimal to allow these countries to rollover their debt on the conditional belief that a bailout might occur in the future. The fiscal cost to g of an immediate default may have exceeded the expected costs from possible future bailouts. Notice however, that we specify the optimal policy such that $D(b_{opt};\pi) = x_0^i$. In other words, while g is willing to let i roll over its debts, it is still able to avoid risk-shifting, in the sense of avoiding excessive debt issuance at period 0.

Summarizing the main points of the baseline model. The previous analysis makes a number of interesting points for the analysis:

- First, if the probability of bailout $1-\pi$ is sufficiently small, there is no 'risky' equilibrium and the only possible solutions are either to issue safe debt (when rollover needs are small enough) or issue the amount necessary to exactly roll over the debt (i.e. $c_0^i = 0$). In other words, when the probability of bailout is too small, there is no risk shifting equilibrium anymore.

- when π is sufficiently small (high probability of bailout), as long as the funding needs are not too high, country i chooses a unique level of debt b_{opt} regardless of the funding needs. We also know that this optimal level of debt is such that $\bar{b} \leq b_{opt} < b_{\max}$, i.e. it occurs for levels of debt sufficiently elevated that default might occur.

6 Debt monetization (incomplete)

Debt monetization is an alternative to default which we have excluded so far. Even though article 123 of the Treaty of the European Union forbids ECB direct purchase of public debt, debt monetization can still take place through inflation. In this section, we concentrate on how the interaction of transfers and debt monetization affects the probability of default and how the ECB may be overburdened when transfers are excluded. To facilitate the analysis we simplify the model by assuming a zero recovery rate ($\rho = 0$) and by focusing on two polar cases where transfers are always possible ($\pi = 0$) and where transfers are excluded ($\pi = 1$).

There are now three players: i, g and the ECB. In addition to g's decision on the transfer, i's decision on default, the ECB decides how much and whether to monetize the debt. The timing of decisions of the ECB and g is not important.

In our model we assume the ECB can choose the inflation rate for the monetary union as a whole. This would be the case for example with Quantitative Easing (QE) which generates higher inflation and euro depreciation that both reduce the real value of public debt. Importantly, all public debts are inflated away at the same rate in the monetary union so that g also stands to benefit from it. However, both countries also suffer from the inflation distortion cost that are proportional to output. If z is the inflation rate, the distorsion cost is $\delta z y_1^i$ for i and $\delta z y_1^g$ for g. The inflation rate is capped at $\overline{z}$ because above this rate the distortion cost is infinite.

The ECB can also implement targeted purchases of public debt. In this case, it would be possible to buy public debt of a specific country without any inflation cost for example if it was sterilized by sales of other eurozone countries debt. The Outright Monetary Transactions (OMT) program announced in September 2012 is close to such a description. This program however resembles a transfer in the sense that part of the debt of i is taken off the market and that to sterilize this intervention the ECB would sell g debt. A condition of the OMT program is that the country needs to have received financial sovereign support from the eurozone's bailout funds EFSF/ESM. This strengthens our interpretation of the OMT program as a financial support program, i.e. a transfer. Remember that the OMT was never put into place but remains a possibility. The Securities Markets Programme (SMP) program was put into place in May 2010 by the ECB and terminated in September 2012 to be replaced by OMT. The aim was to purchase sovereign bonds on the secondary markets. At its peak, the programme's volume totalled around ff210 billion. The Eurosystem central banks that purchased sovereign bonds under this programme hold them to maturity. The programme initially envisaged that central bank money created from the purchase of securities would be sterilised. This description suggests that the (never implemented) OMT and the (now terminated) SMP programmes are close to the way we interpret transfers. However, the OMT rules imply that such a transfer can not take place without support from the eurozone's bailout funds EFSF/ESM. Hence, we keep the assumption that the transfer τ_1 is decided by g. On the other hand, debt monetization at the inflation rate z is the sole responsibility of the ECB.

We first analyze the decision to default of i for a given transfer and inflation/monetization rate. If i repays the ECB chooses the rate z and if i defaults it chooses the rate $\hat{z}$. The budget constraint in period 1 of the i households becomes:

$$\begin{aligned} c_1^i &= y_1^i - T_1^i + \left(b_1^{i,i} + b_1^{g,i}\right)(1-z) - \delta z y_1^i + b_1^{u,i} \quad \text{if } i \text{ repays} \\ c_1^i &= y_1^i(1-\Phi) - T_1^i + b_1^{g,i}(1-\hat{z}) - \delta \hat{z} y_1^i + b_1^{u,i} \quad \text{if } i \text{ defaults} \end{aligned}$$

Government i constraint in $t = 1$ is:

$$\begin{aligned} T_1^i + \tau_1 &= b_1^i(1-z) \quad \text{if } i \text{ repays} \\ T_1^i &= 0 \quad \text{if } i \text{ defaults} \end{aligned}$$

Consolidating the private and public budget constraints, we again proceed by backward induction. At $t = 1$, i can decide to default after the shock ϵ_1^i has been revealed and the transfer τ_1 announced. Taking b_1^i and τ_1 as given, i repays if and only if:

$$y_1^i\left[\Phi - \delta(z-\hat{z})\right] \geq b_1^i\left(1-\alpha^{i,i}\right)(1-z) + (z-\hat{z})\, b_1^g \alpha^{g,i} - \tau_1 \tag{15}$$

For g, the budget constraint is:

$$\begin{aligned} c_1^g &= y_1^g - T_1^g + \left(b_1^{i,g} + b_1^{g,g}\right)(1-z) - \delta z y_1^g + b_1^{u,g} \quad \text{if } i \text{ repays} \\ c_1^g &= y_1^g(1-\kappa) - T_1^g + b_1^{g,g}(1-\hat{z}) - \delta \hat{z} y_1^g + b_1^{u,g} \quad \text{if } i \text{ defaults} \end{aligned}$$

and g government constraint in $t = 1$ is:

$$\begin{aligned} T_1^g - \tau_1 &= b_1^g (1 - z) \quad \text{if } i \text{ repays} \\ T_1^g &= b_1^g (1 - \hat{z}) \quad \text{if } i \text{ defaults} \end{aligned}$$

Hence, inflation looks like a partial default, except that the total cost for the eurozone is $\delta z \left(y_1^i + y_1^g\right)$ in case of inflation and $\Phi y_1^i + \kappa y_1^g$ in case of default. We reasonably assume that Φ and κ are larger than δz, meaning that,in proportion to output, the costs of default are both larger than the marginal distortionary cost of inflation.

6.1 The case with transfers

We first analyze the case where transfers by g are possible and not subject to political risk i.e. $\pi = 0$. Remember that in presence of transfers by g to i, g captures the entire surplus of i not defaulting: g's transfers are ex-post efficient from the joint perspective of g and i. This implies that the objective of the ECB and g are perfectly aligned if, as we assume, the ECB maximizes the whole EMU welfare. Hence, the ECB will choose either zero or maximum inflation rate $\overline{z}$ depending whether the marginal benefit of inflating the eurozone debt held in the rest of the world is below or above its marginal distortion cost. In the case of no default, this will be the case if:

$$b_1^i \alpha_1^{i,u} + b_1^g \alpha_1^{g,u} < \delta \left(y_1^i + y_1^g\right) \tag{16}$$

In case of default, given that there is no i debt to inflate the condition is:

$$b_1^g \alpha_1^{g,u} < \delta \left(y_1^i + y_1^g\right) \tag{17}$$

This defines two thresholds for the ECB decision. In case of no default, the ECB chooses a zero inflation rate if i output realization is such that:

$$\epsilon_1^i > \frac{b_1^i \alpha_1^{i,u} + b_1^g \alpha_1^{g,u}}{\delta \overline{y}_1^i} - \frac{y_1^g}{\overline{y}_1^i} \equiv \overline{\overline{\epsilon}} \tag{18}$$

In case of default, the condition becomes:

$$\epsilon_1^i > \frac{b_1^g \alpha_1^{g,u}}{\delta \overline{y}_1^i} - \frac{y_1^g}{\overline{y}_1^i} \equiv \hat{\epsilon} \tag{19}$$

We can compare different cases with different degrees of fiscal dominance. **Fiscal dominance** would apply if the ECB inflates the eurozone debt even if i defaults so that only g debt remains. This is not a very interesting or plausible case so we ignore it and assume $\hat{\epsilon} < \epsilon^{min}$ which means that we concentrate as before on relatively low levels of debt to GDP levels in g and relatively high levels of the distortion costs δ. This implies that $\hat{z} = 0$. Another polar case is one of **monetary dominance**. This is a situation with low levels of g debt relative to GDP and high distortion costs δ. A sufficient condition is: $\overline{\overline{\epsilon}} < \epsilon^{min}$. The ECB never inflates the debt in a situation where transfers are possible because transfers are sufficient and the ECB would never want to avert a default if it was not in g interest which is also the interest of the Eurozone as whole. This case is identical to the one analyzed in section (4) where the role of the ECB was ignored.

