

# Financial Engineering and the Macroeconomy\*

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May 13, 2018

## Abstract

The volume of financial engineering has grown markedly across the world over the past few decades, for at least two reasons. On the supply side, technological improvement and regulation arbitrage have made complex security creation more cost effective. On the demand side, appetite for safe assets has increased. We describe a dynamic model of security creation where the macroeconomic impact of those changes can be characterized and quantified. We find that those shocks can cause large increases in costly security creation volumes. But the resulting impact on output, capital formation and TFP is generally small, and may well be negative. While financial engineering serves a fundamental and socially valuable role in our model, the impact of financial engineering booms on macroeconomic aggregates is limited.

Keywords: Endogenous Security Markets; Financial Engineering; Macroeconomic Aggregates

JEL codes: E44; O11

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\*We thank Julio Suarez at AFME, Sharon Sung at SIFMA and research assistance from Tristan Young. We also thank Roozbeh Hosseini, Robert Townsend, Vincenzo Quadrini, Gianluca Violante, André Silva, Grace Li, as well as seminar participants and attendees at the 2017 Midwest Macro Meetings, the 2017 Society for Economic Dynamics meetings, the 2017 Society for the Advancement of Economic Theory meetings, the 2017 North-American and European Econometric Society meetings, the University of Georgia and the Federal Reserve Bank of Atlanta for their generous comments. Pedro Amaral: [pamaral@fullerton.edu](mailto:pamaral@fullerton.edu), Dean Corbae: [dean.corbae@wisc.edu](mailto:dean.corbae@wisc.edu), Erwan Quintin: [equintin@bus.wisc.edu](mailto:equintin@bus.wisc.edu).

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# 1 Introduction

The volume of financial engineering – by which we mean the transformation of cash-flows to create securities that cater to the needs of heterogenous investors – has grown markedly across the world over the past few decades. In the United States, the cash-flows created by the real estate sector, as well as corporate assets and liabilities such as receivables and business loans are now routinely pooled and tranced into securities with different risk and liquidity characteristics. Figure 1 illustrates the growing importance of financial engineering by plotting the outstanding volume of Asset-Backed Securities (ABS), excluding housing-related securities. It also shows that within this asset class, Collateralized Loan Obligations (CLOs) – which are securities backed by business loans – have grown from virtually non-existent in the mid-1990’s to over half a trillion dollars in 2017.<sup>1</sup>

At least two concurrent phenomena have fueled the rise of financial engineering activities. First, technological improvements and regulatory arbitrage have made the activity cheaper.<sup>2</sup> Second, demand for the securities created via financial engineering – appetite for highly rated assets, in particular – has increased.<sup>3</sup> In this paper, we lay out an environment in which supply and demand shocks cause changes in the volume of costly security creation and use the resulting model to take on a simple question: What should we expect the impact of financial engineering booms to be on macroeconomic aggregates such as GDP, capital formation, and total factor productivity (TFP)?

Our model is a dynamic extension of [Allen and Gale \(1988\)](#)’s optimal security design model in which the production side of the economy aggregates up to a standard neoclassical model with aggregate uncertainty. The economy contains investors (households) who are

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<sup>1</sup>Although only shorter data is available for the Asset-Backed Commercial Paper (ABCP) market, we know that its volume doubled to reach over 1.2 trillion dollars between 2000 and 2007. That market collapsed in 2008 and has yet to recover. Figure 1 shows that the CLO market was much less affected by the crisis and has doubled in size since then.

<sup>2</sup>See [Allen and Gale \(1994\)](#), for an early review of factors behind the boom in financial innovation over the past few decades.

<sup>3</sup>See [Bernanke \(2011\)](#). As they put it, “Given the strength of demand for safe U.S. assets, it would have been surprising had there not been a corresponding increase in their supply.”

risk-neutral as well as investors who are highly risk-averse and have a high willingness to pay for safe securities. Absent transaction costs, it would be optimal for producers to sell the safe part of the stochastic cash-flows they generate to risk-averse agents and the residual claims to risk-neutral agents. But splitting cash-flows in this fashion is costly. Given this cost, producers choose which securities to create taking their market value – i.e the willingness by households to pay for these securities – as given. Given the resulting security menu at each possible history, households choose a consumption policy which in turn, pins down their willingness to pay for securities. In equilibrium, the resulting pricing kernel has to coincide with the kernel assumed by producers. [Allen and Gale \(1988\)](#) show that this fixed point problem always has a solution in their static environment. We show that the same holds in our dynamic extension.

We go on to fully characterize optimal security creation policies. First, it only makes sense to sell risk-free securities to risk-averse households, and producers who do choose to issue safe assets always issue as much of it as they can. Second, producers either retain (consume, literally speaking, in our model) residual cash-flows or sell them to risk-neutral households when the value of doing so exceeds the security creation cost. In our model, as in recent US data, security creation activities result in the production of safe securities backed by risky assets. Not surprisingly then, we find that lowering security creation costs or increasing the fraction of risk-averse agents result in an increase in costly security creation activities and, in particular, increased issuances of safe securities.

The production side of the model economy aggregates up to a standard neoclassical production function where conventionally-measured TFP is the average productivity of active producers. Keeping prices fixed, when security creation costs fall, the set of active producers is unchanged. This is because marginal producers, those just indifferent between operating or not, issue just one type of security, not both. It follows that only general equilibrium effects – endogenous changes in interest rates and wages resulting from the change in security creation costs – affect the set of active producers. As such effects tend to be small, this immediately implies that the connection between financial engineering booms and TFP is quantitatively

limited, which we illustrate via calibrated numerical simulations of our model.

We also find that while lowering security creation costs has a large impact on the fraction of producers who engage in security creation and the volume of securities so created, the resulting effect on output and capital formation is small. Again, producers who choose to engage in security creation after costs fall are, for the most part, producers who would have been active anyway. When there is a change in producer participation following the change, it tends to be small as it is the result of the transpiring general equilibrium effects.

Perhaps more surprisingly, the impact of lowering security creation costs on capital formation and output can even be negative. In our model, spending on securities is allocated to capital formation, producer rents, and security creation costs. While total spending on securities always rises following a decrease in the cost, so can the resources spent on security creation as more producers engage in it. Capital formation, hence GDP, can fall. We also find that when average output and capital formation do rise as financial engineering activities increase, TFP tends to fall. Put another way, when more financial engineering is associated with more output, it is also associated with lower TFP. This is because the increase in capital formation increases the participation of marginal producers, and those producers drag average TFP down.

Increases in financial engineering caused by an increase in the fraction of risk-averse agents also have small macroeconomic impacts. Furthermore, these demand-driven booms in financial engineering are even more likely to have a negative impact on output. In our first, supply-side experiment, when costs fall and more engineering takes place, the smaller cost per producer offsets the fact that more producers choose to bear the security creation cost. When the shock comes from the demand side, that offsetting effect is no longer active and financial engineering booms must imply that more resources are spent on security creation.

The two comparative statics experiments make very different predictions when it comes to security prices. The decrease in security creation costs causes the risk-free rate to go up, while a higher appetite for risk-free assets by investors causes the risk-free rate to fall. Given the steady fall in safe yields observed in the past two decades, these intuitive findings confirm

the view championed for instance by [Bernanke \(2011\)](#) that the recent rise in securitization in the United States has been largely demand-driven. Our model predicts that given this, one should not expect this rise to have contributed large increases in output, if any.

[Gennaioli, Shleifer, and Vishny \(2013\)](#) present a model where more demand for safe assets results in more securitization, more investment and more output when investors have rational expectations. In their model, security creation is free so that expanding financial engineering has no impact on resource use. Their main point, however, is that when investors fail to take into account small probability events (a behavior they term neglected risk, and a violation of rational expectations), the impact of financial engineering booms on output becomes ambiguous. They do lead to more investment and more output during expansions but, on the other hand, results in greater leverage by financial intermediaries which makes recessions more severe. We find that even when investors have rational expectations, booms in financial engineering are unlikely to be associated with large output gains.

Our paper is also related to the recent empirical literature that argues that the effect of financial development on growth and productivity becomes weaker, if not negative, at high levels of financial development.<sup>4</sup> [Arcand, Berkes, and Panizza \(2015\)](#), for instance, make the empirical case that once private credit reaches 100% of GDP, additional increases in private intermediation have a negative impact on growth. The standard explanation for this tapering proposed in those papers is that once the allocative benefits of better credit markets are exhausted, the nature of financial activity expansion changes. Whereas at early stages of development credit expansion leads to the funding of new and highly productive projects, eventually financial development emphasizes security engineering activities. Based for instance on the aforementioned paper by [Gennaioli, Shleifer, and Vishny \(2013\)](#), or classical arguments formalized by, e.g., [Tobin \(1984\)](#) that large financial sectors inefficiently draw skilled human capital away from the production sector,<sup>5</sup> they argue that too much finance

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<sup>4</sup>See [Sahay, Cihak, N'Diaye, Barajas, Pena, Bi, Gao, Kyobe, Nguyen, Saborowski, Svirydzenka, and Yousefi \(2015\)](#) for a recent review of the empirical literature.

<sup>5</sup>[Philippon and Reshef \(2013\)](#) make the case that skilled workers in Finance earn excessive rents.

may be detrimental to growth.

While they are consistent with a weak correlation between financial engineering activities and output at high level of development, the inference one should draw from our findings is quite different. Financial engineering serves a clear, beneficial social role in our framework. When the fraction of risk-averse agents increases, the economy optimally responds by creating more safe assets, even though this is a costly activity. A social planner who must bear the same security creation costs as our producers would respond in the same fashion. As in [Allen and Gale \(1988\)](#) (or in [Gennaioli, Shleifer, and Vishny \(2013\)](#), when investors have rational expectations) our equilibria are constrained-efficient. The points we make in this paper are strictly positive: there is no reason to expect a large positive impact of financial engineering booms on macroeconomic aggregates.

## 2 The environment

Consider an economy in which time is discrete. Each period, a mass one of two-period lived households is born. Each household is endowed with a unit of labor which they deliver inelastically in the first period of their life for a competitively determined wage. There are two types of households – type  $A$  and type  $N$  – that differ in terms of how they value consumption plans, as we will explain below. Denote the fraction of type  $A$  households by  $\theta$  while  $1 - \theta$  denotes the fraction of type  $N$  households born each period.

