Delegating Resource Allocation: Multidimensional Information vs. Decisions

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Abstract

This paper studies the problem of how to delegate the allocation of finite resources across multiple categories to an agent who has better information on their benefits. It focuses on a tractable, natural class of delegation policies that impose a floor or cap on the allocation to each category, a generalization of Holmström's (1977) "interval controls" to multidimensional settings. The paper characterizes the optimal policies and shows that they can impose distorting limits on categories which cause no conflict of interest with the agent, so as to curb how the conflict in other categories affects his overall allocation. Such limits are more likely to be optimal when the conflict is weaker and the agent also has specific information on categories causing no conflict. These solutions to the trade-off between rules and discretion differ substantively from those in settings with unidimensional decisions or information. The paper discusses applications to delegation within firms, the design of fiscal constitutions and policies, and individual commitment problems.

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KEYWORDS: delegation, multidimensional decision, multidimensional information, resource constraint, cap, floor.

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1 Introduction

In many settings, one party (principal, she) has to delegate to another, better informed party (agent, he) the allocation of a finite amount of resources across multiple categories—for example, money across projects or goods and time across tasks. Besides the presence of a fixed resource budget, often these problems have two other distinctive features. First, not only the agent's decision but also his information involves multiple dimensions—the value of each project, good, or task. Second, for some dimension of choice the agent's goals are in conflict with the principal's, but for others they are not—the agent may favor only some projects, goods, or tasks.

For such problems, this paper investigates how the principal designs her delegation policies trading off rules vs. discretion, and how these policies depend on the degree of conflict between the parties as well as the kind of information observed by the agent. The paper shows that the principal can benefit from imposing distorting restrictions on dimensions along which her preference *agrees* with the agent's, so as to limit how the conflict along other dimensions affects his overall allocation. Perhaps surprisingly, such restrictions are more likely to be optimal when the conflict is *weaker*, and when the agent also has information that is specific to dimensions causing no conflict of interest. In these situations, one may instead think that rules are *less* valuable than discretion, and that the agent should always be allowed to respond to specific information on dimensions which cause no conflict (hereafter, agreement dimensions).

For the sake of concreteness, I present the analysis in terms of a fiscal-constitution problem between society and the government.[1] Society delegates the government to allocate the economy's resources (its GDP) between private consumption and public spending. Their goals disagree, however: The government always favors higher public spending than does society.[2] Public spending finances multiple services under the control of the government (national security, law enforcement, infrastructures, etc.). In this paper, although the government is biased in favor of public spending, it does not favor any specific service more than others.[3] Thus, given any level of public spending, both parties agree on how to allocate it across services. Though strong, this assumption helps to highlight the roles of conflict and agreement dimensions in delegation problems. Before the government chooses an allocation, it observes non-contractible information (called state) of two types: one affects the overall trade-off between private consumption and public spending, the other the social benefit of each service. For example, the first type of information may be the state of the overall business cycle, the second may be threats to national security. Due to these different goals and information, society faces a typical trade-off between rules and discretion when designing a fiscal constitution, i.e., a delegation policy specifying which allocations the government can choose.

[1] This paper's model is similar to that in Amador et al. (2006), but differs from it by considering a richer allocation problem as well as information structure.

[2] This hypothesis is supported by theoretical as well as empirical work in the political-economy literature (Niskanen (1975), Romer and Rosenthal (1979), Peltzman (1992), Funk and Gathmann (2011)).

[3] The assumption is consistent with some empirical evidence. For example, Peltzman (1992) finds that "voters do not much care how the Federal government allocates its spending. Basically, every extra dollar is equally bad."

Similar delegation problems arise in many other settings. Shareholders delegate the allocation of financial resources between marketing multiple products and R&D to CEOs, who may be overly concerned about the company's short-run performance and hence assign relatively less importance to R&D. Managers delegate time allocation across multiple tasks to workers, who may value unproductive activities like surfing the internet or chatting with colleagues relatively more. Universities delegate time allocation between research projects and teaching to professors, who may value relatively more research than teaching. Individuals delegate income allocation across multiple consumption goods and savings to their future (or short-run) selves, who may be present biased.[4] An analogous problem may arise for governments with regard to current spending and borrowing.[5]

As is well known (see below), multidimensional decisions and information make mechanism-design problems hard to analyze. Therefore some structure needs to be imposed on the environment in order to make any progress. This paper allows for general distributions of the agent's information, but limits the externalities across the dimensions of his decisions. It also leverages the structure that the resource constraint gives to the problem. Finally, it mostly focuses on a tractable class of delegation policies: the multidimensional version of Holmström's (1977) "interval controls." Such policies "are simple to use with minimal amount of information and monitoring needed to enforce them" and "are widely used in practice" (Holmström (1977), p. 68).[6] Concretely, society can design fiscal constitutions that set either a *cap* or a *floor* on how much the government is allowed to allocate to private consumption and to each public service.[7]

To build intuition, Section 4 examines how restricting one dimension of the government's allocation at a time affects society's payoff. Since the government tends to consistently allocate too few resources to private consumption, we may expect society to design a constitution that sets a floor on private consumption (or equivalently an *aggregate* cap on public spending). Indeed, *if all society can do* is to limit how much the government is allowed to assign to private consumption, a floor that binds in some states strictly improves on a full-discretion policy. When binding, the floor prevents the government from splurging on public spending, but does not affect how it divides the available resources across services, a decision which raises no conflict with society. Moreover, society sets the floor strictly above the lowest level of private consumption that is optimal according to its preference. This result is related to those in Amador et al. (2006), but differs in important aspects explained in the paper.

The effects of caps on each public service are more subtle. When binding, such a

[4]See Thaler and Shefrin (1981), Laibson (1997), Gul and Pesendorfer (2004), Amador et al. (2006), Fudenberg and Levine (2006).

[5]See Halac and Yared (2014).

