FOR ONLINE PUBLICATION
PERSISTENCE OF POWER: REPEATED MULTILATERAL BARGAINING

MARINA AGRANOV CHRISTOPHER COTTON CHLOE TERGIMAN

1. Mathematical proofs for Section 3

Proof to Proposition 1. We first show that, when players care enough about future rounds, exclusion of a player from future coalitions is a credible threat, sustainable as part of a SPE. Let Z denote an arbitrary subgame of one of our games. We say that player i is "excluded in subgame Z" if in every period t along the path of play of the subgame, $x_i^t = 0$. An excluded player receives zero payoffs in all future periods.

Lemma 1. *Consider any subgame Z starting at any time t in any of our three games. If discount factor γ is sufficiently large, then for any player $i \in \{1, ..., n\}$, there exists SPE of Z in which player i is excluded. When $m \geq 2$, such an equilibrium exists for every $\gamma > 0$.*

Proof. Suppose that player i is the excluded player. Let $\bar{\mathbf{x}}^j = (\bar{x}_1^j, ..., \bar{x}_n^j)$ be an allocation rule, denoting x^t for each period t that player j serves as AS_t. Each player $j = 1, ..., n$ including $j = i$ has such an allocation rule, with $\bar{\mathbf{x}} = (\bar{x}^1, ..., \bar{x}^n)$, and where $\bar{x}_i^j = 0$. For each $j = 1, ..., n$, let K^j denote a coalition of m players excluding j.

Consider the following strategy profile starting at any time $\hat{t}$. The strategy profile excludes player i along the path of play:

- In every t beginning with $\hat{t}$, as long as no player deviated from the equilibrium strategy in any earlier period starting with $\hat{t}$,
  - AS_t proposes $\mathbf{x}^t = \bar{\mathbf{x}}^{AS_t}$
  - Each player $j \in K^{AS_t}$ votes in favor of the period t proposal (and in *Majority Support* votes in favor of AS_t) if and only if $\mathbf{x}^t = \bar{\mathbf{x}}^{AS_t}$.
  - Each player $j \notin K^{AS_t}$ votes against the period t proposal (and in *Majority Support* votes against AS_t).
- If in any period starting with $\hat{t}$ any player j deviates from the above profile, then beginning in the next period, the game switches to a subgame equilibrium in which the deviating player (rather than the previously excluded player i) is excluded. If both the AS and another player deviate (sequentially), then beginning in the next period, the game switches to a subgame equilibrium in which the player who deviated most recently (the voting player) is excluded. The general equilibrium structure of the new subgame equilibrium is identical to the one

Date: November 14, 2016.

described here, except we denote the allocation rule by $\hat{\mathbf{x}} = (\hat{x}^1, ..., \hat{x}^n)$ rather than $\bar{\mathbf{x}}$, and that player j is now the excluded player.[1]

We determine when such a strategy profile constitutes an equilibrium, deriving a specific threshold value of γ such that the lemma is satisfied when γ is larger than this threshold. The condition is the same for each game

$$\Gamma \in \{Baseline, Vote\ of\ Confidence, Majority\ Support\}$$

Thus, when those conditions are met, there exists a SPE of each subgame in which any player i is excluded for the duration of the game. The condition will not only ensure that the above strategy profile is a subgame equilibrium, but will ensure that the threat of exclusion is credible, as it simply triggers another equilibrium of the same form with a different player being excluded.

In all three games $(Baseline, Vote\ of\ Confidence, Majority\ Support)$, the AS each period has no incentive to deviate from offering $\mathbf{x}^t = \bar{\mathbf{x}}^{AS_t}$. If she deviates to offer something besides this, then everyone votes against the proposal and it fails, and the AS is excluded in future periods, returning a NPV of current and future periods equal to 0. Even if the AS receives nothing herself, she has no incentive to deviate as she receives 0 given any deviation.

In all three games $(Baseline, Vote\ of\ Confidence, Majority\ Support)$, no player who is expected to vote in favor of a proposal has an incentive to vote against it. For every $j \in K^{AS_t}$ in period t, voting against $\bar{\mathbf{x}}^{AS_t}$ causes the proposal to fail, and causes the game to enter a subgame equilibrium in which j is excluded. This leads to a NPV of current and future payoffs equal to 0. Even a player who is excluded in the current equilibrium has no incentive to deviate, as she also receives 0 given any deviation.

In all three games $(Baseline, Vote\ of\ Confidence, Majority\ Support)$, no player who is expected to vote against the equilibrium allocation has an incentive to deviate, as voting in favor of the equilibrium allocation will not change the current period outcome and will lead to the deviating player being excluded for the duration of the game.

It remains to show that no player would choose to vote in favor of a proposal other than $\bar{\mathbf{x}}^{AS_t}$ in any period t. When $m \geq 2$, no player has an incentive to vote in favor of $\mathbf{x}^t \neq \bar{\mathbf{x}}^{AS_t}$, as a single player cannot pass a proposal on his own. Therefore, when $m \geq 2$, deviating to vote for an off equilibrium path proposal does not change the current outcome, but leads to the deviating player being excluded in future periods. Thus, when $m \geq 2$, no player will ever deviate from the strategy of voting against any $\mathbf{x}^t \neq \bar{\mathbf{x}}^{AS_t}$.