The economy also contains a large mass of two-period lived producers born at each date  $t$ . In the first period of their life, each producer can choose to operate a project whose activation requires an investment of one unit of the consumption good at the start of any period. An active project operated by a producer of skill  $z_t > 0$  yields gross output

$$z_t^{1-\alpha} n_t^\alpha$$

at the end of period  $t$ , where  $\alpha \in (0, 1)$  and  $n_t$  is the quantity of labor employed by the

project.

The skill level,  $z_t$ , of a particular producer is subject to aggregate uncertainty. Producers must decide whether to activate their project before knowing whether aggregate conditions  $\eta \in \{B, G\}$  are good ( $G$ ) or bad ( $B$ ). The aggregate shock follows a first-order Markov process with known transition function  $T : \{B, G\} \rightarrow \{B, G\}$ . Producer types, therefore, are a pair,  $z = (z_B, z_G) \in \mathbb{R}_+^2$  of skill levels. A producer of type  $(z_B, z_G)$  is of productivity  $z_B$  during bad times and  $z_G$  during good times. The mass of producers in a given Borel set  $Z \subset \mathbb{R}_+^2$  is  $\mu(Z)$  in each period. We assume that  $\mu$  has continuous derivatives<sup>6</sup> and that producer types are public information.

Producers have linear preferences and can either consume at the beginning of the first period of their life or at the beginning of the second period, although they heavily discount late consumption. Specifically, a consumption profile for producers born at date  $t$  is a triplet  $(c_{y,t}^P, c_{o,t+1}^P(B), c_{o,t+1}^P(G))$  where  $c_{y,t}^P$  is their consumption at the start of the first period of their life while  $(c_{o,t+1}^P(B), c_{o,t+1}^P(G))$  is their second-period consumption, which may depend on the realization of the aggregate shock. They rank those consumption profiles according to:

$$c_{y,t}^P + \epsilon E(c_{o,t+1}^P(\eta) | \eta_{t-1}),$$

where  $\epsilon$  is a small but positive number.

After the aggregate shock is realized, conditional on having activated a project, and taking the wage rate,  $w_t$ , as given, a producer of talent  $z$  chooses her labor input by solving

$$\Pi(w_t; z) \equiv \max_{n>0} z^{1-\alpha} n^\alpha - n w_t,$$

where  $\Pi$  denotes net operating income. Let

$$n^*(w_t; z) \equiv \arg \max_{n>0} z^{1-\alpha} n^\alpha - n w_t$$

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<sup>6</sup>This is for simplicity only. The case where  $\mu$  features positive mass points can be handled by introducing lotteries, as in [Halket \(2014\)](#).

denote the profit-maximizing labor used, given values of the aggregate shock and the wage. We note, for future reference, that  $n^*$  is linear in the realized level  $z$  of skill.

Active producers finance the investment of capital they need by selling securities, i.e. claims to their end-of-period output, to households. Selling one security is free, but selling two different types of securities carries a fixed cost  $\zeta > 0$ . One interpretation of this cost is that the agent types are physically separated from one another. Producers must decide whether to locate near one type or near the other. Delivering payoffs to the closer type is free – this is a mere normalization – delivering payoffs to the more distant type is more costly.

As in [Allen and Gale \(1988\)](#), producers are small hence, when considering which securities to issue, they take as given households' willingness to pay for marginal investments in the associated payoffs. Formally, let  $q_{N,t}(x_B, x_G)$  be the price at which a marginal amount of a security with payoffs  $(x_B, x_G) \geq (0, 0)$  at date  $t$  can be sold to type  $N$  households, where payoffs may depend on aggregate conditions. Similarly, let  $q_{A,t}$  be the price at which contingent securities can be sold to type  $A$  households. Active producers of type  $(z_B, z_G)$  choose non-negative security payoffs and consumption profiles to maximize:

$$q_{A,t}(x_{A,t}(B), x_{A,t}(G)) + q_{N,t}(x_{N,t}(B), x_{N,t}(G)) - 1 - \zeta \mathbf{1}_{\{x_{A,t} \neq 0, x_{N,t} \neq 0\}} + \epsilon E(c_{o,t}^P(\eta) | \eta_{t-1})$$

subject to:

$$\begin{aligned} x_{A,t}(B) + x_{N,t}(B) + c_{o,t+1}^P(B) &\leq \Pi(w_t(B); z_B), \\ x_{A,t}(G) + x_{N,t}(G) + c_{o,t+1}^P(G) &\leq \Pi(w_t(G); z_G), \\ q_{A,t}(x_{A,t}(B), x_{A,t}(G)) + q_{N,t}(x_{N,t}(B), x_{N,t}(G)) &\geq 1 + \zeta \mathbf{1}_{\{x_{A,t} > 0, x_{N,t} > 0\}}, \end{aligned}$$

where the indicator  $\mathbf{1}_{\{x_{A,t} > 0, x_{N,t} > 0\}}$  takes value one when a non-zero payoff is sold to each household type. The last condition simply says that proceeds from selling securities must cover funding needs at the start of the period. Clearly, producers become active when that constraint can be met since in that case (and only in that case) they enjoy non-negative



consumption.

Securities, therefore, are mappings from the aggregate state to a non-negative dividend. Allowing negative dividends would be formally similar to allowing households to short-sell securities. As is well known, doing so can lead to non-existence, even in one-period versions of the environment we describe. More importantly perhaps, financial engineering could not generate private profits if short-sales were unlimited, since any value created by splitting cash-flows could be arbitrated away in the traditional Modigliani-Miller sense.<sup>7</sup> As a result, no costly security creation would take place in equilibrium.

Households take as given the set of securities available at the start of a particular period. From their point of view, the menu of securities is a set of gross returns

$$R_{i,t}(z, \eta) = \frac{x_{i,t}(z, \eta)}{q_{i,t}(x_{i,t}(z, B), x_{i,t}(z, G))}$$

on the security issued by producers of type  $z = (z_B, z_G) \in \mathbb{R}_+^2$  for household type  $i \in \{A, N\}$  with the convention that  $R_{i,t}(z) = 0$  if type  $z$  is inactive.

Consider a household of type  $N$  born at date  $t$ . They earn wage  $w_t$  when young. They consume a part  $c_{y,t}^N$  of those earnings and enter the second period of their life with wealth  $w_t - c_{y,t}^N$ . They allocate that wealth to the securities available at that time by choosing a quantity  $a_t^N(z) \geq 0$  to invest in the securities produced by each producer type  $z$ . In particular notice that investment decisions are made before uncertainty is realized. Once that happens, at the end of the second period of their life, they consume the proceeds  $\int a^N(z) R_{N,t}(z, \eta) d\mu$  where  $\eta$  is the realization of the aggregate shock. Formally, given  $w_t$ , type  $N$  households born at date  $t$  solve:

$$\max_{a_t^N(z), c_{y,t}^N, c_{o,t+1}^N \geq 0} \log(c_{y,t}^N) + \beta \log \left\{ E(c_{o,t+1}^N(\eta) | \eta_t) \right\}$$

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<sup>7</sup>See [Allen and Gale \(1988\)](#) for the formal version of this argument.

subject to:

$$\begin{aligned} w_t &= \int a_t^N(z) d\mu + c_{y,t}^N, \\ c_{o,t+1}^N(B) &= \int a_t^N(z) R_{N,t}(z, B) d\mu, \\ c_{o,t+1}^N(G) &= \int a_t^N(z) R_{N,t}(z, G) d\mu, \end{aligned}$$

where  $\beta > 0$ .

Given these preferences, type  $N$  households consume a fixed fraction of their earnings in the first period of their life. Once they become old, they have risk-neutral preferences over the remaining consumption plans. As a result, old type  $N$  agents invest all their wealth in those securities whose expected return is highest. Therefore, letting

$$\bar{R}_{N,t} = \max_z T(B|\eta_{t-1}) R_{N,t}(z, B) + T(G|\eta_{t-1}) R_{N,t}(z, G),$$

old risk-neutral agents are willing to pay:

$$q_{N,t}(x(B), x(G)) = \frac{T(B|\eta_{t-1}) x(B) + T(G|\eta_{t-1}) x(G)}{\bar{R}_{N,t}}$$

for a marginal investment in a security with payoff  $(x(B), x(G))$  at date  $t$ .

Similarly, type  $A$  agents born at date  $t$  solve

$$\max_{a_t^A(z), c_{y,t}^A, c_{o,t+1}^A \geq 0} \log(c_{y,t}^A) + \beta \log \left\{ \min \{ c_{o,t+1}^A(B), c_{o,t+1}^A(G) \} \right\}$$

subject to:

$$\begin{aligned} w_t &= \int a_t^A(z) d\mu + c_{y,t}^A, \\ c_{o,t+1}^A(B) &= \int a_t^A(z) R_{A,t}(z, B) d\mu, \\ c_{o,t+1}^A(G) &= \int a_t^A(z) R_{A,t}(z, G) d\mu. \end{aligned}$$

Old agents of type  $A$ , in other words, are infinitely risk-averse and try to maximize the value of worst-case scenario consumption. Their preferences are also such that they save a fixed fraction of their earnings when young.