[6]Discussing multidimensional delegation, Armstrong (1995) writes, "in order to gain tractable results it may be that *ad hoc* families of sets such as rectangles or circles would need to be considered, and that [...] simple results connecting the dispersion of tastes and the degree of discretion could be difficult to obtain. Moreover, in a multidimensional setting it will often be precisely the *shape* of the choice set that is of interest." (p.20, emphasis in the original). The present paper addresses these considerations.

[7]Alonso and Matouschek (2008) and Amador and Bagwell (2013b) provide conditions for Holmström's "interval controls" to be fully optimal in settings with unidimensional decision and information. No such conditions have been derived for the multidimensional case yet.

cap mitigates the government's overspending *in aggregate*, but by freeing up resources, it also exacerbates overspending on all other services. It thus distorts the allocation for dimensions along which both parties' preferences are not in conflict. Despite these drawbacks, capping services can benefit society: Even if it can impose a cap on only one of them, doing so strictly dominates granting the government full discretion. Intuitively, the implied higher private consumption yields a first-order gain for society, as it values this dimension relatively more. On the other hand, the distortions in the provision of public services yields a second-order loss for society, as its preference on these dimensions agrees with the government's.

The paper then examines policies that combine multiple caps and floors. Service-specific caps can be combined to impose, de facto, a target floor on private consumption. Such a combination, however, can implement the same allocations as using an actual floor if and only if information affects only the trade-off between private consumption and public spending. This is because specific caps cannot ensure that public spending stays below a certain level, and at the same time allow the government to freely vary how it allocates this level across services in response to their idiosyncratic information.

Specific caps can also be used *on top of* an aggregate one (or a floor on private consumption). Does society ever benefit from doing so? The answer turns out to depend both on the strength of the government's bias and on the type of its information.

On the one hand, everything else equal, if the government's bias is sufficiently *weak*, then an optimal fiscal constitution must involve service-specific caps. That is, it leads to distortions along agreement dimensions as an optimal way to control the government's bias in other dimensions. This is because, when information also affects the value of each service, the government spends the most on one service in the states where its value is much higher than for the other services, whereas it spends the most in aggregate in states where *all* services have high values. An aggregate cap deals with the latter states. However, if it is set relatively high—which is what society wants to do when the bias is weak—it may not curb overspending in the former states. To overcome this issue, society can add service-specific caps that bind only when the aggregate one does not.

On the other hand, if the government's bias is sufficiently *strong*, a policy featuring only an aggregate cap on public spending (or floor on private consumption) is optimal. This is because it turns out that society can never benefit from letting the allocation to private consumption fall below the lowest optimal level from its viewpoint. But this limit will always bind for a strongly biased government, even if it faces service-specific caps. Since such caps then do not improve private consumption and create distortions along agreement dimensions, they can only harm society. For strong enough biases, it is also optimal for society to commit ex ante to a fixed level of public spending and grant the government full discretion in allocating it across services.

A general message emerges here. In a concrete setting, we may observe richer delegation policies with more rules than in other settings and infer that this is because the principal faces an agent with a stronger bias. The paper shows, however, that the correlation between complexity and conflict of interest may actually be negative, as richer policies can work better when the agent's bias is weaker and simpler ones when his bias is stronger.

A simple policy with only an aggregate cap is also more likely to be optimal when the government's service-specific information decreases. Section 6 considers the limit case in which the information affects only the trade-off between private consumption and public spending (i.e., it is unidimensional). Despite this, the multiplicity of services continues to play a key role. If the government is forced to allocate high spending levels inefficiently across services—for example, using specific caps—it will find such levels less attractive and hence will tend to penalize private consumption less. Another way to deter the government from doing so, however, is money burning, namely forcing it to "throw away" part of what it does not allocate to private consumption.[8] The paper shows that when money burning is allowed and fiscal constitutions can be *any* set of feasible allocations—not just generalized intervals—it is without loss to restrict attention to policies that regulate only private consumption and aggregate spending, but not single services.

This simplification helps us understand when policies imposing only a consumption floor are optimal for any degree of the government's bias. If we focus on *all* sets of feasible allocations in terms of public spending and private consumption only, Amador et al.'s (2006) main result implies that those floor policies are optimal if and only if the information distribution satisfies a simple, weak condition. This condition is then sufficient (but not necessary) for the same policies to be optimal among all generalized-interval ones. However, it happens to be more likely to hold when the bias is weak, which corresponds to settings where policies with only a consumption floor are no longer optimal if information is multidimensional. Amador et al. (2006) argue that, when their condition fails, money burning can be part of an optimal policy. The present paper shows that, by requiring distorted allocations across public services, society can achieve the same payoff while burning less money. In some settings, money burning is even superfluous. This is the case, for instance, if welfare falls significantly when some public service receives very few resources, which seems plausible for cases like national security, law enforcement, or criminal detention. This is useful if in practice money burning is infeasible or illegal. In short, even with unidimensional information, treating public spending as a monolithic entity can be restrictive, and distortions along agreement dimensions may again be part of an optimal delegation policy.

Section 7 discusses the implications of these results for the other settings with similar delegation problems mentioned before, which relate to the literatures on public finance, individual commitment problems, corporate governance, and organization design.

2 Related Literature

This paper contributes to the literature on the trade-offs between rules (commitment) and discretion (flexibility) and its numerous applications.[9] It shows how a more realistic

[8]Papers that study money burning as a tool to shape incentives in delegation problems include Amador et al. (2006), Ambrus and Egorov (2009), Ambrus and Egorov (2013), Amador and Bagwell (2013a), Amador and Bagwell (2013b).