Weak fiscal dominance, which we concentrate on, applies when the ECB, for low levels of i output realizations, decides to inflate the debt only in the case of no default of i. There are several conditions on output realizations and parameters for such a situation to exist:

$$
\begin{aligned}
\epsilon_1^i &< \overline{\overline{\epsilon}} \\
\epsilon_1^i &> \frac{\alpha_1^{i,u} b_1^i (1-z) - \alpha_1^{gu} b_1^g \overline{z} - y_1^g (\kappa - \delta \overline{z})}{(\Phi - \delta \overline{z}) \overline{y}_1^i} \equiv \underline{\epsilon}' \\
\epsilon_1^i &< \frac{\left(1-\alpha_1^{i,i}\right) b_1^i (1-\overline{z}) + \alpha_1^{g,i} b_1^g \overline{z}}{(\Phi - \delta \overline{z}) \overline{y}_1^i} \equiv \tilde{\epsilon} \\
\hat{\epsilon} &< \epsilon^{min} < \underline{\epsilon}' < \overline{\overline{\epsilon}} < \tilde{\epsilon}
\end{aligned}
$$

The first condition says that the output realization is such that the ECB sets $z = \overline{z}$, the second that g prefers no default and transfer and the third that indeed i requires a transfer when $z = \overline{z}$. These conditions apply for intermediate levels of the output realization i. The last condition on the ranking of thresholds requires in particular intermediate levels of debt (see appendix for details). In this case, the transfer is the minimum that leaves i indifferent between default and no default:

$$
\tau_1 = b_1^i \left(1-\alpha_1^{i,i}\right) (1-\overline{z}) - y_1^i \left[\Phi - \delta \overline{z}\right] + \overline{z} b_1^g \alpha^{g,i} \tag{20}
$$

We can compare the transfer with monetization and without monetization ($\overline{z} = 0$). The first element on the right hand side reduces the required transfer because debt monetization weakens the incentive of i to default. However, the second term, the inflation distorsion (proportional to y_1^i) must be compensated by a higher transfer given that in default there is no such inflation distortion. The last term is the inflation tax on the g debt held by i which also must be compensated

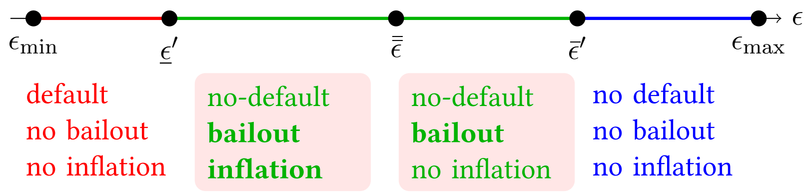


Figure 7: Bailout and Inflation under Weak Fiscal Dominance

by a higher transfer. Hence, debt monetization allows to reduce the transfer for low levels of g debt and low inflationary distortion costs which is the case we concentrate on.

It can also be shown that ECB monetization, if it takes place, always reduces the likelihood of default in the sense that $\frac{\partial \underline{\epsilon}'}{\partial \bar{z}} < 0$, i.e. the output realization below which i defaults falls with debt monetization. The condition for this to be true is that $\underline{\epsilon}' < \bar{\bar{\epsilon}}$ which is indeed the case when debt is monetized and i does not default. The intuition is that the net gain of inflating the debt for the eurozone is eliminated when default occurs. Hence, monetization, because it taxes agents from outside the eurozone, produces an additional gain of not defaulting. This lower default probability due to monetization increases the welfare of g but does not affect i which is left indifferent between defaulting and not defaulting.

A related result the whole benefit of debt monetization, if it occurs, is captured by g. This can be checked by computing consumption in g in the case of transfer (no default) and debt monetization:

$$c_1^g = y_1^g + \Phi y_1^i - b_1^i \alpha_1^{i,u} - b_1^g \left(1 - \alpha_1^{g,g}\right) + b_1^{u,g} + \bar{z}\left[b_1^i \alpha_1 a^{i,u} + b_1^g \alpha_1^{g,u} - \delta\left(y_1^i + y_1^g\right)\right]$$

The last term is the net benefit of i and g debt monetization which is always strictly positive if it is optimal for the Eurozone as a whole to inflate the debt. Hence, under weak fiscal dominance, the possibility of debt monetization is always at the benefit of g. All the surplus of monetization of the whole eurozone debt held by the rest of the world is thus captured by g.

It can be shown (see appendix) that as i output realizations deteriorate, the equilibrium moves from a situation with 1) no default, no transfer, no inflation, ; 2) no default, transfer, no inflation; 3) no default, inflation, transfer; 4) default, no inflation, no transfer. This is shown in Figure 7.

6.2 When transfers are excluded: the overburdened ECB

The situation we described is one where a fiscal union or a strong cooperative agreement exists such that fiscal transfers are possible with full discretion ($\pi = 0$). This meant that there were two instruments for two objectives: transfers to avoid default and inflation to monetize the debt held outside the eurozone. There is an efficient use of these two instruments.

These transfers may actually be hard to implement for political and legal reasons which we captured in the previous analysis with $\pi > 0$. They may not be possible also because of the difficulty to get an agreement with multiple eurozone creditor countries who share the cost of the transfer and its benefit, i.e the absence of default. Such a situation would generate a prisoner's dilemma because avoiding i default is a public good. The Nash equilibrium may be characterized by the absence of transfers. We analyze the simplest version of this situation with $\pi = 1$,. This is also a situation where we assume that the ECB cannot perform OMT/SMP type of debt purchases.

Contrary to the situation where transfers are feasible, the ECB, which maximizes the Eurozone welfare, may now use monetary policy to avert a costly default. We find that it prefers to avert default if, for a positive inflation rate $z < \overline{z}$:

$$\Phi y_1^i + \kappa y_1^g + z\left(b_1^g \alpha^{g,u} + b_1^i \alpha^{i,u}\right) > b_1^i \alpha^{iu} + \delta z\left(y_1^i + y_1^g\right) \tag{21}$$

where we have assumed that parameters are such that the ECB would choose zero inflation in a situation of default. Italy will choose not to default (without transfer) if:

$$\overline{z} > z \geq \frac{b_1^i\left(1-\alpha^{i,i}\right) - \Phi y_1^i}{b_1^i\left(1-\alpha^{i,i}\right) - \delta y_1^i - b_1^g \alpha^{g,i}} \equiv \tilde{z} \tag{22}$$

This defines the minimum inflation rate for i not to default. The maximum inflation rate that the ECB is willing to accept to avert a default is:

$$z \leq \frac{\Phi y_1^i + \kappa y_1^g - b_1^i \alpha^{i,u}}{\delta\left(y_1^i + y_1^g\right) - b_1^i \alpha^{i,u} - b_1^g \alpha^{g,u}} < \overline{z} \tag{23}$$

The conditions for this to be feasible are 1) that the minimum inflation rate for i not to default lies below the maximum rate acceptable by the ECB or:

$$\begin{aligned} & b_1^i\left(1-\alpha^{i,i}\right) y_1^g(\kappa-\delta) + \kappa b_1^g y_1^g \alpha^{g,i} - b_1^i b_1^g\left[\left(1-\alpha^{i,i}\right)\alpha^{g,u} + \alpha^{g,i}\alpha^{i,u}\right] > \\ & y_1^i\left[\Phi b_1^g\left(1-\alpha^{g,g}\right) - (\Phi-\delta) b_1^i \alpha^{i,g} - \delta y_1^g(\Phi-\kappa)\right] \end{aligned}$$

Note that with the assumption that $\Phi > \kappa > \delta$ and for b_1^g small enough this inequality is always verified.

Condition 2) is the one that binds in the case of low g debt: The minimum inflation rate that

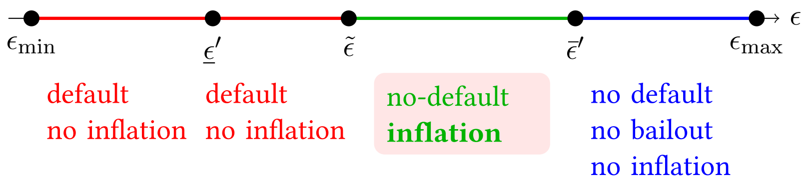


Figure 8: Bailout with Overburdened Central Bank

makes i choose not to default must be lower than $\overline{z}$. This will be the case if:

$$\epsilon_1^i \quad > \quad \frac{\left(1-\alpha^{i,i}\right) b_1^i \left(1-\overline{z}\right) + \alpha^{gi} b_1^g \overline{z}}{\left(\Phi - \delta \overline{z}\right) \overline{y}_1^i} \equiv \tilde{\epsilon}$$

To simplify the comparison with a situation where transfers are possible, we restrict ourselves to the case we called "monetary dominance" so that in presence of fiscal transfers the ECB chooses zero inflation. The situation where transfers are not possible, so that the ECB is the only institution that can act to avert a default, is one where there are more output realizations with default (for $\underline{\epsilon}' < \epsilon_1^i < \tilde{\epsilon}$) and more output realizations with positive inflation at the rate $\tilde{z}$ (for $\tilde{\epsilon} < \epsilon_1^i < \overline{\epsilon}'$). Hence, the impossibility to use transfers forces the ECB to inflate the debt at rate $\tilde{z}$ in equation (22),to avert the default. We refer to this case as the "overburdened ECB". The level of debt monetization $\tilde{z}$ varies between 0 and $\overline{z}$. Note that the numerator is positive because otherwise i could repay without transfer or inflation. Given that the inflation rate is positive, the denominator is also positive. This implies that the inflation rate increases with the debt of i held outside i and decreases with the output realization of i. The reason is that the incentive to default increases with the first element and decreases with the second one. Note also that the inflation rate increases with the debt level in g held by i although the only objective is to avert the default of i. The reason is that i must be compensated for the monetization of g debt it holds.

For output realizations such that $\underline{\epsilon}' < \epsilon_1^i < \overline{\epsilon}'$ (see figure 8), consumption in g is lower when transfers are excluded either because of too much inflation or of a default that could be avoided with transfers. This implies that g is the sole victim of an "overburdened ECB".

6.3 Optimal debt choice with potential inflation

The appendix analyzes in detail the optimal choice of debt when debt can be monetized by inflation.

6.3.1 The case with transfers

Fiscal revenues $D(b_1^i) = b_1^i / R^i$ raised by the government of country i in period $t = 0$:

$$\begin{aligned} D(b_1^i) = \frac{b_1}{R^i} &= \beta \mathcal{P} b_1^i + \overline{\lambda}^i \\ &= \beta b_1^i \left(1 - \pi_d\right)\left(1 - E\left(z\right)\right) + \overline{\lambda}^i \end{aligned}$$

where again we assumed zero recovery to simplify and where $E\left(z\right)$ is the expected inflation rate. Remember also that in this case we assume zero political risk so that $\pi = 0$. The characterization of the Laffer curve for different levels of debt is done in the Appendix. For relatively low levels of debt where there is no default risk, expected potential debt monetization can only increase the yield as it cannot reduce the probability of default. At higher levels of debt, an increase in the monetization rate increases the debt threshold above which default becomes possible. The reason is that higher monetization reduces the transfer that g needs and is willing to give to i to avoid a default. Hence, at intermediate levels of debt, expected debt monetization may actually reduce the yield of the debt issued by i.