Consider an old household of type  $A$  alive at date  $t$ . Define

$$\bar{R}_{A,t} = \frac{\min \{c_{o,t}^A(B), c_{o,t}^A(G)\}}{a_{A,t_1}}$$

as the effective return these agents realize on their investment at the optimal solution to their problem. If  $c_{o,t}^A(B) < c_{o,t}^A(G)$  at the optimal solution, their willingness to pay for a marginal investment in a security with payoffs  $(x(B), x(G))$  is

$$q_{A,t}(x(B), x(G)) = \frac{x(B)}{\bar{R}_{A,t}}.$$

Indeed, they only value marginal payoffs in the lowest consumption state in that case. The symmetric property must hold when  $c_{o,t}^A(B) > c_{o,t}^A(G)$ . When  $c_{o,t}^A(B) = c_{o,t}^A(G)$ , which we will soon argue must hold in equilibrium at all dates,

$$q_{A,t}(x(B), x(G)) = \frac{\min(x(B), x(G))}{\bar{R}_{A,t}}.$$

Having stated every agent's optimization problem, we can now define an equilibrium. Old households of type  $i \in \{A, N\}$  enter date 0 with wealth  $a_{i,-1} > 0$ . The aggregate state of the economy at date 0 is fully described by  $\Theta_0 = \{a_{A,-1}, a_{N,-1}, \eta_{-1}\}$  where  $\eta_{-1} \in$

$\{B, G\}$  is the aggregate shock at date  $t = -1$ . An equilibrium, then, is a list of security payoffs  $\{x_{i,t}(z, \eta)\}$  for each household type, producer type and aggregate shock, the associated returns  $\{R_{i,t}(z, \eta)\}$ , consumption profiles  $(c_{y,t}^P, c_{o,t+1}^P(B), c_{o,t+1}^P(G))$  for each producer type and a corresponding set  $Z_t$  of active producers, wage rates  $\{w_t(\eta)\}$  for each  $\eta \in \{B, G\}$ , consumption plans and security purchases  $\{c_{y,t}^i, c_{o,t+1}^i, a_t^i(z)\}$  for each household type and, finally, pricing kernels  $\{q_{A,t}, q_{N,t}\}$  such that, at all dates and for all possible histories of aggregate shocks:

1. Security purchases and consumption plans solve the household's problem;
2. Security menus and consumption plans solve each producer's problem;
3. The market for labor clears:

$$\int n^*(w_t(\eta); z) d\mu = 1 \text{ for } \eta \in \{B, G\};$$

4. The market for each security type clears:

$$\int \theta a_t^A(z) R_{A,t}(z, \eta) d\mu = \int x_{A,t}(\eta) d\mu$$

$$\int (1 - \theta) a_t^N(z) R_{N,t}(z, \eta) d\mu = \int x_{N,t}(\eta) d\mu$$

for  $\eta \in \{B, G\}$ ;

5. Pricing kernels are consistent with the household's willingness to pay for marginal payoffs, i.e.:

- (a)  $q_{N,t}(x(B), x(G)) = \frac{T(B|\eta_{t-1})x(B) + T(G|\eta_{t-1})x(G)}{R_{N,t}}$ ,
- (b)  $q_{A,t}(x(B), x(G)) = \frac{\min(x(B), x(G))}{R_{A,t}}$  if  $c_{o,t}^A(B) = c_{o,t}^A(G)$ ,
- (c)  $q_{A,t}(x(B), x(G)) = \frac{x(G)}{R_{A,t}}$  if  $c_{o,t}^A(B) > c_{o,t}^A(G)$ ,
- (d)  $q_{A,t}(x(B), x(G)) = \frac{x(B)}{R_{A,t}}$  if  $c_{o,t}^A(B) < c_{o,t}^A(G)$ ,

for all possible securities  $(x(B), x(G)) \geq (0, 0)$  where:

$$\bar{R}_{N,t} = \max_z T(B|\eta_{t-1}) R_{N,t}(z, B) + T(G|\eta_{t-1}) R_{N,t}(z, G),$$

while

$$\bar{R}_{A,t} = \frac{\min\{c_{o,t}^A(B), c_{o,t}^A(G)\}}{a_{A,t-1}}.$$

The final equilibrium condition is similar to the consistency condition imposed by [Allen and Gale \(1988\)](#). Because type A households have Leontieff preferences, we cannot simply write as they do that pricing kernels are marginal rates of substitutions but the economic content of the condition is exactly the same. Producers take pricing kernels as given and choose securities to maximize their profits. Consumers, given this menu of securities, choose an optimal consumption plan which implies their marginal willingness to pay of securities. The implied kernels have to coincide with the kernels assumed by producers.

### 3 Properties of equilibria

The state of the economy at the start of a period is fully described by the wealth of old households  $a_{i,t-1} > 0$  for  $i \in \{A, N\}$  and the most recent aggregate shock  $\eta_{t-1}$ . For every possible value of these three objects we need to find pricing kernels  $(q_{A,t}, q_{N,t})$  as well as wages rate  $(w_t(B), w_t(G))$  for each possible realization of the aggregate shock, such that all markets clear and the Allen-Gale condition (equilibrium condition 5) is satisfied. This is a static problem which we characterize in this section. Since households simply save a fixed fraction of their wages in each period, a simple law of motion will then fully describe an equilibrium.

#### 3.1 Security space

The following result greatly simplifies the analysis.

**Lemma 1.** *In any equilibrium, the consumption of risk-averse agents is risk-free and they only purchase risk-free securities. Furthermore, in any equilibrium,*

$$\bar{R}_{N,t} \geq \bar{R}_{A,t}$$

*with a strict inequality whenever  $\zeta > 0$  and a positive mass of producers issue two securities.*

*Proof.* Assume, by way of contradiction, that an equilibrium exists in which, in a given period, the consumption bundle  $(c_B, c_G)$  of old risk-averse agents is such that  $c_B > c_G$ . Then, given their preferences, risk-averse agents pay nothing for the bad-realization payoff on any security, as their marginal valuation of consumption in bad times is zero. Moreover, in order for  $c_B > c_G$  to hold, a positive mass of securities with higher payoffs in the bad state than in the good state must be sold to risk-averse agents. But those producers would be strictly better off either selling the bad state payoff to risk-neutral agents, or simply consuming it themselves. The case in which  $c_B < c_G$  can be similarly ruled out.

To see why risk-neutral agents must earn a premium assume that the opposite holds. Then, producers would earn strictly more on any security sold to risk-neutral agents. But this would contradict the fact that the supply of securities to risk-averse must be strictly positive, since they always have strictly positive wealth. Finally, if producers bear the cost in order to sell two securities, the benefit of doing so, compared to selling everything to risk-neutral agents, must be strictly positive.  $\square$

Knowing this, it must be in any equilibrium that

$$q_{A,t}(x(B), x(G)) = \frac{\min(x(B), x(G))}{\bar{R}_{A,t}}$$

where

$$\bar{R}_{A,t} = \frac{\min\{c_{o,t}^A(B), c_{o,t}^A(G)\}}{a_{t-1}}.$$

Furthermore, since it only makes sense to issue risk-free securities to risk-averse agents, pro-

ducers choose a risk-free payoff  $x_A \geq 0$ , risky-payoffs  $x_N$  for type  $N$  agents, and an end of period consumption plan  $c_o^P$  to maximize:

$$\frac{x_A}{\bar{R}_{A,t}} + \frac{T(G|\eta_{t-1})x_N(G) + T(B|\eta_{t-1})x_N(B)}{\bar{R}_{N,t}} - 1 - \zeta 1_{\{x_A > 0 \text{ and } x_N > 0\}} + \epsilon E(c_2^P | \eta_{t-1}),$$

where feasibility, i.e., the non-negativity restriction on security payoffs imposes:

$$\begin{aligned} x_A &\leq \min \{ \Pi(w(B); z_B), \Pi(w(G); z_G) \} \\ x_A + x_N(B) + c_o^P(B) &\leq \Pi(w(B); z_B), \\ x_A + x_N(G) + c_o^P(G) &\leq \Pi(w(G); z_G). \end{aligned}$$

The first restriction says that risk-free payoffs must be risk-free hence have to be deliverable even under the worst-case realization of profits. The other two restrictions are feasibility conditions for each possible realization of the aggregate state.

To ease notation in the statement of our next result, write

$$\underline{\Pi}(z) = \min \{ \Pi(w(B); z_B), \Pi(w(G); z_G) \}$$

as short-hand notation for the lowest possible realization of profits for a particular producer at a particular history, and denote the state where the lowest profit is realized as  $\underline{\eta}(z)$ . By the same token, let

$$\bar{\Pi}(z) = \max \{ \Pi(w(B); z_B), \Pi(w(G); z_G) \}$$

be short-hand for the highest possible realization of profits, and  $\bar{\eta}(z)$  denote the state where the highest possible profit is realized.

The following proposition states that the solution to the producer problem satisfies a simple bang-bang property. Producers that tranche cash flows and issue two types of securities do so while issuing as much risk-free securities as possible, that is, paying off their lowest possible profit non-contingently.

**Proposition 2.** *In an equilibrium where a positive mass of producers pays the security creation cost  $\zeta$  such producers set  $x_A(z, \eta) = \underline{\Pi}(z)$*

*Proof.* Consider a producer that paid creation cost  $\zeta$  in a particular period. It should be clear that, in light of lemma 1, any solution must involve  $x_A > 0$ . Therefore, consider any feasible choice  $(x_A, x_N, c^P)$  such that  $x_A > 0$  but  $x_A < \underline{\Pi}(z)$ . Then, an increase in  $x_A$  would increase the producer's objective by

$$\frac{1}{\bar{R}_{A,t}} - \max \left\{ \epsilon, \frac{T(G|\eta_{t-1}) + T(B|\eta_{t-1})}{\bar{R}_{N,t}} \right\} > 0.$$

Indeed, lemma 1 guarantees the inequality with respect to the second element of the max operator. Moreover, it must also be the case that  $\frac{1}{\bar{R}_{A,t}} > \epsilon$  (and that  $\frac{1}{\bar{R}_{N,t}} > \epsilon$  for that matter), otherwise it would not make sense to pay the security creation cost in the first place, as the producer could simply sell one type of securities and consume the remainder.  $\square$

These results follow from a fundamental feature of environments in the spirit of [Allen and Gale \(1988\)](#) such as ours: producers take state prices as given, hence have a linear objective defined over a convex set, which, leads to bang-bang financial policies. This has nothing to do with the fact that our agents are either fully risk-neutral or fully risk-averse. Producer problems solve a linear problem simply because they are small, hence their actions have no impact on pricing kernels. When producers choose to create some risk-free debt, they maximize the production of such debt.