[9]See also Athey et al. (2005), Ambrus and Egorov (2013), Amador and Bagwell (2013b).

and detailed description of the decision problem can open the door to identifying superior (yet simple) commitment policies. The closest papers are Amador et al. (2006) and Halac and Yared (2014). In the first paper, information has one dimension, allocations have two (only total public spending matters for payoffs), and the agent has a known bias against one of them. As the above examples suggest, in many settings allocations as well as information involve more dimensions. The present paper shows that Amador et al.'s (2006) main result does not generalize to such richer settings. Also, in contrast to Amador et al. (2006), this paper has to restrict the class of delegation policies and rely on different techniques, due to the well-known intricacies of multidimensional mechanism design (discussed below). In the dynamic settings of Halac and Yared (2014), society and the government disagree on how to trade off consumption between the present and the future, but not across future periods; in each period both consumption and information is unidimensional, but the latter is correlated over time. As a result, optimal fiscal constitutions may distort future consumption, even though it causes no conflict of interest once its current level is fixed. This is because society can relax the government's current incentive constraints by exploiting the link, created by information, between the values of present and future allocations, as in other dynamic mechanism-design problems.[10] By contrast, in the present paper agreement dimensions may be distorted to exploit the link with other dimensions created by the resource constraint.

More generally, this paper relates to the literature on optimal delegation in principal-agent settings following Holmström (1977, 1984). Delegation problems naturally involve multidimensional decisions and information. Few papers, however, have examined such problems—and not of the kind considered here.[11] In Koessler and Martimort (2012), the agent's information has one dimension and uniform distribution, his decision is bi-dimensional and unconstrained, and payoffs are quadratic. This causes payoffs to ultimately depend on the decision's mean and spread across dimensions; also, the latter can be used as an imperfect pseudotransfer to screen the agent's information.

In Frankel (2014) and (2015), both information and decisions are multidimensional. In Frankel (2014), the agent has the *same* bias for all dimensions, but the principal is uncertain about its properties (strength, direction, etc.). The paper characterizes the delegation policies that maximize the principal's payoff for the worst-case bias (max-min policies). Frankel's (2014) "budget" policies should not be confused with caps and floors in this paper: They require the *average* decision across dimensions to satisfy a preset value (called budget). Frankel (2015) considers policies that set caps and floors not directly on decisions, but "against the agent's bias:" They limit the gap, due to the bias, between the agent's and principal's total utilities from the implemented decisions. Such policies are fully optimal if payoffs are quadratic and information is normally, i.i.d. distributed across dimensions. More generally, they ensure that, relative to the first best, the principal's loss *per dimension* vanishes as the number of dimensions grows while information remains independent across them. By contrast, the present paper

[10] See, for example, Courty and Li (2000), Battaglini (2005), Pavan et al. (2014).

[11] In Alonso et al. (2013), a fixed amount of resources has to be allocated across multiple dimensions, but each dimension is controlled by a different agent with a unidimensional piece of information. This makes the class of problems they study fundamentally different from that of the present paper.

allows for general distributions and considers a different class of delegation policies. It shows that policies which constrain dimensions for which the agent is *not* biased may strictly dominate policies which do not—like Frankel's (2015). It also provides useful result for settings in which the agent's decision has a small, fixed number of dimensions.

Finally, this paper relates to the literature on multidimensional screening (see Stole and Rochet (2003) for a detailed survey). This literature allows for transfers between the principal and the agent, a key difference from delegation problems which prevents us from applying the lessons from that literature and forces us to consider a restricted class of policies. In the present paper we could use the dual approach and other methods in Rochet and Choné (1998) to summarize the agent's incentive constraints and simplify the principal's objective. The fixed resource budget, however, adds a state-wise constraint to the problem. Although general techniques exist for such problems (Luenberger (1969)), in our case they do not help to characterize the optimal unrestricted mechanisms, which need not follow the same logic of the unidimensional case—as suggested by Rochet and Choné (1998)—or for that matter of monopolistic screening. One benefit of this paper's approach is that its results are insensitive to details of the information structure. By contrast, such details can significantly affect the solution of multidimensional screening problems (see Manelli and Vincent (2007)).

3 Model

For the sake of concreteness, I present the model in terms of a fiscal-constitution problem between society and the government. Section 3.1 discusses other interpretations and applications. The setting is similar to that studied in Amador et al. (2006) (hereafter, AWA), except for two aspects which I will indicate shortly.

Society delegates to the government the choice of how to allocate the economy's known resources, I, between private consumption, $x_0 \geq 0$, and public spending, $y \geq 0$, subject to the constraint $x_0 + y \leq I$.[12] Society and the government systematically disagree on how I should be allocated: The government always favors higher public spending than does society.[13] Following AWA, if y leads to a payoff $w(y)$ and x_0 to a payoff $v(x_0)$, then society's overall welfare is $\theta w(y) + v(x_0)$ and the government's payoff is $\theta w(y) + bv(x_0)$. The variable $\theta > 0$ represents non-contractible information that arrives before the government chooses an allocation and determines the importance of public spending vs. private consumption. For instance, θ could capture the state of the business cycle: In a recession public spending may become more valuable, corresponding to a higher θ. The parameter $b \in (0, 1)$ captures in a tractable way the preference conflict by systematically biasing the government in favor of public spending.

A key difference between private consumption and public spending is that the government controls how the amount y is allocated across multiple services—national security, law enforcement, infrastructures, health care, etc. Arguably, different allocations of the same amount y yield different payoffs for society. To capture this feature, let $n > 1$

[12] In the proofs, society is denoted by P for principal and the government by A for agent.

[13] See Footnote 2 for theoretical and empirical justifications of this assumption.

be the number of public services and x_i be the amount of resources allocated to service i. This multidimensionality of public spending is the first departure from AWA. Since $y = \sum_{i=1}^n x_i$, the resource constraint becomes

$$B = \{(\mathbf{x}, x_0) \in \mathbb{R}_+^{n+1} : \sum_{i=0}^n x_i \le I\}.$$

For simplicity, normalize I to equal 1. We can then interpret each x_i as the share of resources allocated to service i.