When $m = 1$, however, a single player can pass a proposal on his own. We must rule out that possibility that any player is willing to vote in favor of an off equilibrium path proposal giving him any share $x_j \leq 1$ where $\mathbf{x} \neq \bar{\mathbf{x}}^{AS_t}$. It is sufficient to determine when players are unwilling to accept an off equilibrium proposal offering them $x_j = 1$, as it will imply that j is also unwilling to accept any deviant offer giving him $x_j < 1$. Accepting a proposal with $x_j = 1$ leads to a payoff to player j of 1 in the current period, and to j being excluded in future periods. Thus, the NPV of current and future period

[1]We can adopt any rule to determine which legislator is excluded in the event that two or more non-AS players deviate at the same time. The is because we only need to show that unilateral deviation by a single player is never optimal.

payoffs is simply equal to the current period payoff of 1. Rejecting such a proposal leads to an equilibrium in which the current AS is excluded rather than player j, as the current AS had deviated from the equilibrium to make the deviant proposal. This leads to a NPV future payoffs equal to

$$\frac{1}{n}\sum_{k=1}^{n}\hat{x}_j^k\left(\delta+\frac{\gamma}{1-\gamma}\right)$$

where $\frac{1}{n}\sum_{k=1}^{n}\hat{x}_j^k$ is the expected allocation provided to player j with a random selection of a new period AS. In *Baseline*, there is a random draw of a new AS each period. In *Vote of Confidence* and *Majority Support*, there is a random draw in the next period, and this draw will determine the equilibrium allocation for all future periods thereafter.

We require that in period t, no player $j \neq AS_t$ has an incentive to deviate:

$$\frac{1}{n}\sum_{k=1}^{n}\hat{x}_j^k\left(\delta+\frac{\gamma}{1-\gamma}\right)\geq 1.$$

This requires that for each $j \neq AS_t$, $\frac{1}{n}\sum_{k=1}^{n}\hat{x}_j^k$ is positive, and γ is sufficiently large.

Next, we calculate the threshold for γ for the existence of an exclusion equilibrium of the form described above. To do so, notice that when $\frac{1}{n}\sum_{k=1}^{n}\hat{x}_j^k = 1/(n-1)$ for all $j \neq AS_t$, the left hand side of the above expression is maximized for the j with he minimum value of the expression. We can rewrite the equilibrium conditions as

$$\frac{1}{n-1}\left(\delta+\frac{\gamma}{1-\gamma}\right)\geq 1.$$

Which holds as long as $\gamma \geq \bar{\gamma}$, where

$$\bar{\gamma} \equiv \frac{n-1-\delta}{n-\delta} \in (0,1).$$

As long as $\gamma \geq \bar{\gamma}$, there exists a SPE of any subgame in which any player i is excluded indefinitely. □

Such equilibria are sustained by the threat to non-excluded players of themselves being excluded in the future if one deviates from their respective strategies in the exclusion equilibrium. This threat can incentivize players to reject even generous proposals from currently excluded players, for example.

Because exclusion can be maintained as part of a SPE of every subgame, it can serve as a credible threat to any player who does not play in accordance with a broader class of equilibrium strategies. Since a player can be no worse off than when he is excluded, the threat of future exclusion offers a powerful incentive to sustain cooperation amongst players in our games. With the threat of exclusion, any allocation can be maintained within SPE of our three games, when deviation from a SPE strategy by a AS or a coalition partner expected to vote in favor of the proposal triggering the exclusion of the deviating player in all future periods.

In the remaining proofs, a "subgame equilibrium in which i is excluded" will refer to the subgame equilibrium described in the proof to Lemma 1, and where $\bar{x}_k^j = 1/(n-1)$ for all j and all $k \neq i$. That is, in the subgame equilibrium where i is excluded, the other players split the allocation evenly each round. We have shown that such an equilibrium exists when $\gamma \geq \bar{\gamma}$.

For each round $r = 1, 2, ...$ and each player $j = 1, ..., n$, let $\mathbf{a}^{r*} = (a_1^{r*}, ..., a_n^{r*})$ be a feasible allocation rule such that $a_i^{r*} \in [0, 1]$ for each i and $\sum_i a_i^{r*} = 1$. Let $K^{r,j}$ be a coalition of m players excluding j. Consider the following partial description of the strategy profile starting from period $t = (1, 1)$:

- For every $r = 1, 2, ..., p = 1, 2, ...$ and corresponding $t = (r, p)$, as long as no player previously deviated from the equilibrium strategy,
  - AS_t proposes $\mathbf{x}^t = \mathbf{a}^{r*}$
  - Each player $i \in K^{r,AS_t}$ votes in favor of the period t proposal (and in *Majority Support* votes in favor of AS_t) when $\mathbf{x}^t = \mathbf{a}^{r*}$.
  - Each player $i \notin K^{r,AS_t}$ votes against the period t proposal (and in *Majority Support* votes against AS_t).
- If in any period t' any player j deviates from the above profile, then beginning in the next period, the game switches to a subgame equilibrium in which the deviating player is excluded (as described in the proof to Lemma 1). If both the AS and another player deviate (sequentially), then beginning in the next period, the game switches to a subgame equilibrium in which the player who deviated most recently (the voting player) is excluded.

From the proof to Lemma 1, we already know that excluding a player who does not play the equilibrium-path strategies is always consistent with SPE when $m \geq 2$, and is consistent with SPE when $m = 1$ as long as $\gamma \geq \bar{\gamma}$, where

$$\bar{\gamma} \equiv \frac{n - 1 - \delta}{n - \delta} \in (0, 1).$$

When this is the case, the treat of exclusion is credible.