To better understand what characterizes producers issuing both types of securities, recall from lemma 1 that  $\bar{R}_{N,t} > \bar{R}_{A,t}$  so that producers earn strictly more revenues from by selling to both agent types rather than simply dealing with risk-neutral agents. That gain in revenue must exceed fixed cost  $\zeta$ . Their expected profit net of capital costs is:

$$\frac{T(\bar{\eta}(z)|\eta_{t-1}) (\bar{\Pi}(z) - \underline{\Pi}(z))}{\bar{R}_{N,t}} + \frac{\underline{\Pi}(z)}{\bar{R}_{A,t}} - \zeta,$$

while a producer that sells exclusively to risk neutral agents has expected profit net of capital



costs of:

$$\frac{T(\bar{\eta}(z)|\eta_{t-1})\bar{\Pi}(z) + T(\underline{\eta}(z)|\eta_{t-1})\underline{\Pi}(z)}{\bar{R}_{N,t}},$$

which implies that a producer will prefer to issue two securities to just catering to risk neutral agents when  $\underline{\Pi}(z) \left( \frac{1}{\bar{R}_{A,t}} - \frac{1}{\bar{R}_{N,t}} \right) \geq \zeta$ . Intuitively, this happens when the security creation cost is sufficiently low, when the difference between the returns paid to the two types is small enough, and importantly, when the worst possible profit is large enough. Note, in particular that the decision between tranching cash flows or issuing risky free securities does not depend on the highest possible profits  $\bar{\Pi}(z)$ . This can be seen in Figure 2, showing producer security policies, where the frontier between producers issuing both security types (tranching) and those issuing only risky securities (risky) are straight lines, that do not depend on the producer's productivity in the state when they realize their highest profit.

To complete the picture, let us examine the difference between the expected profit associated with issuing both security types and issuing only riskless assets. The latter is:

$$\frac{\underline{\Pi}(z)}{\bar{R}_{A,t}} + \epsilon (\bar{\Pi}(z) - \underline{\Pi}(z)),$$

implying that issuing both types of securities is preferable when

$$\left( \frac{T(\bar{\eta}(z)|\eta_{t-1})}{\bar{R}_{N,t}} - \epsilon \right) (\bar{\Pi}(z) - \underline{\Pi}(z)) \geq \zeta.$$

Intuitively, the producers that issue safe securities in exclusivity are those whose expected profits are sufficiently similar across states, as illustrated, for example, by the diagonal swaths on the top two panels of Figure 2. These considerations will play a key role in interpreting the outcome of our upcoming simulations.

### 3.2 Aggregation and GDP accounting

The aggregate production function that results from adding up the individual projects' production plans takes a familiar neoclassical form. In order to derive it, let  $Z_\Theta \subseteq \mathbb{R}_2^+$  denote the set of types that operate projects (an equilibrium object) given the aggregate state,  $\Theta$ , of the economy, where we dispense with time subscripts to reduce clutter. Let  $K$  denote the aggregate quantity of capital used to operate active projects in a given period. In equilibrium this has to equal the measure of projects activated:

$$K = \int_{Z_\Theta} d\mu.$$

It will be useful to define the average productivity among active projects when the realization of the aggregate state is  $\eta \in \{B, G\}$ :

$$\bar{z}(\eta) \equiv \frac{\int_{Z_\Theta} z_\eta d\mu}{\int_{Z_\Theta} d\mu},$$

and to note that this implies  $K\bar{z}(\eta) = \int_{Z_\Theta} z_\eta d\mu$ .

In equilibrium, the measure of labor supplied is one at all dates, but generalizing to other, off-equilibrium employment levels, let  $N$  denote the total mass of employment. Then, for the labor market to clear, and using the solution to the projects' labor choice problem, we must have that for each possible realization,  $\eta$ , of the aggregate shock:

$$\begin{aligned} N &= \int_{Z_\Theta} n^*(z_\eta, w(\eta)) d\mu \\ &= n^*(1, w(\eta)) \int_{Z_\Theta} z_\eta d\mu \\ &= n^*(1, w(\eta)) K \bar{z}(\eta). \end{aligned}$$

We can now write the aggregate production function given aggregate capital, aggregate

labor and the aggregate productivity shock:

$$\begin{aligned}
F(\eta, K, N) &= \int_{Z_\Theta} z_\eta^{1-\alpha} n^*(z_\eta, w)^\alpha d\mu \\
&= \int_{Z_\Theta} z_\eta n^*(1, w(\eta))^\alpha d\mu \\
&= \int_{Z_\Theta} z_\eta \left( \frac{N}{K \bar{z}(\eta)} \right)^\alpha d\mu \\
&= \left( \frac{N}{K \bar{z}(\eta)} \right)^\alpha \int_{Z_\Theta} z_\eta d\mu \\
&= \bar{z}(\eta)^{1-\alpha} K^{1-\alpha} N^\alpha.
\end{aligned} \tag{3.1}$$

This is a standard-looking neoclassical production function, where the term  $\bar{z}(\eta)^{1-\alpha}$  plays the role of measured TFP, which in this environment is a function of the efficiency of activated projects.

As we will discuss in more depth in section 4, this expression immediately implies that the effects of making security creation cheaper on TFP must be ambiguous. Unlike in traditional models of financial development, there are no untapped efficiency gains at the project level. The net impact of any change in the environment on TFP boils down to whether new entrants are more or less productive than already active and exiting producers. If anything, and as we will confirm via numerical simulations later, new entrants following a drop in security creation costs are more likely to be relatively low-productivity producers. Simply put, highly productive producers are active regardless of whether security creation is cheap or expensive.

The set of equilibrium conditions defined above implies an aggregate feasibility constraint that must hold every period. On the expenditure side, define aggregate consumption as the sum of each agent type's consumption,

$$C_t \equiv \theta (c_{y,t}^A + c_{o,t}^A) + (1 - \theta) (c_{y,t}^N + c_{o,t}^N) + c_{o,t}^P + c_{y,t}^P$$

where  $c_{o,t}^P$  is the second-period consumption of producers who born at date  $t - 1$  while  $c_{y,t}^P$  is

the first period consumption of producers who born at date  $t$ .

Aggregate investment is the sum of next period's capital and the expenditures intermediaries incur in creating new securities:

$$I_t = K_{t+1} + \int_{Z_\Theta} \zeta 1_{\{x_A > 0 \text{ and } x_N > 0\}} d\mu.$$

The result is that we can express the aggregate feasibility constraint in a familiar form,

$$C_t + I_t = Y_t.$$

That is, GDP equals the sum of aggregate consumption and investment.

### 3.3 Existence

In a given period and in any current state – a triplet  $\{a_A, a_N, \eta_{-1}\}$  of wealth for the two household type and aggregate shock in the most recent period – existence of an equilibrium will be guaranteed if rates of return  $(\bar{R}_A, \bar{R}_N)$  and wages  $w(B), w(G)$  can be found so that security and labor markets clear. The associated law of motion for wealth simply follows from the fact that wealth levels at the start of a given period are the previous period's wage, times the fixed savings rate implied by the preferences we have specified.

**Proposition 3.** *An equilibrium exists.*

*Proof.* Let  $\{a_1, a_2, \eta_{-1}\}$  be the starting state of the economy at a particular date. Start with a guess  $(R_N, R_A, w(B), w(G))$  for the four equilibrium prices we need, and compute the corresponding set  $Z(R_N, R_A, w(G), w(B)) \subset \mathbb{R}_+^2$  of active producers. Next, compute excess demand for each security type and labor for each of the two possible realizations of the aggregate shock in the current period. Specifically, starting with risky securities created

for risk-neutral agents:

$$ED_N(R_N, R_A, w(G), w(B)) = a_N - \int_{Z(R_N, R_A, w(G), w(B))} \frac{E(x_N(z))}{R_N} d\mu,$$

where  $E(x_N(z))$  is the expected payoff of risky securities created by producers of type  $z \in Z(R_N, R_A, w(G), w(B))$ . As for risk-averse agents:

$$ED_A(R_N, R_A, w(G), w(B)) = a_A - \int_{Z(R_N, R_A, w(G), w(B))} \frac{x_A(z)}{R_A} d\mu,$$

where

$$x_A(z) \in \{0, \underline{\Pi}(z)\}$$

is the risk-free payoff selected by producers of type  $z \in Z(R_N, R_A, w(G), w(B))$ . Excess demand for labor when the aggregate state is good is

$$ED_G^L = \int_{Z(R_N, R_A, w(G), w(B))} n^*(w_G, z_G) d\mu - 1$$

and the corresponding expression defines  $ED_B^L$  for the case where the aggregate shock is bad. We need to prove that  $(ED_N, ED_A, ED_B^L, ED_G^L)$  is zero for at least one four-tuple of prices.

Holding other prices fixed, each element of the  $ED$  demand vector is continuous and strictly monotonic in its own price. It also diverges without bound as each price goes to zero. Existence of the zero we need follows from classical arguments. To see this, for all  $n \in \mathbb{N}$ , define  $A_n = [\frac{1}{n}, n]^4$ . Then define  $G_n : ED(A_n) \mapsto A_n$  by

$$G_n(y_1, y_2, y_3, y_4) = \arg \max_{R_N, R_A, w(G), w(B) \in A_n} w(B)y_3 + w(G)y_4 - R_N y_2 - R_A y_1,$$

Roughly speaking,  $G$  raises wages when there is an excess demand for labor and lowers rates of returns when there is an excess demand for securities. The Theorem of the Maximum implies that  $G_n$  is non-empty, upper hemi-continuous and convex-valued. It follows that  $G_n \times ED$

has a fixed point on  $ED(A_n) \times A_n$ .

Letting  $n$  go to  $+\infty$  gives a sequence of prices. That sequence must have a bounded subsequence. To see why, assume for instance that  $R_A$  diverges to  $+\infty$ . Then at least one wage must fall to zero.<sup>8</sup> Say that  $w(B)$  goes to zero. So, then, must  $w(G)$  since otherwise there would eventually be an excess supply of labor in the good state, which, given our mapping, would mean that  $w(G)$  follows  $\frac{1}{n}$  at least along a subsequence. According to the same mapping, collapsing wages require that excess supply for labor remain positive in both states, which means that aggregate labor demand is bounded above, which means that profits are bounded above (since they are linear in the wage bill.) But then, a diverging  $R_A$  would imply that excess demand for safe securities is eventually positive, which would imply that  $R_A$  eventually follows  $\frac{1}{n}$  at least along a subsequence. Arguments are similar for the other three prices.