Besides the information on the value of public spending vs. private consumption, the government may also observe non-contractible information on the social return of each public service. This idiosyncratic information is captured by a variable $\mathbf{r} \in \mathbb{R}^n$, where r_i refers to service i. For instance, r_i can measure the intensity of national-security threats or of natural disasters requiring public relief. This multidimensionality of information is the second departure from AWA. Given $(\theta, \mathbf{r})$, society's and the government's payoffs become

$$\theta u(\mathbf{x}; \mathbf{r}) + v(x_0) \quad \text{and} \quad \theta u(\mathbf{x}; \mathbf{r}) + bv(x_0).$$

Although the government favors public spending over private consumption relative to society, it does not favor any specific public service more than others.[14] This simplifying assumption is of course strong, but it helps to highlight the role of dimensions of (more) conflict and of (more) agreement in delegation problems. To add clarity and tractability to the model, assume that

$$u(\mathbf{x}; \mathbf{r}) = \sum_{i=1}^n u^i(x_i; r_i) \quad \text{with} \quad u_{12}^i > 0 \text{ for all } i.$$

This additive-separability assumption may rule out realistic externalities across public services, but it will help to isolate the mechanisms of interest for this paper. This assumption is superfluous for some of the results below, which I will point out. I expect that the other results are robust to moderate externalities (see below).

Our information structure involves some degree of redundancy, as both an increase in θ and an increase in all components of $\mathbf{r}$ render public spending more valuable. This approach, however, has several benefits. First, it clarifies the conceptual distinction between service-specific information and information affecting the overall trade-off with private consumption. Second, it will allow us to keep the same model throughout the entire analysis and hence focus on the core messages of the paper. Third, it simplifies the comparison with the literature.

The following are mostly technical assumptions:

- *Information*: Let $S = [\underline{\theta}, \overline{\theta}] \times [\underline{r}_1, \overline{r}_1] \times \cdots \times [\underline{r}_n, \overline{r}_n]$, where $0 < \underline{\theta} < \overline{\theta} < +\infty$ and $-\infty < \underline{r}_i < \overline{r}_i < +\infty$ for all $i = 1, \dots, n$. The joint distribution of $(\theta, \mathbf{r})$ is represented by the probability measure G which has full support over S; that is, $G(S') > 0$ for every open $S' \subset S$.[15]

[14] See Footnote 3 for some evidence supporting this property.

[15] This holds, for instance, if G has a strictly positive and continuous density function over S.

- *Differentiability, monotonicity, concavity*: The function $v : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable with $v' > 0$ and $v'' < 0$. For all $i = 1, \ldots, n$, the function $u^i : \mathbb{R}_+ \times [\underline{r}_i, \overline{r}_i] \to \mathbb{R}$ is twice differentiable with $u^i_1(\cdot; r_i) > 0$ and $u^i_{11}(\cdot; r_i) < 0$ for all $r_i \in [\underline{r}_i, \overline{r}_i]$; also, u^i_1 and u^i_{11} are continuous in both arguments.

- *Boundary conditions*: $\lim_{x_0 \to 0} v'(x_0) = +\infty$ and $\lim_{x_i \to 0} u^i_1(x_i; r_i) = +\infty$ for all $r_i \in [\underline{r}_i, \overline{r}_i]$ and $i = 1, \ldots, n$. This will allow us to focus on interior solutions.

As usual in delegation problems, here society faces a trade-off between rules and discretion. On the one hand, it benefits by letting the government act on its information; on the other, it may want to limit the government's freedom to choose how to allocate the economy resources. To do so, society may design a fiscal constitution dictating restrictions and guidelines that an allocation must satisfy. Formally, such a constitution is a nonempty subset C of the resource constraint B, containing the allocations the government is allowed to implement. For example, if $C = B$, the government has full discretion; if C contains only one element, it has no discretion at all. I will be more precise below about the class of fiscal constitutions examined in this paper.

The timing is as follows. First, society designs and commits to a fiscal constitution C. Then, the government observes information $(\theta, \mathbf{r})$ and chooses an allocation $(\mathbf{x}, x_0)$ in C. The allocation is implemented and payoffs realize. Society designs C to maximize its expected payoff.

The rest of the paper uses the following simplifying notation and definitions. Denote every element of S by $\mathbf{s} = (\theta, r_1, \ldots, r_n)$, which we will call state. Let society's and the government's payoff in each $\mathbf{s}$ from allocation $(\mathbf{x}, x_0)$ be

$$U(\mathbf{x}, x_0; \mathbf{s}) = \hat{u}(\mathbf{x}; \mathbf{s}) + v(x_0) \quad \text{and} \quad V(\mathbf{x}, x_0; \mathbf{s}) = \hat{u}(\mathbf{x}; \mathbf{s}) + bv(x_0), \tag{1}$$

where $\hat{u}(\mathbf{x}; \mathbf{s}) = \theta \sum_{i=1}^n u^i(x_i; r_i)$. For each $\mathbf{s}$, let $\pi^*(\mathbf{s})$ be the allocation society would like the government to choose in that state and $\alpha^*(\mathbf{s})$ the allocation it actually chooses under full discretion:

$$\pi^*(\mathbf{s}) = \arg\max_B U(\mathbf{x}, x_0; \mathbf{s}) \quad \text{and} \quad \alpha^*(\mathbf{s}) = \arg\max_B V(\mathbf{x}, x_0; \mathbf{s}). \tag{2}$$

The functions π^* and α^* satisfy useful properties, summarized in the following lemma for ease of reference.