For the above to be a SPE, no player must have an incentive to unilaterally deviate from the above strategy profile. For the same reasons as in the previous proof, the AS will have no incentive to deviate from proposing $\mathbf{x}^t = \mathbf{a}^{r*}$, no player voting in favor of or against $\mathbf{x}^t = \mathbf{a}^{r*}$ has an incentive to change their vote, and no player has an incentive to vote for any $\mathbf{x}^t \neq \mathbf{a}^{r*}$ when $m \geq 2$.

It remains to show that when $m = 1$, no player will want to vote for any $\mathbf{x}^t \neq \mathbf{a}^{r*}$. If the AS proposes an allocation giving player i share $\hat{x}_j \neq a_j^{r*}$, then player j can accept the proposal, earning $\hat{x}_j$ this period, but being excluded in future periods. Or the player can reject the proposal, and the game reverts to an equilibrium in which the current period AS is excluded, and the other players each earn $1/(n-1)$ each period. Player j has no incentive to deviate as long as

$$\hat{x}_j \leq \frac{1}{n-1}\left(\delta + \frac{\gamma}{1-\gamma}\right).$$

The greatest possible incentive that player j has to deviate is when $\hat{x}_j = 1$. Plugging this into the above inequality and solving for γ gives

$$\gamma \geq \frac{n - 1 - \delta}{n - \delta}.$$

This is the same condition as needed for the existence of an exclusion equilibrium. Thus, when the exclusion equilibrium exists, any allocation x^{r*} can be maintained as part of a

SPE in each round, and any $\mathbf{x}^* = (x^{r*})_{r=1}^{\infty}$ can be maintained by a SPE along the path of play.

Proof to Proposition 2: The characteristics of equilibrium follow from the proof to Proposition 1 and the analysis in the body of the paper.

2. Proof to Proposition 3

Consider the proof to Proposition 1. In the embedded proof to Lemma 1, one may limit attention to situations in which the excluded player is the most recent deviant. Given this, the strategy profiles included in the proof to Proposition 1 (and the embedded Lemma 1) rely only on payoff relevant information and the identity of the most recent player to deviate from the equilibrium strategy. Therefore, the identity of the most-recent deviant is the only payoff-irrelevant information from the history of the game that is required to sustain any allocation as part of equilibrium.

3. SSPE without symmetry

We begin by relaxing the symmetry requirement of SSPE. Rather than require that the players' strategies are independent of other player's identities, as is the standard assumption, we allow for stationary strategies which treat other players asymmetrically, specifically when the AS each period chooses which players to include in her MWC. We focus on pure strategy equilibria in this environment.

Let $\mathbf{x}^j = (x_1^j, ..., x_n^j)$ denote player j's equilibrium proposal strategy, which she makes in every period that she serves as AS. Let $\bar{a}_i^j$ denote player i's voting strategy, where i votes for a proposal made by player j in any period t that i serves as AS if and only if $x_i^t \geq \bar{a}_i^j$.

Consider the following stationary, but asymmetric, strategy profile:

- Each player j chooses a MWC K_j made up on m other players. Player j's proposal gives $x_i^j = X$ for each $i \in K_j$, and $x_i^j = 0$ for each $i \notin \{K_j, j\}$.
- Each player i is included in the MWC of exactly m other players.
- Each player i votes in favor of proposal $\mathbf{x}^t$ if and only if $x_i^t \geq X$ when $i \in K_{AS_t}$ and if and only if $x_i^t \geq Y$ when $i \notin K_{AS_t}$.

We determine the values of X and Y such that the above constitutes an asymmetric SSPE.

First, consider such strategies in the context of *Baseline*. Here, the switch from symmetric to asymmetric strategies does not change the incentives that players have to accept or reject proposals each period. A player who is offered $\hat{x}_j$ can accept the proposal and expect a NPV of

$$\hat{x}_j + \left(\frac{1}{n}(1 - mX) + \frac{m}{n}X\right)\frac{\gamma}{1-\gamma} = \hat{x} + \frac{1}{n}\frac{\gamma}{1-\gamma}$$

or he can reject the proposal and expect a NPV of

$$\left(\frac{1}{n}(1 - mX) + \frac{m}{n}X\right)\left(\delta + \frac{\gamma}{1-\gamma}\right) = \frac{1}{n}\left(\delta + \frac{\gamma}{1-\gamma}\right).$$

In equilibrium, $\hat{x}_j = X = Y$, and such an offer leaves a MWC member indifferent between accepting and rejecting the proposal each period. Thus, $X = Y = \frac{\delta}{n}$.