Since the sequence of fixed points above is bounded above and below, it must have a convergent subsequence. None of the associated prices can converge to zero. Assume for instance that  $w(B)$  did converge to zero. Given the mapping we have defined, this requires that aggregate labor demand remains below 1 in the bad state so operating profits in the bad state converge to zero. Since  $w(G)$  cannot diverge to  $+\infty$  as established in the previous paragraph, excess labor demand in the good state has to be non-negative infinitely often which requires that  $w(G)$  also converges to zero at least along a subsequence. But then, either return would have to diverge to infinity, since otherwise, with vanishing wages, there would have to be an excess demand for labor eventually, which is incompatible with declining wages given our mapping. Again, the other prices can be dealt with using similar arguments.

It follows that along the convergent sequence of fixed points introduced above, the price part of the fixed point is eventually in the interior of  $A_n$ . But given the definition of  $G_n$  this is only possible if all excess demands are zero. This completes the proof of existence.  $\square$

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<sup>8</sup>Otherwise, profits are bounded above unless demand for labor diverges to infinity (profits are linear in the wage bill) along a subsequence. If labor demand diverges, wages must converge to zero in at least one state.

The construction implicit in the proof above underlies the computational approach we will employ in our numerical simulations.

### 3.4 Comparative statics: a preview

Having established the existence of equilibria, we can now use our framework to study the relationship between the intensity of financial engineering activities and standard measures of economic development and productivities. There are at least two natural measures of the quantity of financial engineering in our environment. First, we can measure the volume of securities issued by producers who choose to bear the security creation cost. Second, we can measure the amount

$$\int_{Z_\Theta} \zeta 1_{\{x_A > 0 \text{ and } x_N > 0\}} d\mu$$

producers spend on security creation activities. Both measure are, of course, endogenous. We will consider two changes to the environment that may cause these equilibrium quantities to rise.

First and perhaps most naturally, a drop in the security creation cost  $\zeta$ , holding prices constant, can only cause an increase in the fraction of producers who choose to bear that cost. Whether that drop also causes a decline in aggregate security creation costs is ambiguous since, while the cost per producer is cheaper, more producers choose to bear it. Second, we will consider a permanent increase in the fraction of risk-averse agents. We view the second experiment as proxying for the well-documented increase in appetite for safe assets over the past two decades. This second experiment potentially makes the extraction of safe securities from risky projects more profitable. Loosely speaking, our first exercise corresponds to a supply-driven increase in engineering activities while the second corresponds to a demand-driven increase.

Tracing the effects of these shocks is greatly complicated by the fact that they both have an impact on prices in general equilibrium. For instance, we would expect greater demand for safe assets to depress safe returns. In the next section, we resort to calibrated numerical

simulations to quantify the effect of these shocks.

In this section, we preview the results one should expect from these quantitative explorations using a simple parametric example. Assume that producers are scaled up versions of one another in the sense that  $\frac{z_G}{z_B}$  is  $\mu$ -almost surely a constant. Put another way, almost surely,  $z_G = zA_G$  while  $z_B = zA_B$ , where  $z > 0$  is the producer's skill level and  $A_G > A_B > 0$  are aggregate shocks common to all producers. Under those assumptions, the search for market clearing wages becomes one dimensional. The fact that  $Z_\Theta$  is set prior to the realization of the aggregate shock, and hence is the same regardless of that realization, also means that if we know what bad time wages  $w(B)$  are in a particular period, only one value of  $w(G)$  can also clear the labor market during good times. Furthermore, the Cobb-Douglas functional forms we have assumed for production functions imply that  $\frac{w(G)}{w(B)}$  is a constant greater than one. In this case, producer talent is summarized by a scalar  $z_B \in \mathbb{R}_+$ . Security creation policies, as a result, become simple.

**Lemma 4.** *Assume that  $\frac{z_G}{z_B}$  is  $\mu$ -almost surely a constant. Then security creation policies are fully characterized by two thresholds  $\underline{z}_t \leq \bar{z}_t$  in every period. Producers become active when  $z > \underline{z}_t$  and bear the security creation cost when  $z > \bar{z}_t$ .*

The intuition for this result is simple. Only producers whose scale is high enough can generate enough security creation profits to overcome the fixed cost. Since in this parametric example producer types are one-dimensional, only the most qualified producers choose to create different securities for each type.

We can further simplify the example by assuming that, holding other parameters the same,  $\frac{z_G}{z_B}$  is high enough that it is never profitable for any producer to only sell securities to risk-neutral agents. Given  $\zeta$ , when  $\frac{z_G}{z_B}$  is high enough, the gap in profits between good and bad times is so high that producers are always better off selling the excess profits they generate in good times to risk-neutral agents than consuming it. With this assumption, producers whose talent is between the two thresholds described in the lemma sell their entire output to risk-neutral agents.



So consider now a marginal drop in security creation costs  $\zeta$  in a particular period. Holding prices the same, that drop cannot have any effect on the lower threshold since, at that threshold, producers only issue one security. In turn, and once again holding prices the same, wages, output and aggregate TFP cannot change. It follows that, in this example, general equilibrium effects are the only possible source of impact of drops in  $\zeta$  on macroeconomic aggregates. At the original prices and original thresholds, labor markets continue to clear but there is an excess supply of safe securities. So one would expect the risk-free rate to go up and the upper threshold to fall. These, in turn should cause an excess demand for risky securities, which causes the return earned by risk-neutral agents to fall. Holding wages the same, a fall in the return producers have to pay risk-neutral agents causes a fall in the lower threshold, and in turn an increase in labor demand and output.

The fact that changes in security creation costs have no direct, partial equilibrium, effect on producer participation suggests that, quantitatively, their effect on output is bound to be small. Our upcoming numerical simulations will confirm this intuition. More surprisingly, they will show that the effect of increased on security creation can be negative. To understand why this can happen, observe that in our model, we must have, following any change in the environment,

$$\begin{aligned}
 \text{Change in capital formation} &= \text{Change in spending on securities} \\
 &\quad - \text{Change in security creation expenditures} \\
 &\quad - \text{Change in producer consumption/rents.} \quad (3.2)
 \end{aligned}$$

We argued above that the first term on the right-hand-side must go up as  $\zeta$  falls. Security creation expenditures, on the other hand, cannot be monotonic in  $\zeta$  since they are zero when  $\zeta$  is zero and must return to zero once  $\zeta$  is so large that no cash-flow splitting takes place. There must be regions, in other words, where expenditures on creation costs rise as  $\zeta$  falls. Our simulations will show that this effect can be large enough to dominate the behavior of the other components of capital formation. Our simulations will also show that the final term,

producer rents (which equal the sum of all security issuance revenues net of the capital put in place and any security creation costs), can be non-monotonic in  $\zeta$  as well, but it plays a negligible role, quantitatively, in our findings.

## 4 Numerical simulations

To illustrate how the consequences of securitization booms for macroeconomic aggregates may vary depending on what is driving such rise, we run two experiments. In the first, we compare economies that differ only in security creation costs. Starting with an economy with no security creation costs, we increase these costs until no cash-flow splitting takes place in equilibrium. In the second, we compare economies that differ only in the fraction of agents of each type (risk neutral or infinitely risk averse). In particular, in the second experiment, all economies being compared face the same security creation cost.

### 4.1 Parameterization and algorithm

In our model economy, agents live for two periods, working in the first one and living off their savings in the next. Correspondingly, we will think of a period as representing 25 years.

The first set of parameters can be calibrated directly to specific targets. We set the elements of the aggregate state's transition matrix  $T$  so that the probability of remaining in the bad state is  $T_{BB} = 0.2$  and the probability of remaining in the good state is  $T_{GG} = 0.9$ . This implies that the economy spends close to 90 percent of the time in the good state. We set  $\alpha = 0.6$  to match a labor income share of 0.6. This is slightly higher than the usual 70% to account for the fact that some of the profits going to entrepreneurs are rewarding labor. We set  $\theta = 0.5$ , implying an equal share of risk-neutral and risk-averse agents.

We start by making the support set of project productivities  $\mathcal{Z} = [0, 1] \times [0, 1]$ , while its distribution function  $\mu$  is a truncated bivariate normal with mean  $\bar{\mu} = (\mu_G, \mu_B)$  and variance-covariance matrix  $\Phi$ . We normalize  $\mu_B = 1$  and set the terms off the main diagonal in  $\Phi$  to

zero. Furthermore, in setting the variance terms (main-diagonal) we restrict ourselves to a constant coefficient of variation regardless of the state:  $\varsigma = \frac{\sigma_B}{\mu_B} = \frac{\sigma_G}{\mu_G}$ .

These functional assumptions allow us to reduce the number of parameters. We then calibrate three parameters: the discount factor  $\beta = 0.68$ , the mean productivity in good times  $\mu_G = 0.11$ , and the coefficient of variation  $\varsigma = 0.8$  to match three jointly determined moments coming out of our stochastic simulations (described below) for the economy where securitization is costless. To wit, a risk-free rate of 2%; a fall in output of 15% from good to bad times, which is the value that [Gourio \(2013\)](#) uses, and is in the ballpark of values estimated by [Barro and Ursua \(2008\)](#); and a ratio of producer rents to output of 10%, which is reasonable given the approximation for this moment obtained by [Corbae and Quintin \(2016\)](#) using US private corporate sector data.

Standard arguments show that our economies eventually converge to a stochastic steady-state, i.e. an invariant distribution of all endogenous variables in our model.<sup>9</sup> To obtain statistics for all endogenous variables in this stochastic steady-state, we adopt a traditional Markov chain Monte Carlo approach.<sup>10</sup> Specifically, our algorithm is as follows:

1. Given parameters, solve for household and intermediary policy functions for every possible aggregate state of the economy;
2. Draw a 100-period sequence of aggregate shocks  $\{\eta_t\}_{t=1}^{100}$  using the Markov transition matrix  $T$  and record the value of all endogenous variables starting from an arbitrary value of aggregate wealth;
3. After dropping the first 10 periods, so that the assumed initial conditions have at most a negligible effect on the value of endogenous variables, compute average values for all endogenous variables.