Lemma 1.
- *Both π^* and α^* are continuous in $\mathbf{s}$.*
- *For all $i = 0, \ldots, n$, the range of π^*_i (resp. α^*_i) equals an interval $[\underline{\pi}^*_i, \overline{\pi}^*_i]$ (resp. $[\underline{\alpha}^*_i, \overline{\alpha}^*_i]$), with $0 < \underline{\pi}^*_i < \overline{\pi}^*_i < 1$ (resp. $0 < \underline{\alpha}^*_i < \overline{\alpha}^*_i < 1$).*
- *For all $\mathbf{s} \in S$, $\alpha^*_0(\mathbf{s})$ is strictly increasing in b and $\alpha^*_0(\mathbf{s}) < \pi^*_0(\mathbf{s})$ if and only if $b < 1$.*
- *For all $\mathbf{s} \in S$, each $\alpha^*_i(\mathbf{s})$ is strictly decreasing in b.*

These properties follow immediately from (1) the assumptions on U and V, (2) compactness, connectedness, and convexity of S, and (3) standard comparative-statics arguments. Note also that all dimensions of $\mathbf{x}$ are normal goods in the sense that the optimal allocation to each x_i increases as the total resources y available for public spending increase.[16]

[16]Given our assumptions, this property follows, for instance, from Proposition 1 in Quah (2007).

A final observation which will be useful later is that, fixing private consumption, society and the government agree on how to allocate the remaining resources across services: Given $\hat{x}_0 \in [0, 1]$,

$$\underset{\{(\mathbf{x},x_0)\in B : x_0 = \hat{x}_0\}}{\arg\max} U(\mathbf{x}, x_0; \mathbf{s}) = \underset{\{(\mathbf{x},x_0)\in B : x_0 = \hat{x}_0\}}{\arg\max} V(\mathbf{x}, x_0; \mathbf{s}) \quad \text{for every } \mathbf{s}. \tag{3}$$

3.1 Alternative Interpretations and Applications

This section outlines other settings that create delegation problems which can be modeled using the above framework. The reader interested in the results can skip this part.

Intrapersonal commitment problems. In each period an agent has to allocate his income I between savings, x_0, and immediate consumption, y, which involves multiple goods, x_1, x_2, etc. The agent as two selves, called Planner and Doer. Doer is in charge of choosing the allocation and suffers from present bias, which induces him to systematically overweigh immediate consumption (i.e., $b < 1$). Knowing this, Planner would like to limit the effect of Doer's bias by restricting the available allocations. However, some time elapses between when Planner can commit and when Doer chooses. In the mean time, information arrives not only on the relative value of consumption vs. savings (θ), but also on the utility from each consumption good ($\mathbf{r}$). This information may be taste shocks or observation of prices, which determine how each dollar spent on good i, x_i, translates into its physical units.[17] To commit, Planner may force Doer to choose from a subset C of his budget set B.

Workers' time management. In a company, a manager supervises a worker. The worker's contract specifies a workweek of I hours and a fixed wage. The worker is in charge of multiple tasks and chooses how to allocate his time across them (x_1, x_2, etc.). Moreover, he can spend some time x_0 on personal unproductive activities, such as having lunch, chatting with colleagues, going to the bathroom, or surfing the internet. In this case, manager and worker may disagree on the importance of such activities: He is likely to weigh his benefits from x_0 more than does the manager (i.e., $b > 1$ in the model).[18] Being on the shop floor, the worker has firsthand information on which task demands more attention and time at each moment. Given this, the manager would like to let him choose how to allocate his time. However, she may also want to set up some rules to avoid that he spends too much time on unproductive activities. Again, we can model such rules with a subset C of the worker's weekly time budget B.

Corporate governance. The shareholders of a company appoint a CEO, who each year decides how to allocate an overall budget I to R&D, x_0, and sales activities, y (e.g., ad campaigns). The company sells multiple products and the CEO also chooses which

[17]For instance, for all $i = 1, \dots, n$, let $\overline{u}^i(z_i) = \frac{z_i^{1-\gamma_i}}{1-\gamma_i}$ with $\gamma_i > 0$ be the utility from z_i units of good `i and $p_i > 0$ be its ex-ante uncertain price. If we define $x_i = p_i z_i$ for all i, the budget constraint becomes $\sum_{i=1}^n x_i + x_0 \leq 1$. Letting $r_i = p_i^{1-\gamma_i}$, we can also define $u^i(x_i; r_i) = r_i \overline{u}^i(x_i)$, which satisfies all our assumptions. This example can be generalized by allowing each $\overline{u}^i$ to be a smooth, strictly increasing, and strictly concave function.

[18]The paper focuses on the case of $b < 1$, but explains how the analysis can be adapted for the case of $b > 1$.

share of y goes to promoting each of them (x_1, x_2, etc.). Being more concerned with the company's short-run performance, the CEO may assign more importance to current sales than to R&D relative to the shareholders (i.e., $b < 1$). Since he manages the business on a daily basis, he has superior information on the returns from marketing each product and from funding R&D, information the shareholders would like to see incorporated in the allocation of I. However, they may be concerned about the CEO's focus on short-term performance. Therefore, they may want to limit his freedom by requiring him to select from a subset C of all feasible allocations in B.

Research vs. teaching in academia. A university employs a professor to teach and conduct research. Each month, the professor has a total amount of hours I that she can allocate to research, y, or teaching, x_0. Also, she works on several research projects and has to choose how much of y to spend on each (x_1, x_2, etc.). As is often the case, the professor may care more about his research than teaching, relative to the university (i.e., $b < 1$). Nonetheless, she has better information on which activity is more likely to advance her as well as the university's interests on a weekly basis. Thus, the university would like to let the professor choose how to allocate her time across activities. At the same time, it may also want to establish some rules to limit the risk that she overlooks teaching. Again, we can capture such rules with a subset C of the professor's time budget B.