We show that i will always want to issue debt at least at the level of $\overline{\overline{b}}$ where debt is safe because of the bailout and above which inflation may become optimal for the ECB. For $\overline{\overline{b}} \leq b < \overline{\overline{b}}'$, there is no default risk but potential inflation risk. For high $\overline{z}$, the optimal debt is $\overline{\overline{b}} < \overline{b}'$. This implies that the possibility of debt monetization actually induces the country to, ex ante, issue less debt. The reason is that the gains from debt monetization are captured in period 1 by g in the form of lower tranfers. The cost for i is that expected debt monetization $\overline{z}$ increases the cost of borrowing and therefore reduces the gain of issuing debt. Hence, in this case, i will issue debt but at a level such that there is no inflation risk for investors.

For low levels of $\overline{z}$, the optimal debt may be an interior solution with: $\overline{\overline{b}} < b'_{opt} \leq \overline{\overline{b}}'$. The reason is that there are two effects of debt monetization on optimal debt at low levels of $\overline{z}$. One reduces the incentive to issue debt and was explained above: the cost of issuing debt increases for i and the ex-post gains go to g. The other effect is the risk shifting that induces i to raise debt. When default risk does not exist because of transfers, higher expected debt monetization can only reduce the incentive to issue debt.

At higher levels, of debt ($\overline{\overline{b}}' < b_1^i < \overline{\overline{b}}''$) so that both default and inflation are possible depending on the output realization, three mechanisms are at work: (i) risk shifting induces to issue more debt, (ii) the inflation risk increases the cost of issuing debt but (iii) with some default risk, the possibility of monetization also reduces the default risk for some output realizations.

6.3.2 The case without transfers

If for political reasons, ex-post transfers are not possible or there is full commitment to exclude transfers, the budget constraint for i is now different in the case of default and no default:

$$\begin{aligned} c_1^i &= y_1^i - b_1^i\left(1-\alpha^{i,i}\right) + \alpha^{s,i} b_1^s \quad \text{if } i \text{ repays without inflation} \\ c_1^i &= y_1^i\left(1-\delta\tilde{z}\right) - b_1^i\left(1-\alpha^{i,i}\right)\left(1-\tilde{z}\right) + \alpha^{g,i} b_1^g\left(1-\tilde{z}\right) + \alpha^{u,i} b_1^u \quad \text{if } i \text{ repays with inflation} \\ c_1^i &= (1-\Phi) y_1^i + \alpha^{s,i} b_1^s \quad \text{if } i \text{ defaults} \end{aligned}$$

We show in the appendix that because the expected inflation that may be necessary to avoid default is perfectly priced in the interest rate, there is no risk shifting and the optimal level is x_0^i.

7 Conclusion

The objective of our paper was to shed light on the specific issues of sovereign debt in a monetary union. We analysed the impact of collateral damages of default with potential exit and of debt monetization. Because of collateral damages of default, the no bailout clause by governments and the commitment not to monetize the debt are not ex-post efficient. This provides an incentive to borrow by fiscally fragile countries. This is a "German" narrative of the crisis. We showed however that the efficiency benefits of transfers and debt monetization that prevent a default are entirely captured by the creditor country. There is no solidarity" in the transfers made to prevent a default. This is the "Italian" narrative of the crisis. Our model shows that the two narratives are two sides of the same coin. One may think that a policy implication would be to strengthen the no-bailout commitment. We have shown that this may not be the case because doing so may precipitate immediate insolvency. In addition, this may put pressure on the ECB to step in and prevent a default through debt monetization which is less efficient than simple transfers. Some current discussions on eurozone reforms resonate with our analysis. For example, German policy makers and economists have made proposals to introduce orderly restructuring in case of a default in the eurozone. This can be interpreted in the context of our model as lower collateral damage of default for creditor countries that would increase the probability of default because it would reduce the probability of a bailout but also strengthen "market discipline" through a higher yield for fiscally fragile countries.

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Appendices

A Characterizing the Laffer Curve

This appendix provides a full characterization of the Laffer curve in the basic model.

The Laffer curve satisfies :

$$D(b) \quad = \quad \beta b\left(1-\pi_d(b)\right)+\beta\rho\bar{y}_1^i\left(\pi\int_{\underline{\epsilon}(b)}^{\bar{\epsilon}(b)}\epsilon dG\left(\epsilon\right)+\int_{\epsilon_{\min}}^{\underline{\epsilon}(b)}\epsilon dG\left(\epsilon\right)\right)+\overline{\lambda}^i$$

where the cut-offs are defined as:

$$\bar{\epsilon}(b) \quad = \quad \frac{(1-\alpha_1^{i,i})b/\bar{y}_1^i}{\Phi+\rho(1-\alpha_1^{i,i})}$$

$$\underline{\epsilon}(b) \quad = \quad \frac{\alpha_1^{i,u}b/\bar{y}_1^i-\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}}$$

and the probability of default is:

$$\pi_d(b)=G(\underline{\epsilon}(b))+\pi(G(\bar{\epsilon}(b))-G(\underline{\epsilon}(b)))$$

There are a number of cases to consider:

- When $b\le\underline{b}\equiv y_{\min}^i\left(\Phi/(1-\alpha_1^{i,i})+\rho\right)$. In that case $\bar{\epsilon}\le\epsilon_{\min}$ and i's output is always sufficiently high that i prefers to repay even without any transfer from g. This makes i's debt riskless and

$$D\left(b\right)=\beta b+\bar{\lambda}^i$$

- If $\bar{b}\equiv((\Phi+\rho\alpha_1^{i,u})y_{\min}^i+\kappa y_1^g)/\alpha_1^{i,u}\le\hat{b}\equiv y_{\max}^i\left(\Phi/(1-\alpha_1^{i,i})+\rho\right)$. This is a condition on the parameters. It can be rewritten as:

$$\kappa y_1^g/\bar{y}_1^i\le\alpha_1^{i,u}\rho(\epsilon_{\max}-\epsilon_{\min})+\Phi/(1-\alpha_1^{i,i})(\alpha_1^{i,u}\epsilon_{\max}-\epsilon_{\min}(\alpha_1^{i,u}+\alpha_1^{i,g}))$$

  - When $\underline{b}<b\le\bar{b}<\hat{b}$. In that case, we have $\underline{\epsilon}\le\epsilon_{\min}<\bar{\epsilon}<\epsilon_{\max}$. When $b=\bar{b}$, $\underline{\epsilon}=\epsilon_{\min}<\bar{\epsilon}<\epsilon_{\max}$. Default can occur if $\epsilon_1^i\le\bar{\epsilon}$ and ex-post transfers are forbidden. It follows that

$$D\left(b_1\right)=\beta[b_1\left(1-\pi G\left(\bar{\epsilon}\right)\right)+\rho\bar{y}_1^i\pi\int_{\epsilon_{\min}}^{\bar{\epsilon}}\epsilon dG\left(\epsilon\right)]+\bar{\lambda}^i$$

and the slope of the Laffer curve is given by

$$D'(b_1) = \beta\left[1 - \pi G(\bar{\epsilon}) - \frac{\pi\bar{\epsilon}g(\bar{\epsilon})\Phi}{\Phi + \rho(1-\alpha_1^{i,i})}\right]$$

For these intermediate debt levels, default is a direct consequence of the commitment *not* to bail-out country i in period $t = 1$. The derivative of the Laffer curve is discontinuous at $b = \underline{b}$ if the distribution of shocks is such that $g(\epsilon_{\min}) > 0$ and the can write the discontinuity as:

$$\begin{aligned} D'(\underline{b}^+) - D'(\underline{b}^-) &= \beta\left(-\underline{b} + \rho y^i_{\min}\right)\pi g(\epsilon_{\min})\left.\frac{d\bar{\epsilon}}{db}\right|_{b=\underline{b}} \\ &= -\beta\frac{\pi\epsilon_{\min}g(\epsilon_{\min})\Phi}{\Phi + \rho(1-\alpha_1^{i,i})} \leq 0 \end{aligned}$$

The intuition for the discontinuity is that at $b = \underline{b}$, a small increase in debt increases the threshold $\bar{\epsilon}$ beyond $\epsilon_{\min}$, so a default is now possible. This happens with probability $\pi g(\epsilon_{\min})d\bar{\epsilon}$. In that case, investors' discounted net loss is $\beta(-\underline{b} + \rho y^i_{\min})$.

It is possible for the Laffer curve to *decrease* to the right of $\underline{b}$ if $\pi\epsilon_{\min}g(\epsilon_{\min})\Phi/(\Phi + \rho(1 - \alpha_1^{i,i})) > 1$. In that case the increase in default risk is so rapid that the interest rate rises rapidly and i's revenues $D(b)$ decline as soon as $b > \underline{b}$. Given that i can always choose to be on the left side of the Laffer curve by choosing a lower b_1^i, there would never be any default or bailout. We view this case as largely uninteresting.

This case can be ruled out my making the following assumption *sufficient* to ensure $D'(\underline{b}^+) > 0$:

Assumption 1 *We assume the following restriction on the pdf of the shocks and the probability of bailout*

$$\pi\epsilon_{\min}g(\epsilon_{\min}) < 1$$

[Note: (a) this condition cannot be satisfied with a power law and $\pi = 1$ (i.e. no transfers); (b) this condition is satisfied for a uniform distribution if $\pi < \epsilon_{\max}/\epsilon_{\min} - 1$. A sufficient condition for this is $\epsilon_{\min} < 2/3$.[21]]

The second derivative of the Laffer curve is:

$$D''(b) = -\beta\pi\frac{d\bar{\epsilon}}{db}\left[g(\bar{\epsilon}) + \frac{\Phi}{\Phi + (1-\alpha_1^{i,i})\rho}(g(\bar{\epsilon}) + \bar{\epsilon}g'(\bar{\epsilon}))\right]$$

[21] To see this, observe that since $E[\epsilon] = 1$ we can solve for $\epsilon_{\min} < 2/(2+\pi)$.

If we want to ensure that $D''(b) < 0$ a *sufficient* condition is:

Assumption 2 *We assume that g satisfies*

$$\frac{\epsilon g'(\epsilon)}{g(\epsilon)} > -2$$

[Note: we can replace this condition by a condition on the slope of the monotone ratio: $\pi g(\epsilon)/(1-\pi G(\epsilon))$.]

[Note: (a) that sufficient condition is not satisfied for $\rho = 0$ and a power law; (b) it is always satisfied for a uniform distribution since $g'(\epsilon) = 0$.]