Second, consider such strategies in the context of *Vote of Confidence*. Here, the switch to asymmetric strategies does change the incentives that players have to accept or reject proposals. This is because players expect to continue to be included in the MWC of an AS who includes them in her proposal strategy. This means that if player j votes in favor of a proposal giving him $\hat{x}_j$ that is made by an AS such that $j \in K_{AS}$, then j expects a NPV of

$$\hat{x}_j + X\frac{\gamma}{1-\gamma}.$$

Accepting the same proposal made by an AS such that $j \notin K_{AS}$ returns a NPV of only $\hat{x}_j$ to player j, as j does not expect to be included in the future MWCs of that AS. In either case, if j votes against the proposal, he again expects

$$\frac{1}{n}\left(\delta + \frac{\gamma}{1-\gamma}\right).$$

In equilibrium, for proposals made by an AS such that $j \in K_{AS}$, $\hat{x}_j = X$ and this leaves player j indifferent between accepting and rejecting. Thus,

$$X + X\frac{\gamma}{1-\gamma} = \frac{1}{n}\left(\delta + \frac{\gamma}{1-\gamma}\right) \to X = \frac{1}{n}(\delta + \gamma - \delta\gamma)$$

For proposals made by an AS such that $j \notin K_{AS}$, $\hat{x}_j = Y$ and Y leaves player j indifferent between accepting and rejecting. Thus,

$$Y = \frac{1}{n}\frac{\delta + \gamma - \delta\gamma}{1-\gamma}.$$

Given the parameter values, $X < Y$. This means that it is less expensive for an AS to include a player in K_{AS} in her MWC than a non member. Therefore, given the strategies of other players, the AS each period is building making the proposal that results in the highest allocation for herself.

Third, consider such strategies in the context of *Majority Support*. For this game, we must also describe the voting strategies for the players when deciding whether to keep or replace the current period AS. There are two possibilities: either the members of K_j will reelect j when AS each period, or they will not. Those not in K_j have no incentive to reelect player j as AS.

Suppose that we are in an equilibrium of *Majority Support* with high persistence of AS power. Thus, for every AS j, players in K_j vote in favor of player j retaining power whenever j is AS. In this case, the incentives to vote for or against a given proposal are the same as in *Vote of Confidence*, as the AS retains power each period she passes a proposal. As such X and Y are the same as in *Vote of Confidence*. It remains to determine when the members of K_j prefer to reelect the AS rather than to draw a new AS the next period. At the time the players vote for the AS, they are choosing between a favorable vote, which returns NPV of expected future payoffs equal to

$$X\frac{\gamma}{1-\gamma},$$

and an unfavorable vote which returns NPV of expected future payoffs equal to

$$\frac{1}{n}\frac{\gamma}{1-\gamma}.$$

Thus, players in K_j prefer to retain j as AS as long as $X \geq 1/n$. Plugging in for X, this gives

$$\frac{1}{n}(\delta+\gamma-\delta\gamma) \geq \frac{1}{n} \rightarrow \gamma \geq 1.$$

This is a contradiction, as $\gamma \in (0,1)$, ruling out the possibility that such an asymmetric SSPE with high persistence of AS power in *Majority Support* exists.

Suppose that we are in an equilibrium of *Majority Support* with low persistence of AS power. Thus, players vote against the AS in each period. In this case, the incentives to vote for or against a given proposal are the same as in *Baseline*, as there is a new draw of AS power each period. As such X and Y are the same as in *Baseline*, with $X = Y = \delta/n$. It remains to determine when the members of K_j prefer to draw a new AS the next period, rather than reelect the current AS. Our assumption that players ignore weakly dominated strategies means that the players vote as if they were casting the deciding vote. We need the players in K_j to each prefer to vote against the AS. The calculations are the same as in case with AS retention, except with a reversed sign of the inequality. Thus, players in K_j prefer to replace j as AS as long as $X \leq 1/n$. Plugging in for X, this gives

$$\frac{\delta}{n} \leq \frac{1}{n} \rightarrow \delta \leq 1.$$

This condition always holds. Thus, in the asymmetric SSPE of *Majority Support*, the equilibrium resembles that of *Baseline*, with low persistence of AS power.

4. Risk aversion

In the symmetric SSPE of all three games, a player i that votes against a proposed allocation obtains expected net present value of

$$\left(\frac{1}{n}\cdot u_i(1-ma^{\text{Game}})+\frac{m}{n}u_i(a^{\text{Game}})+\frac{n-1-m}{n}u_i(0)\right)\left(\delta+\frac{\gamma}{1-\gamma}\right)$$

where a^{Game} denotes equilibrium share of the coalition partner in a specific game. If, on the contrary, i supports the proposed allocation at time t, she gets

$$u_i(x_i^t)+\frac{\gamma}{1-\gamma}\left[\frac{1}{n}u_i(1-ma^{\text{Game}})+\frac{m}{n}u_i(a^{\text{Game}})+\frac{n-1-m}{n}u_i(0)\right]$$

in the Baseline and Majority Support games, and gets

$$u_i(x_i^t)+\frac{\gamma}{1-\gamma}\cdot\frac{m}{n-1}u_i(a^{\text{Game}})$$

in the Vote of Confidence game.

We assume that players have identical CARA utility functions, with

$$u_i(x) = u(x) = 1-e^{-r\cdot x} \quad \text{for all } i.$$

In the SSPE of the baseline and majority support games, the minimum acceptable offer $\bar{a}$ solves

$$u(\bar{a}) + \tfrac{\gamma}{1-\gamma}\left[\tfrac{1}{n}u(1-m\bar{a}) + \tfrac{m}{n}u(\bar{a}) + \tfrac{n-1-m}{n}u(0)\right] = \left(\tfrac{1}{n}\cdot u(1-m\bar{a}) + \tfrac{m}{n}u(\bar{a}) + \tfrac{n-1-m}{n}u(0)\right)\left(\delta + \tfrac{\gamma}{1-\gamma}\right)$$

This simplifies to

$$1 - e^{-r\cdot\bar{a}} = \left(\frac{1}{n}\cdot(1-e^{-r\cdot(1-m\bar{a})}) + \frac{m}{n}(1-e^{-r\cdot\bar{a}}) + \frac{n-1-m}{n}(1-e^{0})\right)\delta.$$