Public finance. As in Halac and Yared (2014), each year t the government chooses how much to borrow, z^t, and spend, y^t, subject to the intertemporal constraint $y^t \leq \tau + z^t/\rho - z^{t-1}$, where z^{t-1} is the nominal debt inherited from period $t-1$, τ is a fixed tax revenue, and ρ is an exogenous (gross) interest rate. Differently from their model, here the government divides y^t across multiple services, $\mathbf{x}^t$, and its information, $(\theta^t, \mathbf{r}^t)$, is i.i.d. over time. At the beginning of each year, before observing any information, the government evaluates an allocation $(\mathbf{x}^t, z^t)$ using the function $\theta^t u(\mathbf{x}^t; \mathbf{r}^t) + \hat{v}(z^t)$, where $\hat{v}(z^t)$ is the expected payoff from entering period $t+1$ with debt z^t. By contrast, after observing θ^t and $\mathbf{r}^t$, the government chooses an allocation using the function $\theta^t u(\mathbf{x}^t; \mathbf{r}^t) + b\hat{v}(z^t)$ with $b \in (0, 1)$. Halac and Yared (2014) discuss several rationales for the government's present bias; for instance, Jackson and Yariv (2011) show that it arises under natural assumptions when the government aggregates the preferences of heterogeneous citizens, even if they are all time consistent. Anticipating its inconsistency, at the beginning of each year the government may commit to some fiscal rules for that year.[19] To map Halac and Yared's (2014) setting to the present one, we can assume an exogenous upper bound on borrowing $Z < +\infty$, let $x_0^t = -z^t/\rho$ and $I(x_0^t) = \tau + \rho x_0^t$, and define $v(x_0^t) = \hat{v}(-\rho x_0^t)$. The intertemporal constraint becomes $\sum_{i=1}^n x_i^t + x_0^t \leq I(x_0^{t-1})$. At the beginning of each period t, given x_0^{t-1} the government can commit to some budget plan $C \subset B(x_0^{t-1})$.

[19] Due to the i.i.d. assumption, considering fiscal rules that bind only for one period is without loss of generality (see, Amador et al. (2003); Halac and Yared (2014)).

4 Tractable Delegation Policies: Generalized Intervals

In principle, we would like to find society's best policy among all possible fiscal constitutions, that is, all subset C of B. However, as Holmström (1977) noted, "one might want to restrict C to [...] only certain simple forms of [policies], due to costs of using other and more complicated forms or due to the fact that the delegation problem is too hard to solve in general." In his seminal work in which the agent's decision is unidimensional, Holmström (1977) restricted attention to interval policies, noting that such policies "are simple to use with minimal amount of information and monitoring needed to enforce them" and "are widely used in practice." For similar reasons, this paper focuses on a class of policies which correspond to the multidimensional generalization of Holmström's intervals.[20]

Society can design constitutions which impose either a cap or a floor on how much the government can allocate to private consumption and to each public service. Formally, given $\mathbf{f}, \mathbf{c} \in \mathbb{R}_+^{n+1}$ that satisfy $f_i \leq c_i$ for $i = 0, 1, \ldots, n$ and $\sum_{i=0}^{n} f_i \leq 1$, let

$$C_{\mathbf{f},\mathbf{c}} = \{(\mathbf{x}, x_0) \in B : f_i \leq x_i \leq c_i \text{ for all } i\}.$$

Denote the collection of all such delegation policies by $\mathcal{R}$ (for "rectangles"). Given a policy $C_{\mathbf{f},\mathbf{c}}$, a floor (or cap) can constrain the government's decision in some states but not in others. Therefore, when describing a policy from the ex-ante viewpoint, I will refer to a floor (or cap) as *binding* if it constrains the government in a set of states with strictly positive probability. For simplicity, when considering policies in $\mathcal{R}$, I will leave $\mathbf{f}$ and $\mathbf{c}$ implicit unless required by the circumstances. Formally, society has to choose $C \in \mathcal{R}$ so as to maximize

$$\mathcal{U}(C) = \int_S U(\alpha(\mathbf{s}); \mathbf{s}) dG \tag{4}$$

where, for all $\mathbf{s}$,

$$\alpha(\mathbf{s}) \in \underset{(\mathbf{x}, x_0) \in C}{\arg\max}\, V(\mathbf{x}, x_0; \mathbf{s}). \tag{5}$$

This problem has a solution.

Lemma 2. *There exists C that maximizes $\mathcal{U}(C)$ over $\mathcal{R}$.*[21]

Given this result, we can turn to characterizing the optimal delegation policies.

4.1 Restricting Disagreement Dimensions

Society and the government disagree on how much they value private consumption relative to public spending. We may then conjecture that society designs a fiscal constitution which focuses on this one and only dimension of disagreement. Since the government

[20] In settings similar to Holmström's (1977), the literature has identified conditions for intervals to be optimal when no restriction on the delegation policies is imposed (see, for example, Alonso and Matouschek (2008) and Amador and Bagwell (2013b)). Similar conditions are not available, however, for multidimensional settings like in the present paper.

[21] All proofs appear in the Appendix.

tends to allocate too few resources to private consumption, we may expect that society imposes only a floor on this dimension.[22] When binding, the floor prevents the government from splurging on public spending, but never affects how it divides the resources across services, a decision which raises no conflict (recall (3)).

As a first step in examining this conjecture, this section develops the following result: *If all society can do* is to impose limits on how much the government can allocate to private consumption, a floor f_0 that binds in some states improves on the full-discretion policy (i.e., on $C = B$). Moreover, the optimal f_0 is *strictly* higher than the lowest level of private consumption that is optimal from society's viewpoint (i.e., $f_0 > \underline{\pi}_0^*$).[23] These results rely only on two properties of the function $\hat{u}$ in (1): continuity and strict concavity in $\mathbf{x}$. Hence, they would not change if we allowed for general interactions across dimensions of $\mathbf{x}$ and dependence on $\mathbf{s}$.

Proposition 1. *When the only available delegation policies involve a floor on x_0, it is optimal to set f_0 strictly between $\underline{\pi}_0^*$ and $\overline{\pi}_0^*$.*

Note that, in some settings, for practical reasons it may be possible to restrict only dimension x_0. For example, in the application to research vs. teaching in academia, a university can easily request and monitor that a professor allocates at least f_0 hours per week to teaching, but may not be able to constrain the time that she spends on each of her research projects.