The value of $D'(\bar{b}^-)$ is:

$$D'(\bar{b}^-) = \beta \left[1 - \pi G\left(\bar{\epsilon}(\bar{b})\right) - \frac{\pi \Phi \bar{\epsilon}(\bar{b}) g\left(\bar{\epsilon}(\bar{b})\right)}{\Phi + \rho(1-\alpha_1^{i,i})} \right]$$

We can ensure that this is positive (so that the peak of the Laffer curve has not been reached) by assuming that:

$$1/\pi > G\left(\bar{\epsilon}(\bar{b})\right) + \frac{\Phi \bar{\epsilon}(\bar{b}) g\left(\bar{\epsilon}(\bar{b})\right)}{\Phi + \rho(1-\alpha_1^{i,i})}$$

This condition is always satisfied when there is no default ($\pi = 0$). Otherwise, a *sufficient* condition is:

Assumption 3 *We assume that the distribution of shocks satisfies:*

$$1 > G\left(\bar{\epsilon}(\bar{b})\right) + \bar{\epsilon}(\bar{b}) g\left(\bar{\epsilon}(\bar{b})\right)$$

[Note: with a uniform distribution, the condition above becomes $\bar{\epsilon}(\bar{b}) < \epsilon_{\max}/2$. Substituting for $\bar{\epsilon}(\bar{b})$, this can be ensured by choosing $\epsilon_{\min}$ such that

$$\frac{1-\alpha_1^{i,i}}{\Phi + (1-\alpha_1^{i,i})\rho} \frac{(\Phi + \rho \alpha_1^{i,u})\epsilon_{\min} + \kappa y_1^g / \bar{y}_1^i}{\Phi + \rho \alpha_1^{i,u}} < 1 - \frac{\epsilon_{\min}}{2}$$

This can be ensured with $\epsilon_{\min}$ sufficiently small, provided $(\Phi + (1-\alpha_1^{i,i})\rho)\alpha_1^{i,u} > (\Phi + \rho\alpha_1^{i,u})(1-\alpha_1^{i,i})\kappa y_1^g/\bar{y}_1^i$.]

Under assumptions 1 -3, the Laffer curve is upward sloping, decreasing in b, discontinuous at $\underline{b}$ and has not yet reached its maximum at $\bar{b}$.

– When $\bar{b} < b \leq \hat{b}$ then we have $\epsilon_{\min} < \underline{\epsilon} < \bar{\epsilon} \leq \epsilon_{\max}$. It's now possible to default even with optimal transfers and the Laffer curve satisfies

$$D\left(b_1\right) = \beta\left[b_1\left(1 - G\left(\underline{\epsilon}\right) - \pi\left(G\left(\bar{\epsilon}\right) - G\left(\underline{\epsilon}\right)\right)\right) + \rho\bar{y}_1^i\left(\pi\int_{\underline{\epsilon}}^{\bar{\epsilon}}\epsilon dG\left(\epsilon\right) + \int_{\epsilon_{\min}}^{\underline{\epsilon}}\epsilon dG\left(\epsilon\right)\right)\right] + \bar{\lambda}^i$$

with slope:

$$D'\left(b_1\right) = \beta\left[1 - \pi_d - \frac{\pi g(\bar{\epsilon})\bar{\epsilon}\Phi}{\Phi + \rho(1-\alpha_1^{i,i})} - (1-\pi)g(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha_1^{i,u}}\right]$$

One can check immediately that the slope of the Laffer curve is discontinuous at $b = \bar{b}$ as well, if $\pi < 1$ and $g(\epsilon_{\min}) > 0$, with:

$$\begin{aligned} D'(\bar{b}^+) - D'(\bar{b}^-) &= \beta\left(-\bar{b} + \rho y_{\min}^i\right)(1-\pi)g(\epsilon_{\min})\left.\frac{d\underline{\epsilon}}{db}\right|_{b=\bar{b}} \\ &= -\beta(1-\pi)g(\epsilon_{\min})\frac{\Phi\epsilon_{\min} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha_1^{i,u}} \leq 0 \end{aligned}$$

The interpretation is the following: when $b = \bar{b}$, a small increase in debt makes default unavoidable, i.e. default probabilities increase from π to 1, since the debt level is too high for transfers to be optimal. The probability of default jumps up by $(1-\pi)g(\epsilon_{\min})d\underline{\epsilon}$. The discounted investor's loss in case of default is $\beta(-\bar{b} + \rho y_{\min}^i)$.

The second derivative of the Laffer curve is:

$$\begin{aligned} D''(b) &= -\beta\pi\frac{d\bar{\epsilon}}{db}\left[g(\bar{\epsilon}) + \frac{\Phi}{\Phi + (1-\alpha_1^{i,i})\rho}(g(\bar{\epsilon}) + \bar{\epsilon}g'(\bar{\epsilon}))\right] \\ &\quad -\beta(1-\pi)\frac{d\underline{\epsilon}}{db}\left[g(\underline{\epsilon}) + \frac{\Phi}{\Phi + \rho\alpha_1^{i,u}}g(\underline{\epsilon}) + g'(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha_1^{i,u}}\right] \end{aligned}$$

The first term is negative under assumption 2. The second term is also negative under assumption 2, unless $g'(\epsilon)$ becomes too negative.

Assumption 4 *The parameters of the problem are such that $D''(b) < 0$ for $b < \hat{b}$.*

[Note: with a uniform distribution, this condition is satisfied since $g'(\underline{\epsilon}) = 0$.]

We can check that:

$$D'(\hat{b}^-) \quad = \quad \beta\left[(1-\pi)(1-G(\underline{\epsilon})) - \frac{\pi g(\epsilon_{\max})\epsilon_{\max}\Phi}{\Phi+\rho(1-\alpha_1^{i,i})} - (1-\pi)g(\underline{\epsilon})\frac{\Phi\underline{\epsilon}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}}\right]$$

– As $\hat{b} < b \leq \tilde{b}$ where $\tilde{b} \equiv ((\Phi+\rho\alpha_1^{i,u})y_{\max}^i + \kappa y_1^g)/\alpha_1^{i,u}$, we have $\epsilon_{\min} < \underline{\epsilon} \leq \epsilon_{\max} < \bar{\epsilon}$ and now the only way for i to repay its debts is with a transfer from g.

$$D(b) = \beta\left(b(1-\pi)(1-G(\underline{\epsilon})) + \rho\bar{y}_1^i\left(\pi\int_{\underline{\epsilon}(b)}^{\epsilon_{\max}}\epsilon dG(\epsilon) + \int_{\epsilon_{\min}}^{\underline{\epsilon}(b)}\epsilon dG(\epsilon)\right)\right) + \overline{\lambda}^i$$

The derivative satisfies:

$$D'(b) = \beta\left[(1-\pi)(1-G(\underline{\epsilon})) - (1-\pi)g(\underline{\epsilon})\frac{\Phi\underline{\epsilon}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}}\right]$$

Evaluating this expression at $b = \hat{b}^+$, there is an *upwards discontinuity* in the Laffer curve:

$$\begin{aligned} D'(\hat{b}^+) - D'(\hat{b}-) \quad &= \quad \beta\left(\hat{b} - \rho y_{\max}^i\right)\pi g(\epsilon_{\max})\left.\frac{d\bar{\epsilon}}{db}\right|_{b=\hat{b}} \\ &= \quad \beta\pi\frac{\Phi g(\epsilon_{\max})\epsilon_{\max}}{\Phi+\rho(1-\alpha_1^{i,i})} \geq 0 \end{aligned}$$

This upwards discontinuity arises because, at $b = \hat{b}$, an infinitesimal increase in debt pushes $\bar{\epsilon}$ above $\epsilon_{\max}$. The increase in the threshold becomes inframarginal and does not affect the value of the debt anymore (since the realizations where $\epsilon > \bar{\epsilon}$ cannot be achieved anymore).
At $b = \tilde{b}$, the derivative of the Laffer curve satisfies:

$$D'(\tilde{b}^-) \quad = \quad -\beta(1-\pi)g(\epsilon_{\max})\frac{\Phi\epsilon_{\max}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}} \leq 0$$

so the peak of the Laffer curve occurs necessarily at or before $\tilde{b}$.
The second derivative satisfies:

$$D''(b) \quad = \quad -\beta(1-\pi)\frac{d\underline{\epsilon}}{db}\left[g(\underline{\epsilon}) + \frac{\Phi}{\Phi+\rho\alpha_1^{i,u}}g(\underline{\epsilon}) + g'(\underline{\epsilon})\frac{\Phi\underline{\epsilon}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}}\right]$$

which is still negative under assumption 4.
The discontinuity at $\hat{b}$ could be problematic for our optimization problem. Consequently, we make assumptions to ensure that the peak of the Laffer curve occurs at or before $\hat{b}$. A sufficient

assumption is that $D'(\hat{b}^+) < 0$.

Assumption 5 *We assume that the parameters of the problem are such that*

$$D'(\hat{b}^+) = \beta(1-\pi)\left[1 - G(\underline{\epsilon}) - g(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha_1^{i,u}}\right] < 0$$

Under this assumption, the Laffer curve reaches its maximum at $0 < b_{\max} < \hat{b}$ such that $0 \in \partial D(b_{\max})$, where $\partial D(b)$ is the sub-differential of the Laffer curve at b. The peak of the Laffer curve cannot be reached at $\hat{b}$ or beyond since $D'(\hat{b}^-) < D'(\hat{b}^+) < 0$, so $0 \notin \partial D(\hat{b})$ and $D''(b) < 0$ for $b < \tilde{b}$. It follows immediately that $b_{\max} < \hat{b}$.

The economic interpretation of this assumption is that we restrict the problem so that the maximum revenues that i can generate by issuing debt in period 0 do not correspond to levels of debt so elevated that no realization of ϵ would allow i to repay on its own. In other words, the implicit transfer and the recovery value of debt are limited.

– As $b > \tilde{b}$ we have $\epsilon_{\max} < \underline{\epsilon}$ so that default is inevitable, even with transfers and the Laffer curve becomes:

$$D(b) = \beta\rho\bar{y}_1^i + \overline{\lambda}^i$$

which does not depend on the debt level. Note that there is an upwards discontinuity at $\tilde{b}$ since $D'(b) = 0$ for $b > \tilde{b}$.