Plugging in the parameters from the experiment (i.e. m, n, δ) gives

$$7 = 11e^{-r\cdot\bar{a}} - 4e^{-r\cdot(1-\bar{a})}.$$

Solving for $\bar{a}$ gives

$$\bar{a} = \frac{1}{r}\ln\left(-\frac{7}{8}e^{r} + \frac{1}{8}e^{r/2}\sqrt{49e^{r}+176}\right)$$

A similar analysis for the vote of confidence game yields

$$\bar{a} = \frac{1}{r}\ln\left(-\frac{7}{188}e^{r} + \frac{1}{188}e^{r/2}\sqrt{37976+49e^{r}}\right).$$

Using a numerical analysis in Mathematica, we show that these expressions for $\bar{a}$ in the three games are strictly decreasing in r. Thus, as risk aversion increases, the share allocated to the MWC player decreases.

5. Preferences for fair behavior

Here, we incorporate other regarding preferences, as proposed by Fehr and Schmidt (1999). In each period, a player i's period utility is

$$u_i(\mathbf{a}) = a_i - \alpha\frac{1}{n-1}\sum_{j\neq i}\max\{x_i - x_j, 0\} - \beta\frac{1}{n-1}\sum_{j\neq i}\min\{x_j - x_i, 0\},$$

where $\alpha \in (0,1)$ is a cost incurred from others being treated "unfairly" relative to oneself, and $\beta \in [\alpha, 1)$ is a cost incurred by being treated "unfairly" oneself. We focus on the case from the experiment where $n = 3$ and $m = 1$.

Equal division in a grand coalition. First, we determine conditions under which there exists a SSPE in which an equal share is allocated to all players.

Suppose that $\mathbf{a}$ allocates $a_i = 1/3$ for each i. In equilibrium, entering a new period of bargaining gives any player an expected payoff of 1/3. Fairness concerns do not affect payoffs in the case of equal division.

Anticipating a payoff of 1/3 in the next period, if the current period proposal does not pass, a player requires utility of at least $\delta/3$ to vote for the current period proposal. Therefore, if the AS in any given period deviates from equal division, he must offer a MWC player at least $\bar{a}$ for his proposal to pass, where $\bar{a}$ solves

$$\bar{a} - \alpha\bar{a} - \beta(1-2\bar{a}) = \delta/3.$$

Thus,

$$\bar{a} = \frac{3\beta + \delta}{3(1-\alpha+2\beta)}.$$

For equal division to be an equilibrium, the AS must prefer to allocate evenly, earning 1/3 in any period, than to allocate $\bar{a}$ to a single MWC player. This will be the case if

$$1 - \bar{a} - \alpha(2(1-\bar{a}) - \bar{a}) \leq 1/3.$$

Plugging in for $\bar{a}$ and simplifying the expression gives the required parameter condition

$$\alpha \geq 1/3.$$

Therefore, as long as α is sufficiently large, there exists an equilibrium in which the players allocate evenly each period.

5.0.1. *Equal split with MWC.* Next, we consider the possibility that there exists a SSPE in which an AS and a MWC partner split the allocation evenly each period.

In equilibrium, each period the AS and MWC partner receive

$$\frac{1}{2} - \alpha\frac{1}{2}\frac{1}{2} = \frac{2-\alpha}{4},$$

and the excluded player receives

$$-\beta\frac{1}{2}\frac{1}{2} = -\frac{\beta}{4}.$$

From an ex ante perspective, the expected per period utility for each player is

$$\frac{1}{3} - \frac{2}{3}\alpha\frac{1}{2}\frac{1}{2} - \frac{1}{3}\beta\frac{1}{2}2\frac{1}{2} = \frac{2-\alpha-\beta}{6}.$$

Consider the baseline model, where the AS is randomly selected each period. For equal division between an AS and MWC to be an equilibrium, the AS must prefer such an allocation to any alternative.

It is straightforward to show that an AS prefers equal division with a MWC to any success allocation that gives more than 1/2 to a MWC:

$$\frac{2-\alpha}{4} + V_{AS} \geq 1 - a_m - \frac{1}{2}\alpha(1-a_m) - \frac{1}{2}\beta(a_m - (1-a_m)) + V_{AS},$$

where V_{AS} is the expected payoff to the current period AS from future periods, if the current period proposal passes. V_{AS} depends on which one of the three games is being played. This inequality simplifies to

$$2a_m(2 - \alpha + 2\beta) \geq 2 - \alpha + 2\beta.$$

Given that $\alpha, \beta < 1$, this further simplifies to

$$a_m \geq 1/2.$$

Thus, the AS always prefers $a_m = 1/2$ to $a_m > 1/2$ when splitting only with a MWC.