Proposition 1 is clearly related to the results in AWA, but differs in several important respects. In their setting in which both information and public spending are unidimensional, they are able to consider *all* subsets of B as feasible delegation policies and show that the optimal one must indeed involve a binding floor on private consumption. AWA derive their result through a clever application of mechanism-design techniques. As is well known,[24] similar techniques are not available for the present setting with multidimensional information and public spending, which renders finding the optimal policy among all subsets of B a very hard problem. Therefore, this paper first restricts attention to policies which can involve only a floor or cap on private consumption. Second, to prove the optimality of a binding floor, it relies on different techniques from AWA.

The intuition for Proposition 1 is simple. First, society wants to set f_0 at least as high as $\underline{\pi}_0^*$. On the one hand, under full discretion, for some states the government allocates strictly less than $\underline{\pi}_0^*$ to private consumption. But such allocations are never justifiable from society's viewpoint. On the other hand, fixing any x_0, both parties always agree on how $1 - x_0$ should be divided across services. Therefore, even when f_0 is binding, it does not distort the provision of public services. Second, the property that $f_0 > \underline{\pi}_0^*$ follows from an envelope-type argument. For states in which society would choose $x_0 > \underline{\pi}_0^*$, it strictly prefers setting $f_0 = x_0$ and letting the government freely allocate $1 - f_0$ than setting $f_0 = \underline{\pi}_0^*$, which leads to overspending. By contrast, for states in which

[22] Of course, this floor can be equivalently implemented with an aggregate cap on public spending.

[23] Proposition 4 below will imply that it is never beneficial to impose a binding cap on private consumption (i.e., $c_0 < \overline{\alpha}_0^*$).

[24] See, for example, Rochet and Choné (1998) and their discussion on direct and dual approaches to screening problems.

society would choose $x_0 = \underline{\pi}_0^*$, setting $f_0 = \underline{\pi}_0^*$ already perfectly realigns the government's allocation with society's goals in *all* dimensions; therefore, a marginal increase in f_0 has only a second-order negative effect on society's payoff. Finally, society never wants to allocate more than $\overline{\pi}_0^*$ to private consumption, and hence $f_0 \leq \overline{\pi}_0^*$. A similar envelope-type argument explains why this inequality must be strict at the optimum.

Proposition 1 relies on the following lemma, which is also useful to study how f_0 varies as the government's bias b changes. For any floor $f_0 \in [\underline{\alpha}_0^*, \overline{\pi}_0^*]$,[25] for simplicity denote by C_{f_0} the corresponding policy in $\mathcal{R}$.

Lemma 3. *Define* $\overline{S}(f_0) = \{\mathbf{s} \in S : \alpha_0^*(\mathbf{s}) \leq f_0\}$ *and*

$$\mathbf{x}^{f_0}(\mathbf{s}) = \arg \max_{\{\mathbf{x} \in \mathbb{R}_+^n : \sum_{i=1}^n x_i \leq 1 - f_0\}} \hat{u}(\mathbf{x}; \mathbf{s}).$$

The payoff $\mathcal{U}(C_{f_0})$ *is differentiable in* f_0 *over the domain* $[\underline{\alpha}_0^*, \overline{\pi}_0^*]$ *with*

$$\frac{d}{df_0}\mathcal{U}(C_{f_0}) = \int_{\overline{S}(f_0)} \left[v'(f_0) - \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}^{f_0}(\mathbf{s}); \mathbf{s}) \right] dG,$$

for any $i = 1, \ldots, n$.

Using this result, we can easily see which considerations determine society's choice of an optimal floor f_0^*. Of course, f_0^* must satisfy the first-order condition $\frac{d}{df_0}\mathcal{U}(C_{f_0^*}) = 0$. To obtain a simple interpretation of this condition, given $f_0 \in (\underline{\pi}_0^*, \overline{\pi}_0^*)$, let $S_m(f_0)$ contain all states in which society would allocate more than f_0 to private consumption and $S_l(f_0)$ contain all states in which it would allocate less than f_0:

$$S_m(f_0) = \{\mathbf{s} \in S : \pi_0^*(\mathbf{s}) > f_0\} \quad \text{and} \quad S_l(f_0) = \{\mathbf{s} \in S : \pi_0^*(\mathbf{s}) < f_0\}.$$

Also, define the positive function ϕ as

$$\phi(\mathbf{s}) = \begin{cases} v'(f_0) - \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}^{f_0}(\mathbf{s}); \mathbf{s}) & \text{if } \mathbf{s} \in S_m(f_0) \\ \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}^{f_0}(\mathbf{s}); \mathbf{s}) - v'(f_0) & \text{if } \mathbf{s} \in S_l(f_0) \end{cases}, \tag{6}$$

for any $i = 1, \ldots, n$. Intuitively, $\phi(\mathbf{s})$ represents the shadow benefit of raising f_0 in the states in $S_m(f_0)$ and the shadow cost of doing so in the states in $S_l(f_0)$. We can then express the first-order condition as

$$\int_{\overline{S}(f_0^*) \cap S_m(f_0^*)} \phi(\mathbf{s}) dG = \int_{\overline{S}(f_0^*) \cap S_l(f_0^*)} \phi(\mathbf{s}) dG.$$

That is, *when affecting the government's decision with* f_0^*, the expected benefits for the states that demand higher consumption must equal the expected cost for the states that demand lower consumption.

How does the optimal floor change as the government becomes more biased? Everything else equal, society should tighten f_0^*. Indeed, it turns out that $\mathcal{U}(C_{f_0})$ is submodular in (f_0, b), as $\frac{d}{df_0}\mathcal{U}(C_{f_0})$ decreases in b. Intuitively, as the bias worsens, the government penalizes private consumption more; so any f_0 is more likely to bind. This strengthens the

[25] Any other floor is dominated by one in this range.

expected benefit of raising f_0 when society would choose an even higher level of private consumption. But it does *not* change the expected cost of raising f_0 when society would choose a lower level: In such states f_0 binds for any bias, as the government always prefers to allocate less to private consumption than does society. To state the result, let $F(b)$ be the nonempty set of optimal floors on x_0 for bias $b < 1$.