To summarize, under assumptions 1-5, the Laffer curve reaches its peak at $b_{\max}$ with $\bar{b} \leq b_{\max} < \hat{b}$. The Laffer curve is continuous, convex and exhibits two (downward) discontinuities of $D'(b)$ on the interval $[0, b_{\max}]$. Since i will never locate itself on the 'wrong side' of the Laffer curve ($b > b_{\max}$), we can safely ignore the non-convexity associated with the upward discontinuities of the $D'(b)$ at $\hat{b}$ and $\tilde{b}$.

- For the sake of completeness, the remaining discussion describes what happens if $\bar{b} > \hat{b}$ (the reverse condition on the parameters). In that case, as b increases, the country stops being able to repay on its own first. This leads to a somewhat implausible case where the only reason debts are repaid is because of the transfer. We would argue that this is not a very interesting or realistic case.

  – When $\underline{b} < b \leq \hat{b} < \bar{b}$. In that case, we have $\underline{\epsilon} < \epsilon_{\min} \leq \bar{\epsilon} < \epsilon_{\max}$. When $b = \hat{b}$, $\underline{\epsilon} < \epsilon_{\min} < \bar{\epsilon} = \epsilon_{\max}$. Default can occur if $\epsilon_1^i \leq \bar{\epsilon}$ and ex-post transfers are forbidden. It follows that

$$D(b_1) = \beta[b_1(1 - \pi G(\bar{\epsilon})) + \rho\bar{y}_1^i\pi\int_{\epsilon_{\min}}^{\bar{\epsilon}}\epsilon dG(\epsilon)] + \bar{\lambda}^i$$

and the slope of the Laffer curve is given by

$$D'\left(b\right)=\beta\left[1-\pi G\left(\bar{\epsilon}\right)-\frac{\pi\bar{\epsilon}g\left(\bar{\epsilon}\right)\Phi}{\Phi+\rho(1-\alpha_1^{i,i})}\right]$$

As before, default is a direct consequence of the commitment *not* to bail-out country i in period $t=1$. The derivative of the Laffer curve is discontinuous at $b=\underline{b}$ if the distribution of shocks is such that $g\left(\epsilon_{\min}\right)>0$ and $\pi>0$.[22]

Under the same assumptions as before, the Laffer curve slopes up at $b=\underline{b}$.

The second derivative of the Laffer curve is:

$$D''(b)=-\beta\pi\frac{d\bar{\epsilon}}{db}\left[g(\bar{\epsilon})+\frac{\Phi}{\Phi+(1-\alpha_1^{i,i})\rho}(g(\bar{\epsilon})+\bar{\epsilon}g'(\bar{\epsilon}))\right]$$

and we can to ensure that $D''(b)<0$ with:

$$\frac{\epsilon g'(\epsilon)}{g(\epsilon)}>-2$$

– When $\hat{b}<b<\bar{b}$, we have $\underline{\epsilon}\leq\epsilon_{\min}<\epsilon_{\max}<\bar{\epsilon}$. It follows that

$$D(b) \quad = \quad \beta b(1-\pi)+\beta\pi\rho\bar{y}_1^i+\overline{\lambda}^i$$

which has a constant positive slope $\beta(1-\pi)$. At $b=\hat{b}$ the slope is discontinuous, with

$$D'\left(\hat{b}^{-}\right)=\beta\left[1-\pi-\frac{\pi\epsilon_{\max}g\left(\epsilon_{\max}\right)\Phi}{\Phi+\rho(1-\alpha_1^{i,i})}\right]$$

so there is an upwards discontinuity in the slope at $b=\hat{b}$.

– for $\bar{b}<\tilde{b}$ we have $\epsilon_{\min}<\underline{\epsilon}<\epsilon_{\max}<\bar{\epsilon}$ and it is now possible to default even with optimal transfers. The Laffer curve satisfies

$$D\left(b_1\right)=\beta\left[b_1\left((1-\pi)(1-G\left(\underline{\epsilon}\right)\right)+\rho\bar{y}_1^i\left(\pi\int_{\underline{\epsilon}}^{\epsilon_{\max}}\epsilon dG\left(\epsilon\right)+\int_{\epsilon_{\min}}^{\underline{\epsilon}}\epsilon dG\left(\epsilon\right)\right)\right]+\bar{\lambda}^i$$

[22] To see this, observe that: $D'(\underline{b}^{+})=\beta\left[1-\frac{\pi\epsilon_{\min}g(\epsilon_{\min})\Phi}{\Phi+\rho(1-\alpha_1^{i,i})}\right]<\beta$ when $g(\epsilon_{\min})>0$ and $\pi>0$.

with slope:

$$D'(b_1) \quad = \quad \beta(1-\pi)\left[(1-G(\underline{\epsilon})-g(\underline{\epsilon})\frac{\Phi\underline{\epsilon}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}}\right]$$

One can check that the slope of the Laffer curve is discontinuous also at $b = \bar{b}$ as long as $\pi < 1$ and $g(\epsilon_{\min}) > 0$ with:

$$D'(\bar{b}^+) - D'(\bar{b}^-) = -\beta(1-\pi)g(\epsilon_{\min})\frac{\Phi\epsilon_{\min}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}} < 0$$

At $b = \tilde{b}$, the derivative satisfies:

$$D'(\tilde{b}^-) \quad = \quad -\beta(1-\pi)g(\epsilon_{\max})\frac{\Phi\epsilon_{\max}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}} < 0$$

so the peak of the Laffer curve *needs* to occur before $\tilde{b}$.

The second derivative satisfies:

$$D''(b) \quad = \quad -\beta(1-\pi)\frac{d\underline{\epsilon}}{db}\left[g(\underline{\epsilon})+\frac{\Phi}{\Phi+\rho\alpha_1^{i,u}}g(\underline{\epsilon})+g'(\underline{\epsilon})\frac{\Phi\underline{\epsilon}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha_1^{i,u}}\right]$$

which is still negative as long as $g'(\underline{\epsilon})$ is not too negative.

- As $b > \tilde{b}$ we have $\epsilon_{\max} < \underline{\epsilon}$ so that default is inevitable, even with transfers and the Laffer curve becomes:

$$D(b) = \beta\rho\bar{y}_1^i + \overline{\lambda}^i$$

which does not depend on the debt level.

B Optimal Debt

Let's consider the rollover problem of country i. The first order condition is

$$\begin{aligned} 0 \quad &\in \quad \mu_1 + (1-\alpha_1^{i,i})\partial D(b_1^i)(1+\nu_0) - \beta(1-G(\bar{\epsilon}))(1-\alpha_1^{i,i}) \\ &\nu_0 c_0^i = 0 \\ &\mu_1 b_1^i = 0 \end{aligned}$$

We consider first an interior solution and ignore the non-continuity of $D'(b)$ at $\underline{b}$ and $\bar{b}$. The first-order condition becomes:

$$D'(b_1^i) = \beta\,(1 - G(\bar{\epsilon})) \tag{B.1}$$

Both sides of this equation are decreasing in b.

- Consider first the region $0 \leq b_1^i < \underline{b}$. Over that range, debt is safe: $D'(b) = \beta$ and $G(\bar{\epsilon}) = 0$. The first order condition is trivially satisfied: since debt is safe, risk neutral agents price the debt at β and i is indifferent as to the amount of debt it issues as long as it can ensure positive consumption.
- Consider now the interval $\underline{b} < b_1^i < \bar{b}$. We need to consider two cases.
  - when $\pi = 0$, g always bails out i and i's debt is safe. This implies $D'(b_1^i) = \beta$ and

    $$D'(b) - \beta(1 - G(\bar{\epsilon})) = \beta G(\bar{\epsilon}) > 0$$

    so there is no solution in that interval: i would always want to issue more debt.
  - when $\pi = 1$, i defaults when $b > \underline{b}$. Going back to the definition of $D'(b_1^i)$ and $\bar{\epsilon}$ we can check that

    $$D'(b) - \beta(1 - G(\bar{\epsilon})) = -\beta\frac{\Phi}{\Phi + \rho(1 - \alpha_1^{i,i})}g(\bar{\epsilon})\bar{\epsilon} < 0$$

    from which it follows that there is no solution in that interval: i would always want to issue less debt to remain safe.
  - In the intermediate case where $0 < \pi < 1$, it is possible to find a solution to the first-order condition. However, under reasonable conditions the second-order condition of the optimization problem will not be satisfied. This will be the case if $D'(b) - \beta(1 - G(\bar{\epsilon})$ is increasing. A sufficient condition is that g/G is monotonously decreasing. To see this, observe that for $\underline{b} < b \leq \bar{b}$, we have $\underline{\epsilon} < \epsilon_{\min}$ and therefore we can write:

    $$D'(b) - \beta(1 - G(\bar{\epsilon})) = \beta(1 - \pi)G(\bar{\epsilon})\left[1 - \frac{\pi}{1 - \pi}(b - \rho\bar{y}_1^i\bar{\epsilon})\frac{g(\bar{\epsilon})}{G(\bar{\epsilon})}\frac{d\bar{\epsilon}}{db}\right]$$

    The term in brackets is increasing in $\bar{\epsilon}$ when g/G is decreasing. If this condition is satisfied, then there is no solution in the interval $(\underline{b}, \bar{b})$. [Note: this condition is satisfied for a uniform distribution.]
- Consider next the interval $\bar{b} \leq b < \hat{b}$. We already know under the assumptions laid out in section A that we only need to consider the subinterval $(\bar{b}, b_{\max})$ where $b_{\max}$ is the value of the debt that maximizes period 1 revenues. Let's consider the various values of π again:
  - for $\pi = 0$, we have $D'(\bar{b}^-) = \beta$ and $D'(b_{\max}) = 0$. Since $D'(b) - \beta(1 - G(\bar{\epsilon}))$ is continuous over that interval, then there is at least one solution to the first-order condition, possibly at

$b = \bar{b}$. This solution is unique if $D'(b) - \beta(1 - G(\bar{\epsilon}))$ is strictly decreasing over that interval. Recall that over that interval we have:

$$\begin{aligned} D'(b) - \beta(1 - G(\bar{\epsilon})) &= \beta \left[G(\bar{\epsilon}) - G(\underline{\epsilon}) - g(\underline{\epsilon})(b - \rho \bar{y}_1^i \underline{\epsilon}) \frac{d\underline{\epsilon}}{db} \right] \\ &= \beta \left[G(\bar{\epsilon}) - G(\underline{\epsilon}) - g(\underline{\epsilon}) \frac{\Phi \underline{\epsilon} + \kappa y_1^g / \bar{y}_1^i}{\Phi + \rho \alpha_1^i, u} \right] \end{aligned}$$

The condition that $D'(b) - \beta(1 - G(\bar{\epsilon}))$ is decreasing over this range is satisfied for a uniform distribution if $\alpha_1^{i,g}$ is not too high.