He must also prefers such an allocation to any allocation that gives $a_m < 1/2$ to a MWC partner if

$$\frac{1}{2} - \frac{1}{2}\alpha\frac{1}{2} + V_{AS} \geq 1 - a_m - \frac{1}{2}\alpha(2(1-a_m) - a_m) + V_{AS}.$$

This condition simplifies to

$$2a_m(2 - 3\alpha) \geq 2 - 3\alpha,$$

and given that $a_m < 1/2$, it further simplifies to the required condition that

$$\alpha \geq 2/3.$$

The AS must also prefer to allocate evenly with only a MWC rather than to allocate evenly amongst the grand coalition. This is the case if

$$\frac{1}{2}-\frac{1}{2}\alpha\frac{1}{2}+V_{AS}\geq 1/3+V_{AS}\rightarrow\alpha\leq 2/3.$$

This implies that, except for a knife edge case where α is exactly $2/3$, the two conditions cannot be simultaneously satisfied. It is unreasonable to believe that the knife edge condition is satisfied (e.g. assuming that the common α is the realization of any continuous distribution with no mass points implies that $\alpha = 2/3$ is a zero probability event). Therefore, we conclude that incorporating other regarding preferences a la Fehr and Schmidt (1999) does not lead to equal division with a MWC being consistent with SSPE in the baseline game.

Finally, we must establish that the other players would accept an allocation of equal division amongst a grand coalition, if the AS were to deviate from equal division with a MWC to make such a proposal. (Otherwise the AS's preference for such an allocation over equal division with a MWC is not an acceptable deviation.)

A player votes in favor of equal division if

$$1/3+V_i\geq\left(\frac{2-\alpha-\beta}{6}\right)\left(\delta+\frac{1}{1-\gamma}\right),$$

where V_i is the player's expected future payoff from the proposal passing. V_i depends on the game, and whether we are considering symmetric or asymmetric SSPE.

In the baseline game and the symmetric SSPE of the majority support game (where reelection does not occur as part of equilibrium for the same reasons it did not occur originally), $V_i = (2-\alpha-\beta)/6$ and the required inequality simplifies to

$$1/3\geq\frac{2-\alpha-\beta}{6}\delta\rightarrow 2(1-\delta)+(\alpha+\beta)\delta\geq 0\delta,$$

which is clearly satisfied given $0<\alpha,\beta,\delta<1$.

In the asymmetric SSPE of the vote of confidence and majority support games (where the equilibria are of the structure considered in the earlier subsection on asymmetric equilibria), $V_i = (1/2)\gamma/(1-\gamma)$ for the player that is included in the AS's MWC strategy, and $V_i = 0$ for the player that is excluded. The included player will clearly support the equal division within a grand coalition deviation, rather than risk a player that excludes him being selected as AS in the future.

The above analysis rules out SSPE with equal shares to the AS and a MWC for the baseline and majority support games, and for the vote of confidence game assuming asymmetric SSPE. Finally, we need to rule it out for the symmetric SSPE case in the vote of confidence game. To do this, we take a different approach, and show that in this environment, a MWC player will vote against a proposal that splits the allocation evenly between herself and the AS. When the AS divides evenly with a MWC, the two non AS players have an expected value of future payoffs equal to

$$V_i=\left(\frac{1}{2}\left(\frac{2-\alpha}{4}\right)+\frac{1}{2}\left(-\frac{\beta}{4}\right)\right)\frac{1}{1-\gamma}=\frac{2-\alpha-\beta}{8}\frac{1}{1-\gamma}.$$

Let $\bar{a}$ denote the minimum acceptable share by a MWC player in the current period, when the players anticipate that the AS will split the allocation evenly with a randomly chosen player in future periods. $\bar{a}$ solves

$$\bar{a} - \frac{1}{2}\alpha\bar{a} - \frac{1}{2}\beta(1-2\bar{a}) + \frac{2-\alpha-\beta}{8}\frac{1}{1-\gamma} = \frac{2-\alpha-\beta}{6}\left(\delta + \frac{1}{1-\gamma}\right).$$

Substituting in for the values of δ and γ in the experiment and simplifying gives

$$\bar{a} = \frac{98 - 49\alpha + 116\beta}{90(2-\alpha+2\beta)}.$$

When $\alpha = \beta \to 0$, the value $\bar{a}$ achieves its minimum at $\bar{a}_{min} = 98/180 > 1/2$. Thus, a MWC who anticipates accepting allocations of 1/2 in future periods will reject offers of 1/2 in the current period, in hopes of being selected as AS in a new allocation of power.

This rules out the existence of an equilibrium in which the AS and a MWC player split the allocation evenly each period.

6. Instructions for Vote of Confidence treatment

This is an experiment in the economics of decision making. The instructions are simple. If you follow them carefully and make good decisions you may earn a considerable amount of money which will be paid to you at the end of the experiment. The currency in this experiment is called tokens. The total amount of tokens you earn in the experiment will be converted into US dollars: 10 Tokens = $1. You will also get a participation fee upon completion of the experiment.

General Instructions

(1) In this experiment you will be playing 8 Matches. During each Match, you will be randomly assigned an ID and you will be asked to make decisions over a sequence of Rounds.

(2) The number of Rounds in a Match is randomly determined as follows:

You will play every Match in blocks of 4 Rounds. Even though you will complete all 4 Rounds in each block you play, not all Rounds in a block will necessarily count towards your earnings for the Match.

The *first* Round in a Match will always count towards your earnings for that Match. Whether any of the following ones will count will be randomly determined according to the **"70% rule:"** after each Round that counts towards your earnings in a match, there is a 70% chance that the next Round will also count towards your earnings in a Match. The computer will determine this by randomly choosing a number between 1 and 100. If the number is less or equal to 70 then the next Round will also count towards your earnings for this Match.