To facilitate comparisons across economies with different costs, we use the same draw of random aggregate shocks throughout our simulations.

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<sup>9</sup>See [Brock and Mirman \(1972\)](#).

<sup>10</sup>See [Tierney \(1994\)](#).

## 4.2 Varying security creation costs

Figure 2 displays producer policies for four different securitization cost levels. A mass of projects is left inactive because they are unprofitable in expected value terms, regardless of the security structure used to finance them. For any given productivity level in the bad state  $z_B$ , there is a threshold level of productivity in the good state  $\bar{z}_G(z_B)$  above which the expected profits cover the cost of capital and any possible security creation costs and, as a consequence, the project is activated. The threshold  $\bar{z}_G(z_B)$  is weakly decreasing in  $z_B$ . As  $z_B$  falls, producers, regardless of how they finance their activities, need to be (at least weakly) compensated by increases in  $z_G$ .

When security creation costs are zero, issuing risk-free securities is weakly dominated by issuing both types of securities (producers that have exactly the same profits in both states are indifferent between the two). Making costs slightly positive, as we do for  $\zeta = 0.005$ , reveals exactly who these producers are, as issuing risk-free securities becomes slightly more profitable than tranching and, as a consequence, a sliver of active producers starts to do so as shown in panel A of Figure 2. As costs increase further, the measure of risk-free issuing producers expands around the ray where profits are the same in both states, as we argued at the end of section 3.1.

There are, in general, two distinct types of tranching establishments operating, depending on which state they enjoy higher profits. Given a productivity level in the bad state  $z_B$ , producers that have a sufficiently high  $z_G$  (who are vertically above the sliver of riskless producers in panel A of Figure 2) have lower profits in the bad state and therefore only make payments to risk neutral agents in the good state. On the other hand, producers with a sufficiently low  $z_B$  (vertically below the sliver of riskless producers), actually have lower profits in the good state (because wages are higher then) and therefore make payments to risk neutral agents in the bad state.

As long as  $\zeta > 0$ , for low enough  $z_B$  producers prefer to issue risky securities than to tranche and make payments to risk neutral agents in the good state only. This explains the

strip of active producers along the vertical axis issuing risky securities only. An analogous reason explains why it is preferable, for low enough  $z_H$ , to issue risky securities only, as opposed to tranching and making payments to risk neutral agents in the bad state only.

Finally, and obviously, when security creation costs are high enough, no producers engage in it. Producers either issue risk-free securities exclusively, or they issue risky securities exclusively, as seen in panel D of Figure 2.

As security creation costs increase, the share of tranching establishments falls monotonically, as shown in panel B of Figure 3. This puts pressure on the relative issuance of risky securities to increase, which, because the demand for the two security types is constant, means that for markets to clear, the price of risk-free securities needs to rise, leading to a corresponding, fall in the risk-free rate. The same reason explains why risk-neutral agents pay less for risky securities as security creation costs increase, and therefore enjoy a higher rate of return. Both rates of return are shown in panel C of Figure 3.

To understand what happens to capital formation as costs vary, it is instructive to first look at what happens to expenditures in securitization activities. When  $\zeta = 0$ , these expenditures are trivially zero. As  $\zeta$  increases, they start rising, but eventually fall back down to zero, as the share of tranching establishments goes to zero. This results in a Laffer curve-like relationship between security creation costs and expenditures, shown in panel D of Figure 3.

Given our assumption that one unit of capital is needed to operate a project, capital formation equals the share of active projects and is given by total spending on securities net of producer rents and security creation expenditures. Because spending on securities is proportional to output (as it is a fixed fraction of wages that are linear in output) and security creation expenditures initially increase, capital formation falls as security creation costs start increasing (as shown in panel A of Figure 3). Eventually, as costs continue to increase but securitization expenditures start falling, capital formation comes back up, resulting in a non-monotonic relationship with security creation costs. Producer rents are also non-monotonic (panel E of Figure 4), but they are quantitatively less meaningful.

Output displays the same non-monotonic relationship with securitization costs as capital,

as shown in panel A of figure 4. Quantitatively, there is not much difference between the changes in these two variables, resulting in little action in TFP, as shown in panel B of Figure 4. To understand why this is the case, recall the intuition from the one-dimensional example in section 3.4 arguing that the share of operating projects is not affected directly by changes to  $\zeta$ . It turns out this generalizes to our two-dimensional environment. As Figure 2 illustrates, tranching establishments are not marginal in the sense that they are not close to inactivity. Therefore, the operation threshold only moves because of general equilibrium effects, which are quantitatively small. Since TFP is simply the average productivity of active producers, this results in small TFP changes.

### 4.3 Varying the share of each type of agents

In the second experiment, we keep the security creation cost fixed at the intermediate level of  $\zeta = 0.1$ , and vary, instead, the share of the two types of agents, from  $\theta = 0.1$  (10 percent risk-averse agents) to  $\theta = 0.9$ . As the share of risk-averse agent rises, the share of wealth held by those agents increases and so, therefore, does the demand for riskless assets. This puts downward pressure on the risk-free rate and upward pressure on the risky rate, as panel C in Figure 6 shows. As a result, there is a compositional change in the type of active businesses in the direction of catering to the savings of the increasing share of risk-averse agents: the fraction of establishments financed exclusively by risky securities drops monotonically, as the share of those issuing risk-free securities, whether exclusively or by tranching, increases (panel B of Figure 6).

At the same time, as shown in panel A of Figure 6, the overall share of active establishments or, equivalently, capital formation, falls. Initially (compare panels A and B of Figure 5), this decrease is small: as  $\bar{R}_N$  increases and the revenues of the marginal producers exclusively issuing risky securities fall, some switch to issuing riskless securities and some exit. Moreover, some hitherto infra-marginal (inactive), projects become profitable by issuing risk-free securities exclusively, as  $\bar{R}_A$  drops. As  $\theta$  continues to increase (compare panels C and D

of Figure 5), market clearing eventually requires large increases in the equilibrium risky rate of return  $\bar{R}_A$ , driving additional risky establishments out of businesses, while others still start tranching, or financing themselves through the exclusive issuance of risk-free securities. All the while, the fall in the share of active businesses brings down the demand for labor and the wage rate (as shown in panel C of Figure 7).

As the share of active projects fall, so does output, but because it is the relatively less productive establishments that are exiting as  $\theta$  rises, TFP increases. Again, this increase is quantitatively meaningless, but why do projects financed exclusively by risk-neutral agents tend to be less productive, on average? Because if they were productive enough (meaning if their worst-case profits were high enough) they could afford to pay the security creation cost, which would allow them to finance themselves at lower rates by borrowing from risk-averse agents.

Unlike what happened in our previous experiment, the consequences for the use of aggregate resources stemming from the increase in securitization are not mitigated by the fact that per producer cost is reduced. In this case, with fixed per producer securitization costs, as more producers optimally decide to pay the cost and securitize risky cash flows, total expenditures in securitization rise monotonically to reach almost 1% of GDP, as shown in panel D of Figure 6.

These two experiments illustrate how two different mechanisms – one could loosely call the former supply-driven and the latter demand-driven – could have originated the recent securitization boom. Our results show that these two mechanisms imply very disparate outcomes. In particular, the steady fall in risk-free rates we have seen in the last two decades is a crucial piece of evidence, and one that points in the direction of an increase in demand for risk-free securities as being at the origin of the boom.

## 5 Conclusion

By allowing producers or intermediaries to create securities that appeal to investors with heterogeneous tastes, financial engineering leads to more investment broadly defined, which accords well with intuition. Less intuitive is the fact that the resulting securitization boom may not lead to increases in output, capital formation and TFP, as much of the spending on engineered securities may be dissipated into security creation costs and producer rents.

Importantly, we find that regardless of the sign of the change macroeconomic variables experience as a result of the change in securitization costs, such impact is likely to be very small. We conjecture this result is likely to be very robust in environments like ours where producers always operate at their optimal scale and therefore reducing security creation costs only impacts production financing sources. This robustness stems from the fact that, as we argue, the set of operating producers does not change as security creations costs change if prices are held fixed. Changes to the set of operating producers happen indirectly, as a result of the general equilibrium effects the changes in security creations costs instigate. In most environments, such second order effects are likely to be small, and so will the changes in macroeconomic variables they produce.

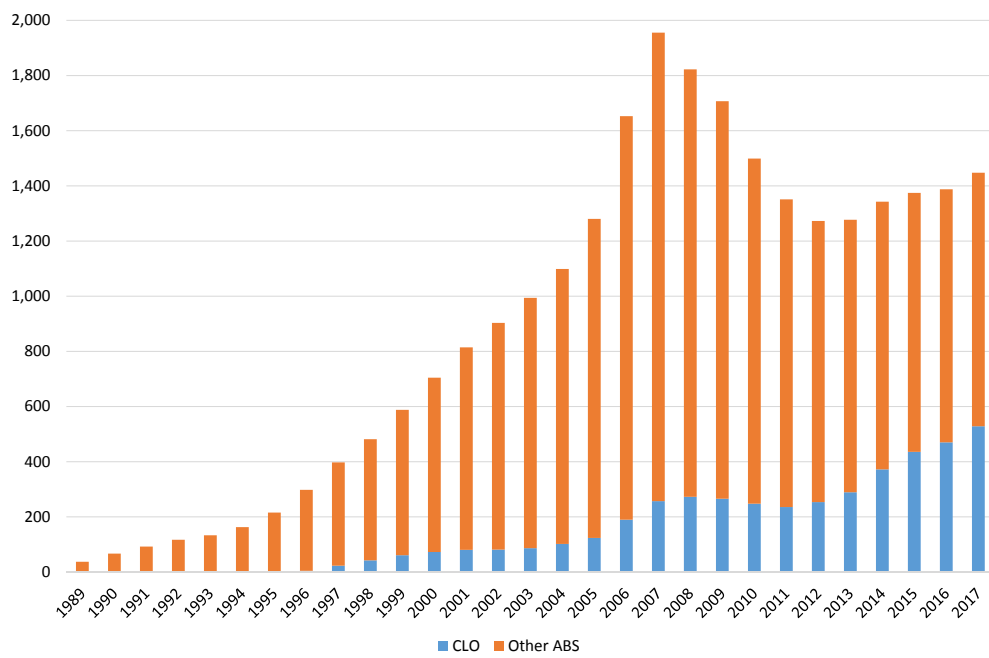
Finally, our model can be used to discern the source of the recent securitization boom the US has experienced. While supply-side changes like a fall in securitization costs leads to a counterfactual increase in the risk-free rate, the opposite happens when the increased securitization is a result of an increase in demand for risk-free securities.