Let's denote the unique solution b_{opt}. If $D'(\bar{b}^+) < \beta(1 - G(\bar{\epsilon}))$ then the solution is $b_{opt} = \bar{b}$.

– for $\pi = 1$ (no bailout), we can check that in that interval we can write

$$D'(b) - \beta(1 - G(\bar{\epsilon})) = -\beta g(\bar{\epsilon})(b - \rho \bar{y}_1^i \bar{\epsilon}) \frac{d\bar{\epsilon}}{db} < 0$$

Since $D'(\bar{\epsilon}^+) < \beta(1 - G(\bar{\epsilon}))$, it follows that there is no solution over that interval.

– For intermediate values of π, as long as π is not too high, we will have a unique solution b_{opt} as before. b_{opt} is decreasing in π for $\pi < \pi_c$. Above this critical value, this equilibrium disappears and the only remaining solutions are for $b \leq \underline{b}$. π_c is characterized by the condition that $D'(\bar{b}^-) = \beta(1 - G(\bar{\epsilon}))$. Substituting, we obtain:

$$\pi_c = \frac{G(\bar{\epsilon})}{G(\bar{\epsilon}) + \frac{\Phi g(\bar{\epsilon}) \bar{\epsilon}}{\Phi + \rho(1 - \alpha_1^{i,i})}}$$

In the case where there is no recovery, the formula for π_c simplifies to

$$\pi_c = \frac{1}{1 + g(\bar{\epsilon}) \bar{\epsilon} / G(\bar{\epsilon})}$$

C Debt Monetization

This appendix provides a full characterization of the different cases that arise with possible debt monetization within a monetary union. They depend on the output shock realization ϵ_1^i and on the ranking of the output thresholds

- **No default, no monetization, no transfer**. Comparison made when $z = 0$ in no default and default. Necessary conditions on output shock:

$$\begin{aligned}
\epsilon_1^i &> \frac{b_1^i \alpha^{iu} - \kappa y_1^g}{\Phi \overline{y}_1^i} \equiv \overline{\epsilon}'' \quad \text{ECB and g prefer no default to default with } z=0 \text{ in both cases} \\
\epsilon_1^i &> \frac{b_1^i \alpha^{iu} + b_1^g \alpha^{gu}}{\delta \overline{y}_1^i} - \frac{y_1^g}{\overline{y}_1^i} \equiv \overline{\overline{\epsilon}} \quad \text{ECB prefers } z=0 \text{ in no default} \\
\epsilon_1^i &> \frac{\alpha^{gu} b_1^g - \delta y_1^g}{\delta \overline{y}_1^i} \equiv \hat{\epsilon} \quad \text{ECB chooses } z=0 \text{ in case of default} \\
\epsilon_1^i &> \frac{b_1^i \left(1-\alpha^{ii}\right)}{\Phi \overline{y}_1^i} \equiv \overline{\epsilon}' \quad i \text{ repays with zero transfer and } z=0
\end{aligned}$$

- **No default, no monetization, positive transfer** Necessary conditions on output shock:

$$\begin{aligned}
\epsilon_1^i &> \overline{\epsilon}'' \quad \text{ECB and g prefer no default to default with } z=0 \text{ in both cases} \\
\epsilon_1^i &> \overline{\overline{\epsilon}} \quad \text{ECB prefers } z=0 \text{ in case of no default} \\
\epsilon_1^i &< \overline{\epsilon}' \quad i \text{ repays only with transfer and } z=0
\end{aligned}$$

- **No default, monetization at maximum rate, no transfer** Comparison made when $t^{ECB} = \overline{t}$ in no default and $t^{ECB} = 0$ in case of default.

$$\begin{aligned}
\epsilon_1^i &< \overline{\overline{\epsilon}} \quad \text{ECB prefers } z=\overline{z} \text{ in no default} \\
\epsilon_1^i &> \frac{\left(1-\alpha^{ii}\right) b_1^i \left(1-\overline{t}\right) + \alpha^{gi} b_1^g \overline{t}}{\left(\Phi - \delta \overline{t}\right) \overline{y}_1^i} \equiv \tilde{\epsilon} \quad i \text{ repays with zero transfer with } z=\overline{t}
\end{aligned}$$

- **No default, monetization at maximum rate, positive transfer** Comparison made when $z = \overline{z}$ in no default and $z = 0$ in case of default.

$$\begin{aligned}
\epsilon_1^i &< \overline{\overline{\epsilon}} \quad \text{ECB prefers } z=\overline{z} \text{ in no default} \\
\epsilon_1^i &> \frac{\alpha^{iu} b_1^i \left(1-\overline{t}\right) - \alpha^{gu} b_1^g \overline{t} - y_1^g \left(\kappa - \delta \overline{t}\right)}{\left(\Phi - \delta \overline{t}\right) \overline{y}_1^i} \equiv \underline{\epsilon}' \quad g \text{ prefers no default, transfer and } z=\overline{z} \\
\epsilon_1^i &< \frac{\left(1-\alpha^{i,i}\right) b_1^i \left(1-\overline{z}\right) + \alpha^{gi} b_1^g \overline{t}}{\left(\Phi - \delta \overline{z}\right) \overline{y}_1^i} \equiv \tilde{\epsilon} \quad i \text{ repays only with transfer with } z=\overline{z}
\end{aligned}$$

In this case, the transfer is the minimum that leaves i indifferent between default and no default (see equation 20).

- **Default, no monetization, no transfer**

  Comparison made when $z = \overline{z}$ in no default and $z = 0$ in case of default.

$$
\begin{aligned}
\epsilon_1^i &< \overline{\overline{\epsilon}} \quad \text{ECB prefers } z = \overline{z} \text{ in no default} \\
\epsilon_1^i &< \frac{\alpha^{iu} b_1^i \left(1 - \overline{z}\right) - \alpha^{gu} b_1^g \overline{z} - y_1^g \left(\kappa - \delta \overline{z}\right)}{\left(\Phi - \delta \overline{z}\right) \overline{y}_1^i} \equiv \underline{\epsilon}' \quad g \text{ prefers default, no transfer} \\
\epsilon_1^i &> \frac{\alpha^{gu} b_1^g - \delta y_1^g}{\delta \overline{y}_1^i} \equiv \hat{\epsilon} \quad \text{ECB chooses } z = 0 \text{ in default}
\end{aligned}
$$

- **Default, monetization, no transfer**

  Comparison made with $z = \overline{z}$ in both cases:

$$
\begin{aligned}
\epsilon_1^i &< \frac{\alpha^{iu} b_1^i \left(1 - \overline{t}\right) - \kappa y_1^g}{\Phi \overline{y}_1^i} \equiv \underline{\epsilon}'' \quad g \text{ prefers default, no transfer and } z = \overline{z} \\
\epsilon_1^i &< \frac{\alpha^{g,u} b_1^g - \delta y_1^g}{\delta \overline{y}_1^i} \equiv \hat{\epsilon} \quad \text{ECB chooses } z = \overline{z} \text{ in default}
\end{aligned}
$$

There are therefore 7 thresholds for output realizations: $\overline{\overline{\epsilon}}; \overline{\epsilon}'; \overline{\epsilon}''; \hat{\epsilon}; \tilde{\epsilon}; \underline{\epsilon}'; \underline{\epsilon}''$. In addition, we assume there is a minimum and maximum output realization ϵ^{max} and ϵ^{min}.

We can rank some of them under the assumption that $\Phi > \kappa > \delta$:

$\underline{\epsilon}' < \overline{\epsilon}'$

$\underline{\epsilon}'' < \overline{\epsilon}''$

$\hat{\epsilon} < \overline{\epsilon}'$

$\underline{\epsilon}'' < \underline{\epsilon}'$

$\tilde{\epsilon} > \underline{\epsilon}'$

To simplify the analysis, we focus on parameter configurations that are most interesting and most plausible for the situation of the eurozone, we rank these thresholds based on the following general assumptions : b_1^g is small relative y_1^g and to b_1^i.

Assumptions on parameters:

- $\hat{\epsilon} < \epsilon^{min}$ which insures that the ECB will choose a zero inflation rate in the case of default. This excludes the case of strong fiscal dominance.

$$
\frac{b_1^g}{y_1^g} < \frac{\delta}{\alpha^{gu}} \left(1 + \frac{y_1^i}{y_1^g} \epsilon^{min}\right)
$$

  The condition on parameters is such that the debt to GDP ratio for g is small enough.

  We then examine two cases: monetary dominance and weak fiscal dominance/.

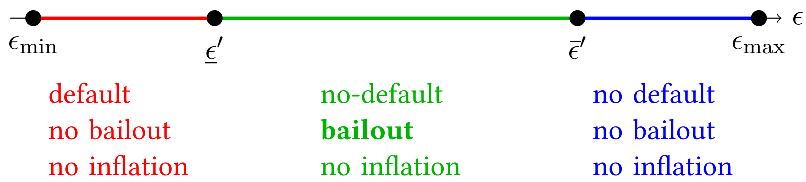

Figure 9: Bailout under Monetary Dominance

- Monetary dominance: If $\overline{\overline{\epsilon}} < \underline{\epsilon}'$, then when transfers are possible, the ECB never chooses positive inflation. This case is valid with high y_1^g and δ, and low b_1^g.
- Weak fiscal dominance: If $\overline{\epsilon}' > \overline{\overline{\epsilon}} > \underline{\epsilon}'$, then when transfers are possible, the ECB may choose positive inflation. This is the case with intermediate levels of y_1^g and δ, and low b_1^g.