Note however, that this random draw is done **"silently."** That is, you will play all four Rounds in a block but you will only find out at the end of the block which Rounds actually count towards your earnings for this Match. If each random draw the computer makes in a block is less or equal to 70, then you will move to the next block of 4 Rounds and so on. **Your earnings for a Match consist of the sum of all your earning over all the Rounds up until the computer drew a number above 70 for the first time in the Match.** The Match ends after the last Round of the block in which the computer drew a number above 70 for the first time.

(3) Once a Match ends, you will be randomly and anonymously rematched with two other people in this room to start a new Match. Each member in the group will again be randomly assigned an ID number. Thus, while your ID remains the same over Rounds *within* a Match, it is very likely to vary from Match to Match and you will not be able to identify who you've interacted with in previous or future Matches.

(4) **What Happens in Each Match**
- In each Match you will be randomly matched into groups of three members. Each member in the group is randomly assigned an ID number. Thus, while your ID remains the same over Rounds *within* a Match, it is very likely to vary from Match to Match.
- At the start of each Match, one of the three members in your group will be randomly chosen to be the Proposer.
- Step 1: The Proposer's task is to propose how to split a budget of 200 tokens between himself and the two other members of his/her group.
- Step 2: Once the Proposer has submitted a budget proposal, all members of your group will observe the budget proposal and will vote on it.
  (a) If a proposal receives a **simple majority of votes** (i.e. two or more members in your group vote in favor of the proposal), then the proposal passes and for this Round the earnings for each of you in the group will correspond to the number of tokens offered to them in that proposal.
  (b) If a proposal receives **fewer than 2 votes** then it is defeated. If a proposal is defeated, you will remain in the same Round, but the computer will then randomly choose one of the three members of your group to be the "new" Proposer. Each member of your group (including the previous proposer) has the same chance of being chosen (1 in 3). Whoever is chosen will submit a new proposal. However, the number of tokens to be divided will be reduced by 20% relative to the preceding proposal and rounded to the nearest integer. Thus, if the first proposal is rejected, then after a "new" Proposer is randomly selected, his/her proposal will involve splitting 160 tokens. If this proposal is rejected, again a "new" proposer will be chosen and his/her proposal will involve splitting 128 tokens, etc... This goes on until a proposed allocation gets 2 or more votes and passes.

Once a proposal receives two or more votes (whether right away or after a delay), you remain in the same group and will move onto the next Round. The Proposer who submitted the successful proposal remains in place, the budget then restarts at 200 tokens and you return to Step 1. This process repeats itself until a Match ends, which is determined by the 70% rule described above. Once a Match ends, you will start a new Match and will be randomly re-matched to form new groups of three. Remember: while your ID remains the same over

Rounds *within* a Match, it is very likely to vary from Match to Match.

(5) **Communication:** In each Round, before the Proposer submits his/her proposal, members of your group will have the opportunity to communicate with each other using a chat box. The communication is structured as follows. On the top of the screen, each member of the group will be told her ID number. You will also know the ID number of the Proposer. Below you will see a box, in which you will see all messages sent to either all members of your group or to you personally. You will not see the chat messages that are sent privately to other members of your group. You can type your own message and send it to one or both members of your group, and only the person(s) you select as recipient(s) will receive your message. The chat option will be available until the Proposer submits his/her proposal. At this moment the chat option will be disabled.

(6) Remember that in each Match subjects are randomly matched into groups and the ID numbers of the group-members are randomly assigned. Thus, while your ID remains the same over Rounds *within* a Match, it is very likely to vary from Match to Match.

(7) **Your Payment:** You will each receive a show-up fee. In addition, at the end of the experiment, the computer will randomly choose one out of 8 Matches that you played. You will be paid for ***all the Rounds that actually counted towards your payment within that Match*** (determined according to the 70% rule).

(8) **Screenshots:** We will now slowly go through different screenshots so you can familiarize yourself with the types of screens you'll be seeing. The examples we are about to go through are not meant to show you what you ought to do in this experiment but are just there to show you on screen the different possible stages of a Match. Please raise your hand if you have any questions about the experiment and/or interface.

7. Walk through screenshots in Vote of Confidence treatment

We are now going to go through what a Match may look like. These screenshots were generated by us and were not the result of actual lab participants. We chose these randomly and nothing you see here is an indication of what you ought to do in this experiment.

We will start by showing you what the screens look like and at the end show you what chat messages may look like.

PICTURE 1 HERE

This screen is the screen that each non-proposer sees. On the top center you will be able to see which Round and Match you are in. You will also be able to see what your member number is. In this particular case, the subject seeing this screen is member 2 and he/she is in Round 1 and Match 2 of the game.

The large box top left is the Message Window. In this message window you will be able to see all the messages that you wrote to someone and all the messages for which you were at least one of the recipients. You will not see the messages that were not sent to you. That is, you will not see the messages that were sent privately between the two other members in your group.

Below the Message Window are a number of other windows. These are the windows you will use if you want to send messages of your own. You will select who to send the message to, here either member 1 or member 3 or both. You select who to send a message to by clicking on the ID number corresponding to that member and then selecting Add. Notice that you will know who the proposer is from this part of the screen because Proposer will be written in parenthesis next to the ID number of the proposer (here Member 3 is the Proposer). If you chose to write to both members you can simply click Add All. You can type your message in the Send Message box. When you are ready you can send the message by clicking send. The person or people you send the message to will then see it appear on his/her/their screens. Only the member(s) you send the message to will see it.

Finally, at the bottom of the screen you will see your entire history of successful proposals. You can return to the history of earlier matches by simply clicking on the tab corresponding to that Match.