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Figure 1: US Asset-Backed Securities Outstanding (USD Billions)



Source: Securities Industry and Financial Markets Association (SIFMA). These volume numbers include all collateral sources except housing-related collateral.

Figure 2: Producer policies: changing securitization costs

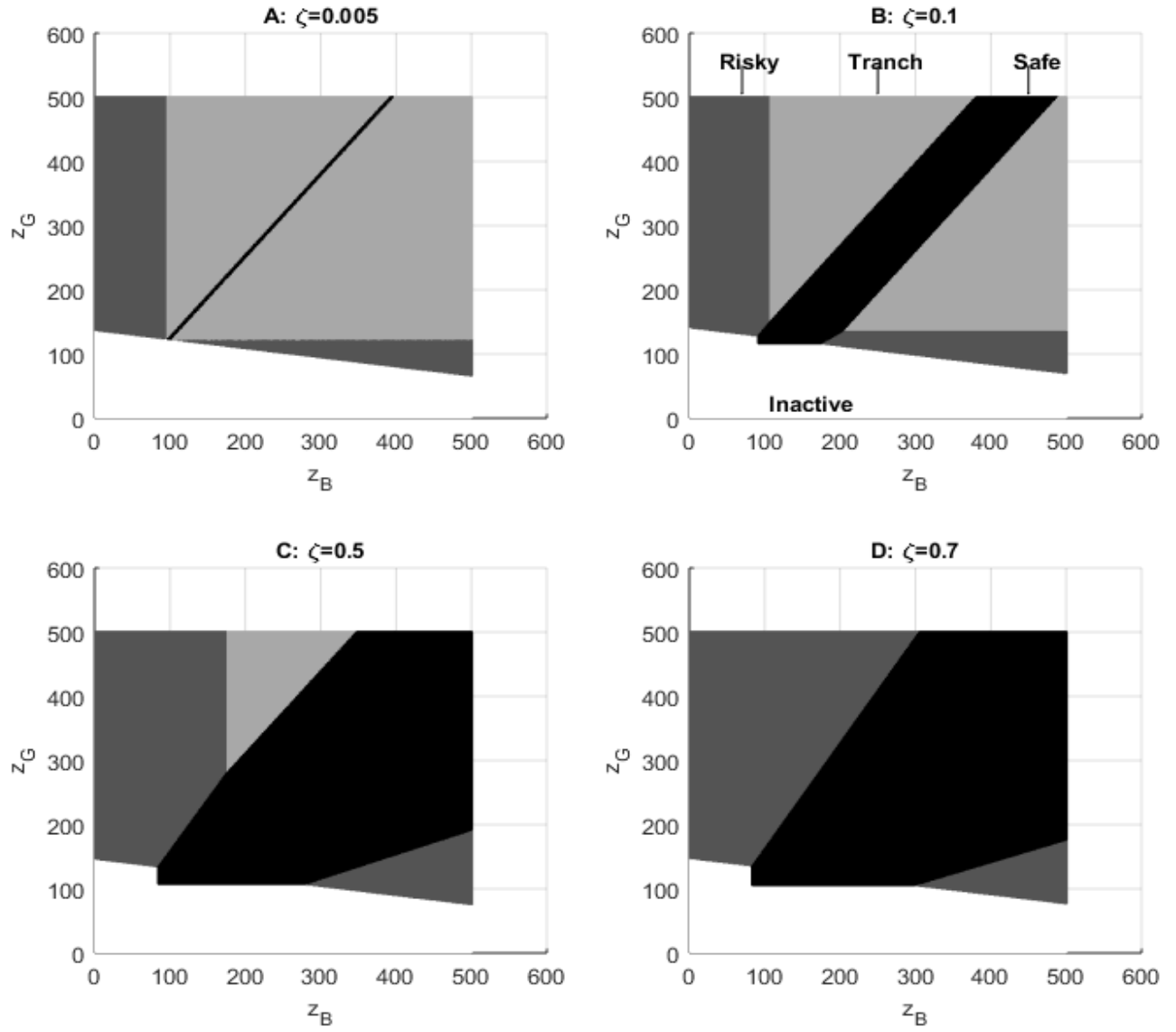