Under monetary dominance, the possible equilibria are shown in figure 9. Only binding thresholds are indicated. Monetary policy does not affect transfers and the decision whether to default or not.

Under weak fiscal dominance, possible equilibria are shown in figure 7. In this case, when output realization in i is sufficiently high ($\epsilon_1^i > \overline{\epsilon}'$), there is no default, no inflation and no transfer. If it is lower, i requires a transfer in order not to default ($\overline{\epsilon}' > \epsilon_1^i > \overline{\overline{\epsilon}}$) but there is no inflation. For $\overline{\overline{\epsilon}} > \epsilon_1^i > \underline{\epsilon}'$), the ECB partly inflates the debt, g makes a transfer to avoid the default. For $\epsilon_1^i < \underline{\epsilon}'$, the default is optimal and there is no more incentive to inflate the debt.

Finally, when transfers are excluded (and $\overline{\overline{\epsilon}} < \underline{\epsilon}'$ so that monetary dominance applies with zero inflation in presence of transfers) the possible equilibria are shown in figure 8. When output realization in i is sufficiently high ($\epsilon_1^i > \overline{\epsilon}'$), there is no default and no inflation. If it is lower, i requires a positive inflation rate in order not to default ($\overline{\epsilon}' > \epsilon_1^i > \tilde{\epsilon}$). For $\epsilon_1^i < \tilde{\epsilon}$), the default is optimal and there is no more incentive to inflate the debt.

C.1 The Laffer curve with debt monetization when transfers are allowed

With potential monetization g debt expected payoff depends on the expected inflation rate chosen by the ECB.

We derive an expression for the fiscal revenues $D(b_1^i) = b_1^i/R^i$ raised by the government of country i in period $t = 0$:

$$
\begin{aligned}
D(b_1^i) = \frac{b_1}{R^i} &= \beta \mathcal{P} b_1^i + \overline{\lambda}^i \\
&= \beta b_1^i \left(1 - \pi_d\right)\left(1 - E\left(z\right)\right) + \overline{\lambda}^i
\end{aligned}
$$

where again we assumed zero recovery to simplify and where $E\left(z\right)$ is the expected inflation rate.

- When the debt level is low (i.e. $b_1^i \leq \underline{b}' \equiv y_{\min}^i \left(\Phi/(1-\alpha_1^{i,i})\right)$ or $\overline{\epsilon}' < \epsilon^{min}$). In that case, the i government would repay its debts even without any transfers for all realizations of output. The debt is safe and there is no default nor monetization risk. This is the same threshold as without monetization (with zero recovery rate)

- When $\underline{b}' < b_1^i \leq \overline{b}' \equiv (\Phi y_{\min}^i + \kappa y_1^g)/\alpha_1^{i,u}$ and $b_1^i \leq \overline{\overline{b}} \equiv \left[\delta\left(y_{\min}^i + y_1^g\right) - b_1^g \alpha^{gu}\right]/\alpha_1^{i,u}$. Or $\overline{\epsilon}' > \epsilon^{min} > \overline{\epsilon}''$ and $\overline{\overline{\epsilon}} < \epsilon^{min}$: In that case, the level of debt is sufficiently low that it is optimal for g to make a transfer in the case of zero inflation. It also sufficiently low at the eurozone level that the ECB indeed prefers zero inflation. The debt is fully safe in this zone. The binding constraint is the zero inflation constraint ($b_1^i \leq \overline{\overline{b}}$) if: $\overline{\overline{b}} < \overline{b}'$ or: $(\Phi - \delta)\, y_{\min}^i - (\kappa - \delta)\, y_1^g + b_1^g \alpha^{gu} > 0$ which applies since we assume that $\Phi > \kappa > \delta$.

- When $\overline{\overline{b}} < b_1^i \leq \overline{\overline{b}}' \equiv \left[(\Phi - \delta\overline{z})\, y_{\min}^i + (\kappa - \delta\overline{z})\, y_1^g + b_1^g \alpha^{g,u}\overline{z}\right] / \left(\alpha_1^{i,u}(1-\overline{z})\right)$ or $\overline{\overline{\epsilon}} > \epsilon^{min} > \underline{\epsilon}'$. In that case, the level of debt is sufficiently low that it is optimal for g to make a transfer. It sufficiently large at the eurozone level that the ECB chooses inflation at rate $\overline{z}$. The debt is safe in this zone in the sense that there is no default but it can be inflated away. This is possible if $\overline{\overline{b}} < \overline{\overline{b}}'$ which is indeed the case for $\Phi > \kappa > \delta$. Hence, in this case:

$$D(b_1^i) = \beta b_1^i \left[1 - \overline{z} G\left(\overline{\overline{\epsilon}}\right)\right] + \overline{\lambda}^i$$

with slope:

$$D'(b_1^i) = \beta \left[1 - \overline{z} G\left(\overline{\overline{\epsilon}}\right) - b_1^i \overline{z} g\left(\overline{\overline{\epsilon}}\right) \frac{\alpha^{i,u}}{\delta \overline{y}_1^i}\right].$$

and $\overline{\overline{\epsilon}} \equiv \frac{b_1^i \alpha^{iu} + b_1^g \alpha^{gu}}{\delta \overline{y}_1^i} - \frac{y_1^g}{\overline{y}_1^i}$. Note that at $\overline{\overline{b}}$ there is a discontinuity. The slope then decreases with b_1^i and could be negative if $\overline{t}$ is too high. Note also that for these relatively low levels of debt where there is no default risk, expected potential debt monetization can only increase the yield as it cannot reduce the probability of default.

- When $\overline{\overline{b}}' < b_1^i < \overline{\overline{b}}'' \equiv \left[\delta\left(y_{\max}^i + y_1^g\right) - b_1^g \alpha^{gu}\right]/\alpha_1^{i,u}$ or $\epsilon^{max} > \underline{\epsilon}' > \epsilon^{min}$. In that case, the level of debt is sufficiently high that both default and inflation are possible depending on the output realization. Hence, in this case:

$$D(b_1^i) = \beta b_1^i \left[1 - \overline{z}\left[G\left(\overline{\overline{\epsilon}}\right) - G\left(\underline{\epsilon}'\right)\right] - G\left(\underline{\epsilon}'\right)\right] + \overline{\lambda}^i \tag{C.1}$$

with slope:

$$D'(b_1^i) = \beta \left[1 - \overline{z} G\left(\overline{\overline{\epsilon}}\right) - (1-\overline{z})\, G\left(\underline{\epsilon}'\right) - b_1^i \overline{z} g\left(\overline{\overline{\epsilon}}\right) \frac{\alpha^{i,u}}{\delta \overline{y}_1^i} - b_1^i g\left(\underline{\epsilon}'\right) \frac{\alpha^{i,u}(1-\overline{z})^2}{(\Phi - \delta\overline{z})\, \overline{y}_1^i}\right]$$

There is another discontinuity at $\overline{\overline{b}}'$ and the slope for levels of debt above this threshold is lower and may be negative for high levels of $\overline{z}$. Note however that an increase of the monetization rate (estimated at $\overline{z} = 0$) increases the debt threshold $\overline{\overline{b}}'$ above which default becomes possible if $\Phi > \delta$ and $\kappa > \delta$. The reason is that a higher monetization reduces the transfer g needs and is willing to give to i to avoid a default for a given output realization. Hence at intermediate levels of debt, expected debt monetization may actually reduce the riskiness (yield) of the debt issued by i. There are two opposite effects of $\overline{t}$: one is that investors ask for a higher yield to be compensated for inflation, the other one is that the default risk falls and this induces lower yields. This second effect is large if the decrease of $\underline{\epsilon}'$ is large when $\overline{z}$ increases. This is potentially an interesting case because it implies that expected inflation makes debt safer but only because it increases the probability of a transfer.

Similarly to the case without inflation, we make assumptions to ensure that the peak of the Laffer curve occurs at or before the level of debt $\hat{b}'$ such that no realization of ϵ would allow i to repay on its own. This requires:

$$D'(\hat{b}'^{+}) = \beta \left[1 - \overline{z} G\left(\overline{\overline{\epsilon}}\right) - (1-\overline{z})\, G\left(\underline{\epsilon}'\right) - b_1^i \overline{z} g\left(\overline{\overline{\epsilon}}\right) \frac{\alpha^{i,u}}{\delta \overline{y}_1^i} - b_1^i g\left(\underline{\epsilon}'\right) \frac{\alpha^{i,u}\,(1-\overline{z})^2}{(\Phi - \delta \overline{z})\, \overline{y}_1^i} \right] < 0$$

- When $\overline{\overline{b}}'' < b_1^i < \overline{\overline{b}}''' \equiv \left[(\Phi - \delta \overline{z})\, y_{\max}^i + (\kappa - \delta \overline{z})\, y_1^g + b_1^g \alpha^{g,u} \overline{z} \right] / \left(\alpha_1^{i,u} (1-\overline{z}) \right)$ or $\overline{\overline{\epsilon}} > \epsilon^{max} > \underline{\epsilon}'$. In that case, the level of debt is sufficiently high that inflation is necessary in case of no default and default is possible depending on the output realization. Hence, in this case:

$$D(b_1^i) = \beta b_1^i \,(1-\overline{z})\,(1 - G\left(\underline{\epsilon}'\right)) + \overline{\lambda}^i$$

with slope:

$$D'(b_1^i) = \beta\,(1-\overline{z}) \left[1 - G\left(\underline{\epsilon}'\right) - b_1^i g\left(\overline{\overline{\epsilon}}\right) \frac{\alpha^{iu}}{\delta \overline{y}_1^i} \right]$$

There is again a discontinuity and the possibility that the Laffer curve is downward sloping.

- When $b_1^i > \overline{\overline{b}}'''$ or $\underline{\epsilon}' > \epsilon^{max}$. In that case, the level of debt is so high that default is optimal for any output realization. Hence, in this case:

$$D(b_1^i) = 0$$

C.1.1 Optimal Debt with monetization.