We are now going to show you the screen that proposers face.

PICTURE 2 HERE

This screen is the screen that each proposer sees. Just like the screen for non-proposers, each Proposer can send messages either to both or only one of the members in his/her group. Just as is the case for non-proposers, the proposer will only see the messages for which he was either the sender or a recipient. Similarly, if the proposer sends a message, only the member(s) that he selected as recipient(s) will see the message. At the bottom of their screens, proposers can also see the history of successful past proposals.

The difference with the screen the non-proposers see is that the proposer also has a space to submit a budget allocation.

On the right-hand side of the screen, the proposer will be reminded of the number of tokens he/she has to divide. In this case it is 200. The proposer will choose how much to allocate to member 1, how much to allocate to member 2 and how much to allocate to him/herself (in this case Member 3). Proposers can directly type their allocations in each box under A1 (amount allocated to member 1), A2 (amount allocated to member 2) and A3 (amount allocated to member3). Proposers can clearly see how much they've allocated to themselves because their box is highlighted in RED.

PICTURE 3 HERE

In this example, the Proposer (Member3) allocated 199 tokens to Member 1, no tokens to member 2 and allocated 1 token to himself.

Note that the box highlighted in RED is the box that corresponds to the allocation to you. Here, since the proposer is Member 3, box A3 is highlighted in RED.

If you are the proposer, when you are done communicating and have decided on a budget allocation you can click on the submit button. Once you click the submit button, all communication stops and all members of the group move onto the voting stage.

[Please note that this particular allocation does not represent what a proposer ought to do in this experiment, this is simply an example meant to illustrate what the different screens will look like. We could have chosen any distribution so long as the three numbers were all greater or equal to zero and summed to 200].

PICTURE 4 HERE

This screen is the type of screen that members see after the proposer has submitted his/her allocation. At this point the message space will be inactive. This particular screenshot is from Member 3.

On the right-hand side of the screen, everyone will see the allocation that was submitted. Because this is the screen of Member 3, the third box is highlighted in RED. The amount offered to you will always be highlighted in RED.

Below this are two buttons, one that says Yes and one that says No. If you support this proposal, click on the Yes button to vote for it. Click on the No button if you do not support this allocation. All members vote (including the proposer).

PICTURE 5 HERE

This screen is the screen that members see after all members have voted on the proposal.

You will be able to see what the votes were (in this case Member 1 voted no, Member 2 voted yes and Member 3 voted yes). If two or more members voted in favor of the proposal as is the case here, then you will be told how much you earned in this particular round. Here, since this is the screen for Member 3, the third number is highlighted in RED. Since two members have voted yes to the proposal, member 1 obtained 199 tokens, Member 2 received 0 tokens and Member 3 received 1 token.

[Please note that here we are just going through screenshots so you can familiarize yourself with the game. Just as the particular proposal was just an example and didn't represent what you ought to do in this experiment, the particular votes we are showing you here also do no represent how you ought to vote in this experiment.]

If the allocation proposal is successful as is the case in this example, you will move onto the next Round. In each group the same Proposer stays in place for the new Round and the budget restarts at 200 tokens.

You would then click "Continue" to move onto the next round.

If however, the proposal had failed (that is, received fewer than 2 votes), you would be still be shown the outcome of the vote but a new proposer would be chosen among the three people in your group (each with 1/3 chance) and you would then move to the following screens.

PICTURE 6 HERE

In this example, we are seeing the screenshot of Member 2, who is a non-proposer.

You can see that now the new proposer is Member 1. You can now communicate until the Proposer (here Member 1) submits an allocation.

The new Proposer sees the following screen.

PICTURE 7 HERE

It is almost identical to the Proposer screen of the previous proposer. There is one difference though: notice that since the previous proposal was rejected, Member 1 now only has 160 tokens to divide among the members of his/her group. If this proposal is rejected, a new proposer will be randomly chosen and this next proposer will only have 128 tokens to divide and so on. (Remember that the budget shrinks by 20% following each rejection until a proposal is passed).

Once a proposal is successful, the Proposer who submitted the successful proposal stays in place for the next Round and the budget restarts at 200.

PICTURE 8 HERE

When a block of 4 Rounds is over, if you are to continue for another block of 4 Rounds you will see something similar to this screen. In this case the Match is to continue for another block of 4 Rounds. You can see the history of play. This is the screen for Member 2. Member 2 was the last successful proposer and so he will continue to be proposer in the next Round.

The next block of 4 Rounds will automatically start shortly after you see this screen. You will play several Rounds until a Match is over, as determined by the 70% rule.

This process repeats itself until all 10 matches are complete.

We will now show you what chats can look like on your screen.

PICTURE 9 HERE

Recall that you only see a chat message if (1) you sent it or (2) you were at least one of the recipients. In this example we are looking at the Screen of Member 3. Member 1 (who is the proposer) has written a message to Member 1. The message content is "hi". Member 3 can see who sent him/her the message and can also see who received it. Here the message "hi" was sent exclusively to her since Member 2 is not listed as a recipient. In this example, Member 2 has no knowledge that this message exists.

PICTURE 10 HERE

Here is an example of what happens when you send a message to someone. In this case, this is the screen of Member 1 and Member 1 sent a message saying "hi" to Member 3. Member 1 sees it in his message window.

In other words, each player in this game will see the messages that he/she sent and also the messages that he or she received. You will not see the messages exchanged privately between the other members of your group.

Are there any questions?