Figure 3: Aggregate outcomes I: changing securitization costs

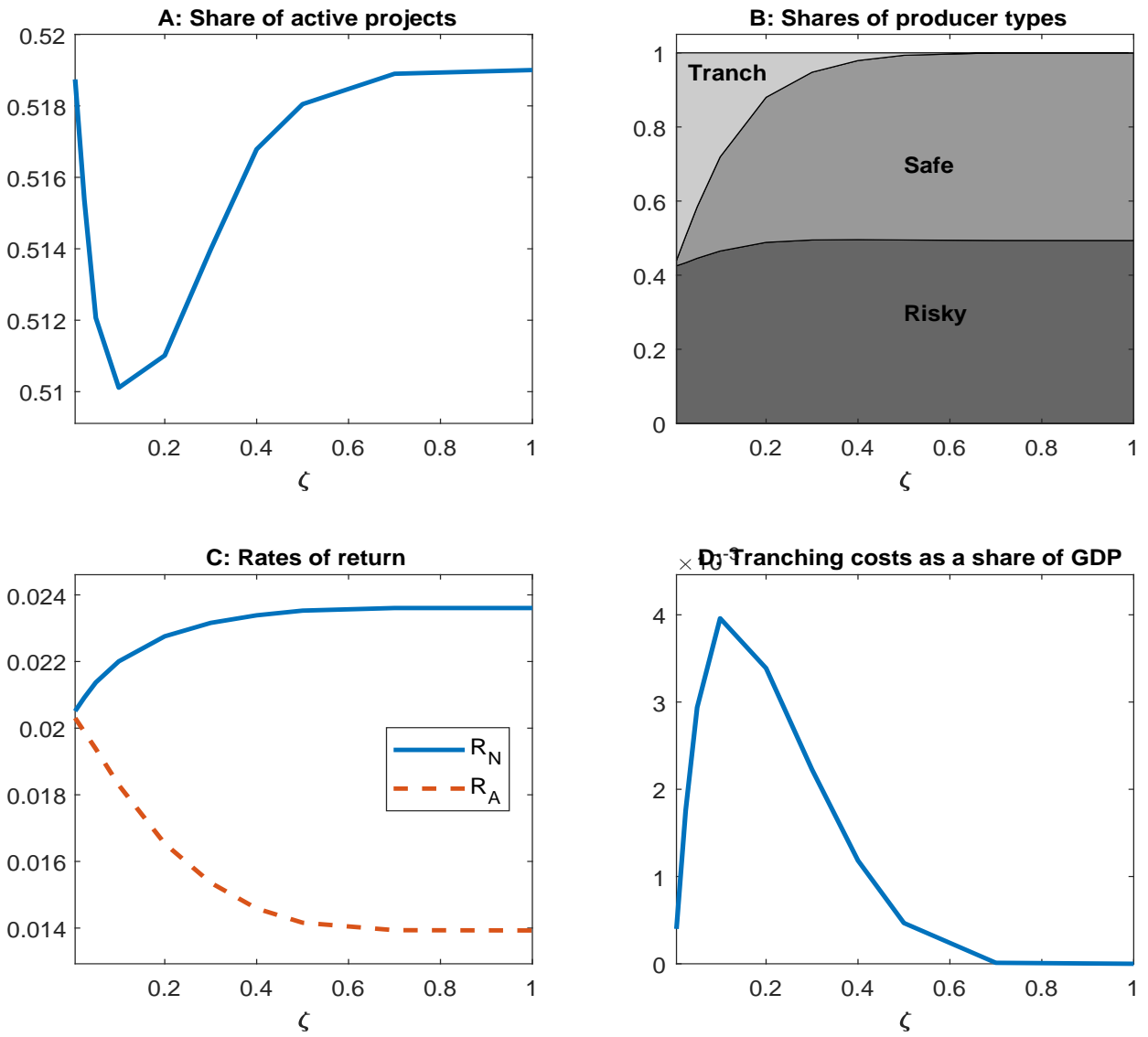


Figure 4: Aggregate outcomes II: changing securitization costs

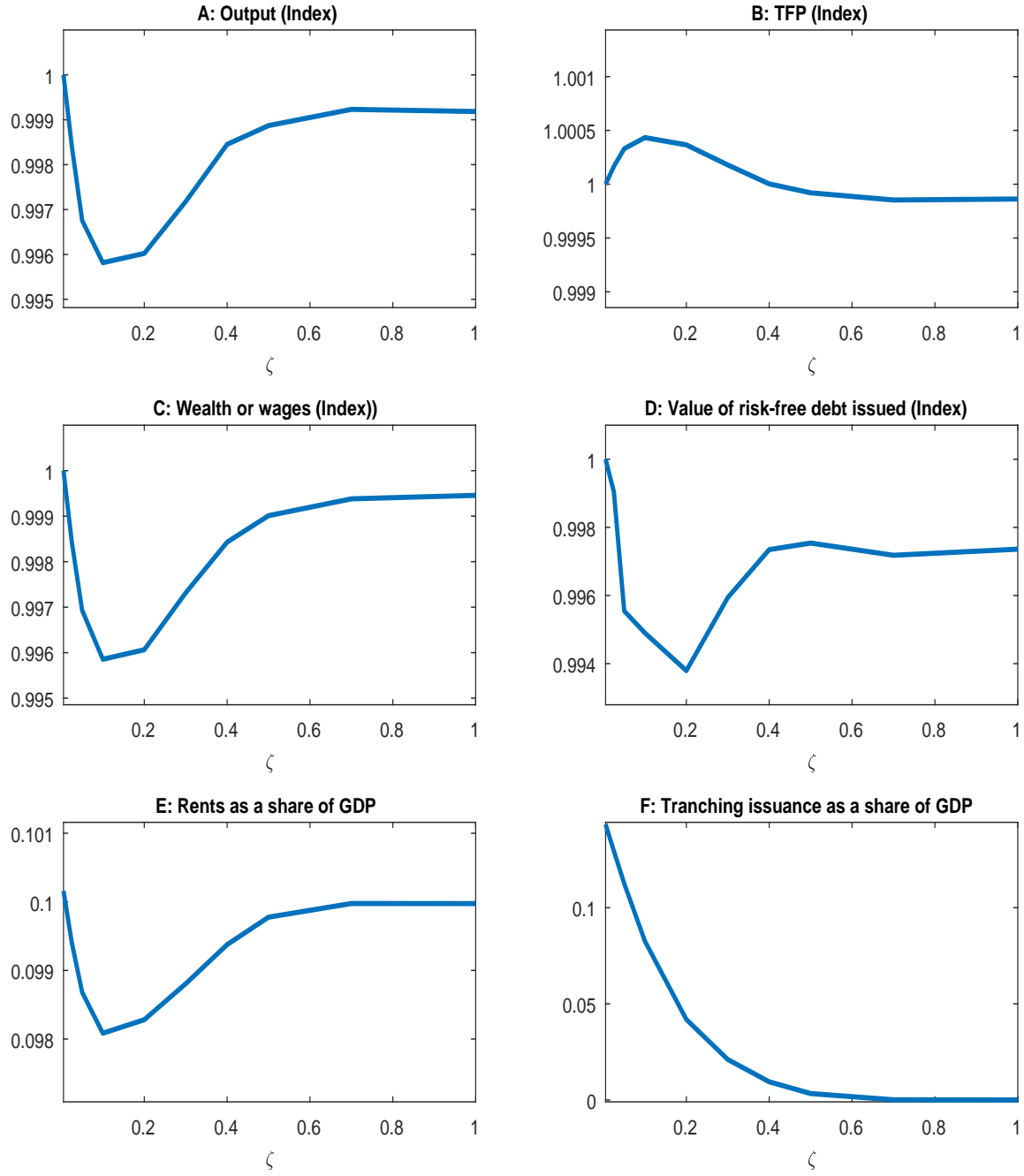


Figure 5: Producer policies: changing types

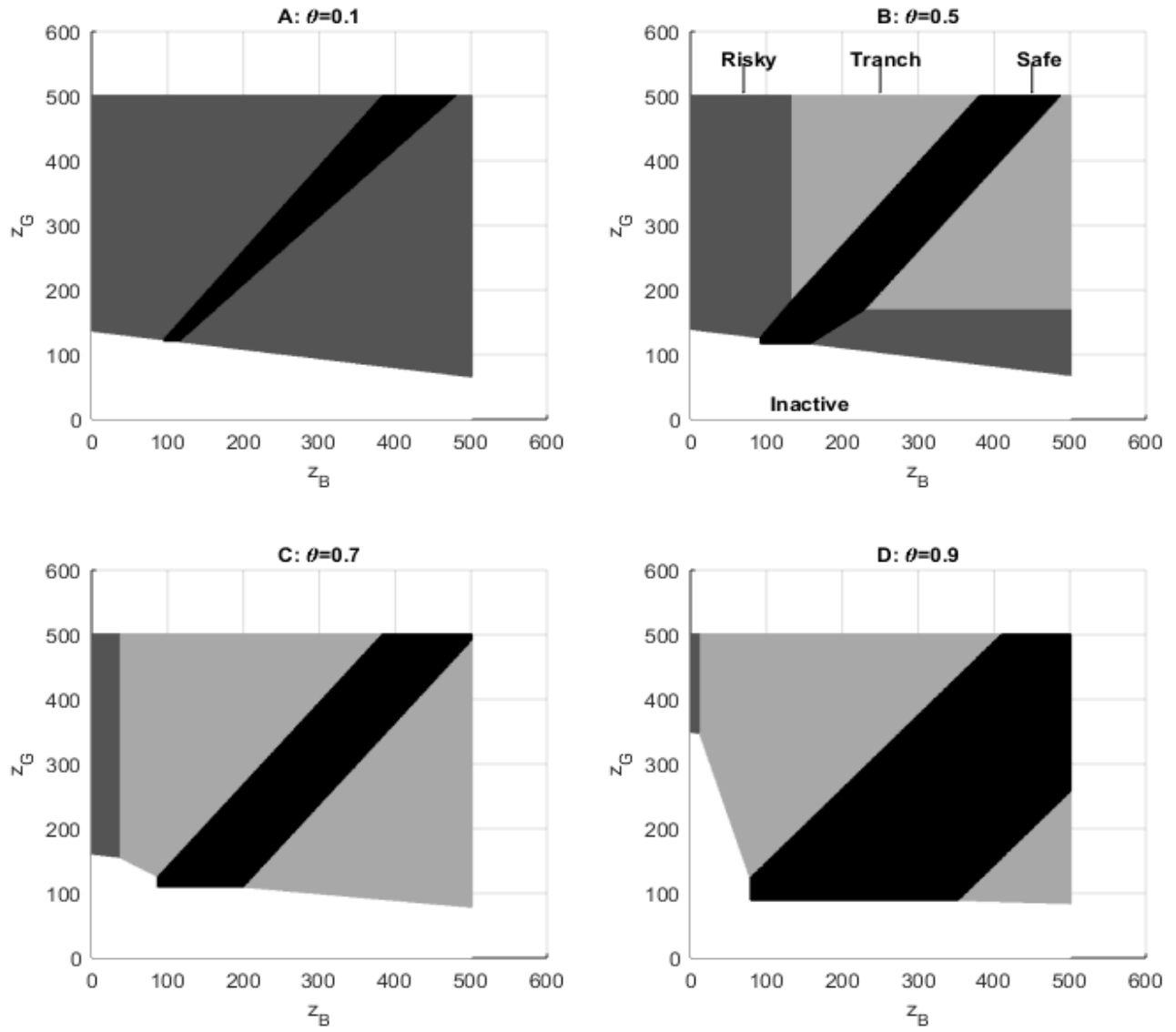


Figure 6: Aggregate outcomes I: changing types

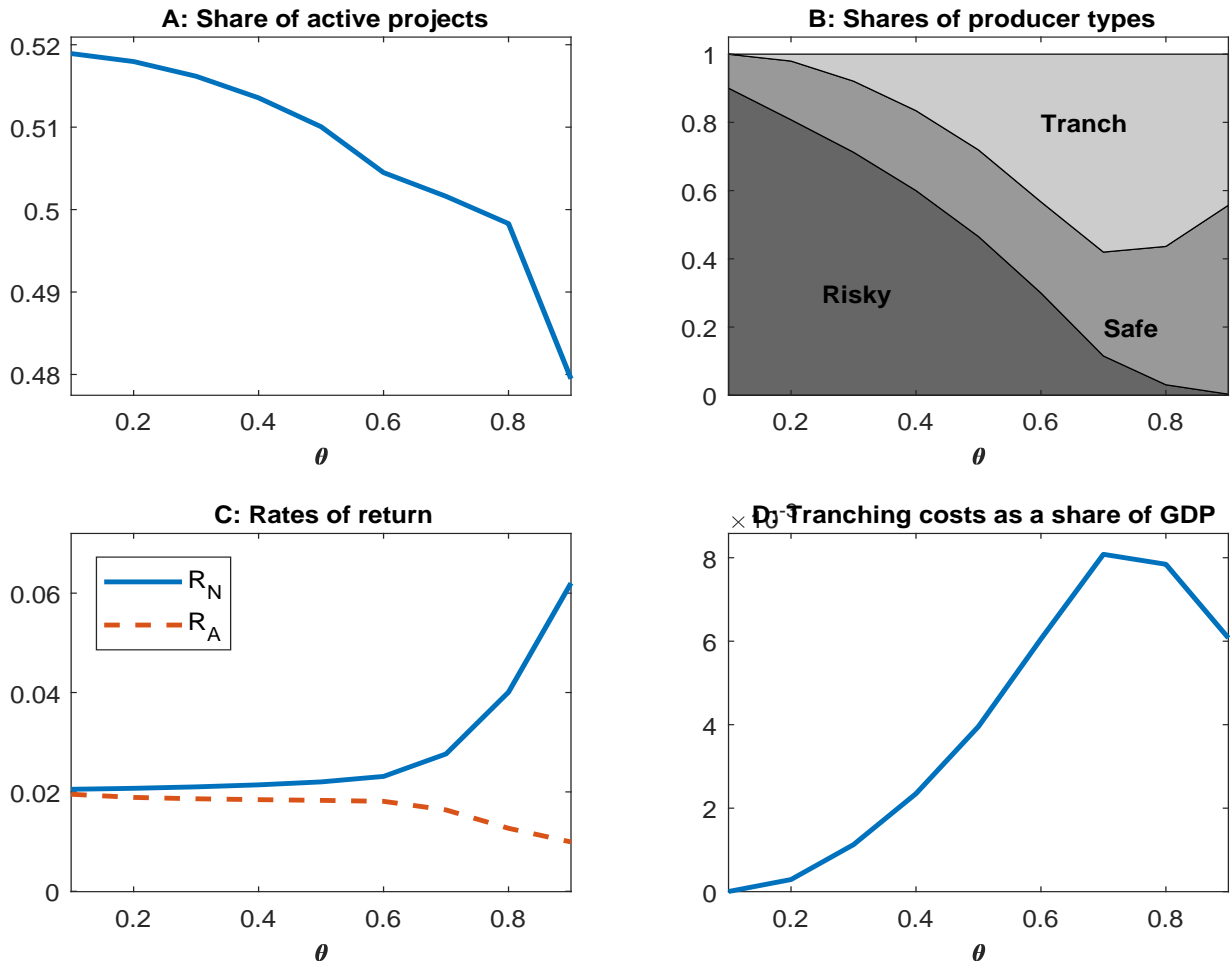


Figure 7: Aggregate outcomes II: changing types