The consolidated budget constraint for i in period 0 is (assuming no inflation in period 0):

$$c_0^i + \alpha_1^{i,i} b_1^i / R^i + \alpha_1^{s,i} b_1^s / R^* \quad = \quad (y_0^i + \tau_0 + b_0^{i,i} - b_0^i + b_0^{s,i}) + b_1^i / R^i$$

Note that with the parameter restrictions we made, the consolidated budget constraint for i in period 0 is exactly as before (with $\rho = 0$) in the sense that the combination of transfer and inflation tax is such that i is indifferent between default and no default so the consolidated budget constraint for period 1 remains identical:

$$c_{1,n}^i = y_1^i - b_1^i(1 - \alpha_1^{i,i}) + \alpha_1^{s,i} b_1^s \qquad \text{if} \quad \epsilon_1^i \geq \bar{\epsilon}' \text{ (i repays, no transfer)}$$
$$c_{1,d}^i = y_1^i(1 - \Phi) + \alpha_1^{i,s} b_1^s \qquad \text{if} \quad \epsilon_1^i < \bar{\epsilon}' \text{ (i defaults or receives a transfer with or without monetization)}$$

where we substituted the optimal transfer and observe that i achieves the same consumption level under default or when receiving a transfer, i.e. when $\epsilon_1^i < \bar{\epsilon}$.

Country i's government solves the following program:

$$\begin{aligned} \max_{b_1^i} \qquad & c_0^i + \beta \left(\int_{\epsilon_{\min}}^{\bar{\epsilon}'} c_{1,d} dG(\epsilon) + \int_{\bar{\epsilon}'}^{\epsilon_{\max}} c_{1,n} dG(\epsilon) \right) \\ \text{s.t.} \qquad & c_0^i \geq 0 \\ & b_1^i / R^i = D(b_1^i) \\ & 0 \leq b_1^i \leq b_{\max} \end{aligned}$$

where c_0^i and c_1^i are defined above. With monetization the FOC is identical:

$$\beta \mu_1 + (1 - \alpha_1^{i,i}) D'(b_1^i)(1 + \nu_0) = \beta(1 - G(\bar{\epsilon}'))(1 - \alpha_1^{i,i})$$

Note that $\bar{\epsilon}'$ does not depend on the monetization rate. We replicate the same analysis of optimal debt as without debt monetization We consider first an interior solution and ignore non-continuities The first-order condition becomes:

$$D'(b_1^i) = \beta \left(1 - G(\bar{\epsilon}')\right) \tag{C.2}$$

Both sides of this equation are decreasing in b.

- Consider first the region $0 \leq b_1^i < \underline{b}'$. Over that range, debt is safe (no default or monetization risk): $D'(b) = \beta$ and $G(\bar{\epsilon}') = 0$. The first order condition is trivially satisfied: since debt is safe, risk neutral agents price the debt at β and i is indifferent as to the amount of debt it issues as long as it can ensure positive consumption.

- Consider now the interval $\underline{b}' < b_1^i < \overline{\overline{b}}$. Remember we assume $\pi = 0$, so g always bails out i if necessary and i's debt is safe. This implies $D'(b_1^i) = \beta$ and i would always want to issue more debt.

- Consider next the interval $\overline{\overline{b}} \leq b < \overline{\overline{b}}'$. In this case (given $\pi = 0$), there is no default risk but potential inflation risk. We have $D'(\overline{\overline{b}}^{-}) = \beta$ and $D'(b_{\max}) = 0$. Since $D'(b) - \beta(1 - G(\overline{\epsilon}))$ is continuous over that interval, then there is at least one solution to the first-order condition, possibly at $\overline{\overline{b}}$. We have:

$$D'(b) - \beta(1 - G(\overline{\epsilon}')) = \beta \left[G(\overline{\epsilon}') - \overline{t} G(\overline{\overline{\epsilon}}) - \overline{z} g(\overline{\overline{\epsilon}}) \frac{\alpha^{i,u}}{\delta \overline{y}_1^i} \right] \tag{C.3}$$

If $D'(\overline{\overline{b}}^{+}) < \beta(1 - G(\overline{\epsilon}'))$ then the solution is $b'_{opt} = \overline{\overline{b}}$.

This is the case for high $\overline{z}$. Hence, for high $\overline{z}$, the optimal debt is $\overline{\overline{b}} < \overline{b}'$. This implies that the possibility of debt monetization actually induces the country to, ex ante, issue less debt. The reason is that the gains from debt monetization are captured in period 1 by g in the form of lower tranfers. The cost for i is that expected debt monetization $\overline{z}$ increases the cost of borrowing and therefore reduces the gain of issuing debt. Hence, in this case, i will issue debt but at a level such that there is no inflation risk for investors.

For low levels of $\overline{z}$, equation (C.3) may be zero or positive so that the optimal debt may be an interior solution with: $\overline{\overline{b}} < b'_{opt} \leq \overline{\overline{b}}'$. The reason is that there are two effects of debt monetization on optimal debt at low levels of $\overline{z}$. One reduces the incentive to issue debt and was explained above: the cost of issuing debt increases for i and the ex-post gains go to g. The other effect is the risk shifting that induces i to raise debt. Note that at this low level of debt (where default risk does not exist), debt monetization can only reduce the incentive to issue debt: the expression in equation (C.3) decreases with $\overline{z}$.

- $\overline{\overline{b}}' < b_1^i < \overline{\overline{b}}''$. In that case, the level of debt is sufficiently high that both default and inflation are possible depending on the output realization. We have :

$$D'(b) - \beta(1 - G(\overline{\epsilon}')) = \beta \left[G(\overline{\epsilon}') - \overline{z} G(\overline{\overline{\epsilon}}) - (1 - \overline{z}) G(\underline{\epsilon}') - \overline{z} g(\overline{\epsilon}') \frac{\alpha^{i,u}}{\delta \overline{y}_1^i} b_1^i - g(\underline{\epsilon}') \frac{\alpha^{i,u} (1 - \overline{z})^2}{(\Phi - \delta \overline{z}) \overline{y}_1^i} b_1^i \right] \tag{C.4}$$

This is a more complex case. There are three mechanisms at work: (i) risk shifting induces to issue more debt, (ii) the inflation risk increases the cost of issuing debt but (iii) with some default risk, the possibility of monetization also reduces the default risk for some output realizations.

C.1.2 The case without transfers

The budget constraint in period 1 of the households is as follows for i :

$$\begin{aligned} c_1^i &= y_1^i - b_1^i\left(1-\alpha^{i,i}\right) + \alpha^{s,i} b_1^s \quad \text{if } i \text{ repays without inflation} \\ c_1^i &= y_1^i\left(1-\delta\tilde{t}\right) - b_1^i\left(1-\alpha^{i,i}\right)(1-\tilde{z}) + \alpha^{g,i} b_1^g (1-\tilde{z}) + \alpha^{u,i} b_1^u \quad \text{if } i \text{ repays with inflation} \\ c_1^i &= (1-\Phi) y_1^i + \alpha^{s,i} b_1^s \quad \text{if } i \text{ defaults} \end{aligned}$$

Importantly, because of the absence of transfers, consumption in i is now different in a situation of default and no default.

- For levels of debt such that $\underline{b}' < b_1^i \leq \tilde{b}' \equiv \frac{y_{\min}^i(\Phi-\delta\overline{z})-\overline{z}\alpha^{g,i}b^g}{(1-\alpha^{i,i})(1-\overline{z})}$, (which corresponds to $\tilde{\epsilon} < \epsilon^{min} < \overline{\epsilon}'$) there is no default risk but an inflation risk. In this case:

$$D(b_1^i) = \beta b_1^i \left[1 - G\left(\overline{\epsilon}'\right)\right] + \beta b_1^i \int_{\epsilon_{\min}}^{\overline{\epsilon}'} \left(1-\tilde{z}(\epsilon)\right) dG(\epsilon) + \overline{\lambda}^i$$

with :

$$\tilde{z} = \frac{b_1^i\left(1-\alpha^{i,i}\right) - \Phi y_1^i}{b_1^i\left(1-\alpha^{i,i}\right) - \delta y_1^i - b_1^g \alpha^{g,i}}$$

Hence:

$$D'(b_1^i) = \beta\left[1 - G\left(\overline{\epsilon}'\right)\right] + \beta \int_{\epsilon_{\min}}^{\overline{\epsilon}'} \left(1-\tilde{z}(\epsilon)\right) dG(\epsilon) - \beta b_1^i \int_{\epsilon_{\min}}^{\overline{\epsilon}'} \frac{d\tilde{z}}{db_1^i} dG(\epsilon)$$

The marginal cost of issuing more debt in this interval is:

$$\beta\left(1-\alpha^{i,i}\right)\left[1 - G\left(\overline{\epsilon}'\right) + \int_{\epsilon_{\min}}^{\overline{\epsilon}'} \left[\left(1-\tilde{z}(\epsilon_i)\right) - b_1^i \frac{d\tilde{z}}{db_1^i} + \frac{\delta\epsilon_i}{1-\alpha^{i,i}} \frac{d\tilde{z}}{db_1^i}\right] dG(\epsilon)\right]$$

In this interval, the difference between the marginal gain and marginal cost is always negative, corresponding to the distortion cost of inflation:

$$-\beta\delta \int_{\epsilon_{\min}}^{\overline{\epsilon}'} \epsilon_i \frac{d\tilde{z}}{db_1^i} dG(\epsilon) < 0$$

as the expected inflation is perfectly priced in the interest rate. Hence, without transfers, even when the central bank can inflate the debt to avoid default, there is no risk shifting and the optimal level is x_0^i

- For levels of debt such that $b_1^i > \tilde{b}'$, (which corresponds to $\tilde{\epsilon} > \epsilon^{min}$) there is both default and inflation risk. In this case, the difference between the marginal gain and marginal cost of increasing debt is also always negative:

$$-\beta \left\{ g(\tilde{\epsilon}) \frac{d\tilde{\epsilon}}{db_1^i} \left(1 - \alpha^{i,i}\right) b_1^i (1 - \bar{z}) + \delta \int_{\epsilon_{\min}}^{\bar{\epsilon}'} \epsilon_i \frac{d\tilde{z}}{db_1^i} dG(\epsilon) + \frac{d\tilde{z}}{db_1^i} \alpha^{g,i} b_1^g \left[G(\bar{\epsilon}') - G(\tilde{\epsilon})\right] \right\} < 0$$