Proposition 2. *$F(b)$ is decreasing in b in the strong set order.*[26] *In particular,* $\max F(b)$ *is decreasing in b and converges to $\underline{\pi}_0^*$ as $b \uparrow 1$. Moreover, there exists $\underline{b} > 0$ such that $F(b) = \{\overline{f}_0\}$ for all $b \leq \underline{b}$, where $\overline{f}_0$ satisfies*

$$\mathcal{U}(C_{\overline{f}_0}) = \max_{f_0 \in [\underline{\pi}_0^*, \overline{\pi}_0^*]} \left\{ v(f_0) + \int_S \hat{u}(\mathbf{x}^{f_0}(\mathbf{s}); \mathbf{s}) dG \right\}.$$

Thus, when the bias is strong enough, society sacrifices entirely the option of letting the government adjust private consumption to information. This happens even though the government does care about x_0 and hence would adjust its level to the state. It is worth pointing out that the threshold $\underline{b}$, whose formula is provided in the Appendix, does not depend on the distribution G.

Case of $b > 1$. In the application to worker's time management, where he overvalues the dimension of conflict with the manager, an analysis similar to that in this section is possible. If the manager can only impose a cap or floor on how much time the worker spends on unproductive activities, she sets a *cap* c_0 which is strictly below the maximum time that she finds acceptable. Moreover, c_0 falls as the worker's tendency to indulge in those activities worsens.

4.2 Restricting Agreement Dimensions

In practice, to implement a floor on private consumption, society can impose a corresponding cap on *aggregate* public spending (i.e., $y \leq 1 - f_0^*$). When public spending is unidimensional, there is nothing else society can do within the class of interval delegation policies. This is no longer true when public spending involves multiple services. Given a target aggregate cap on spending, there are many ways to enforce it using specific caps on each service—the type of constraints we usually see in practice. Moreover, society can employ an aggregate cap *together with* service-specific ones; for example, public spending should never exceed 50% of GDP, but national security alone should be less than 5%. Since in this model society and the government have the same preference when it comes to dividing resources across public services, we may expect specific caps to be at least superfluous, at worst harmful. To check if this is true, this section examines the effects of such caps.

In our setting, service-specific caps can implement the same allocations as a binding cap on aggregate public spending if and only if information affects only the trade-off between public spending and private consumption. That is, the component $\mathbf{r}$ of information

[26]Given two sets F and F' in $\mathbb{R}$, $F \geq F'$ in the strong set order if, for every $f \in F$ and $f' \in F'$, $\min\{f, f'\} \in F'$ and $\max\{f, f'\} \in F$ (Milgrom and Shannon (1994)).

is degenerate (or commonly known from the outset).[27]

Lemma 4. *Fix $f_0 > \underline{\alpha}_0^*$ and let $C_{\mathbf{0},\mathbf{c}}$ be any policy that satisfies $\sum_{i=1}^n c_i = 1 - f_0$ and $c_0 = 1$. If $\mathbf{r}$ is constant, then there exists a $C_{\mathbf{0},\mathbf{c}}$ that implements the same allocations as C_{f_0}. If $\mathbf{r}$ is not constant, then every $C_{\mathbf{0},\mathbf{c}}$ implements allocations that differ with positive probability from those implemented by C_{f_0}.*

We can see the limitations of specific caps as follows. Consider any state $\mathbf{s} = (\theta, \mathbf{r})$ at which f_0 binds and the implemented allocation is $\alpha(\theta, \mathbf{r})$. Let $c_i = \alpha_i(\theta, \mathbf{r})$ for all $i = 1, \ldots, n$. Then, given $C_{\mathbf{0},\mathbf{c}}$ so defined, in state $(\theta, \mathbf{r})$ the government will again choose $\alpha(\theta, \mathbf{r})$ and hence $\alpha_0(\theta, \mathbf{r}) = f_0$.[28] Consider now an $\mathbf{r}'$ that satisfies, say, $r_i' > r_i$ and $r_j = r_j'$ for all $j \neq i$. Under C_{f_0} we must have $\alpha_i(\theta, \mathbf{r}') > \alpha_i(\theta, \mathbf{r})$ and $\alpha_j(\theta, \mathbf{r}') < \alpha_j(\theta, \mathbf{r})$ for all $j \neq i$: When service i becomes more valuable, the government wants to allocate to it more resources, and if f_0 binds before, it must also bind now. Under $C_{\mathbf{0},\mathbf{c}}$, however, this reallocation is not possible because x_i cannot exceed $\alpha_i(\theta, \mathbf{r})$. Recall that under C_{f_0} the allocation across services is always efficient: $\theta u_1^i(\alpha_i(\theta, \mathbf{r}); r_i) = \theta u_1^j(\alpha_j(\theta, \mathbf{r}); r_j)$ for all i, j and $(\theta, \mathbf{r})$. Therefore, trying to implement an aggregate cap on public spending using service-specific caps must involve inefficiencies in the allocation across services—those dimensions for which society and the government do not have conflicting goals.

Given these observations and the fact that society can always impose a single overall cap on public spending (via f_0), one may wonder whether it can ever benefit from imposing specific caps. To answer this question, I first examine the effects of a specific cap in isolation; that is, consider only policies in $\mathcal{R}$ that impose a cap c_i for one $i = 1, \ldots, n$ (i.e., $\mathbf{f} = \mathbf{0}$ and $c_j \geq 1$ for all $j \neq i$). In short, while a binding c_i mitigates the government's *aggregate* overspending, without other constraints it also exacerbates overspending on all other services.[29]

Lemma 5. *Fix $i \neq 0$ and consider policies $C_{\mathbf{0},\mathbf{c}}$ with $c_j \geq 1$ for $j \neq i$. In any state $\mathbf{s}$, if $c_i < \alpha_i^*(\mathbf{s})$, then the government chooses $x_0 > \alpha_0^*(\mathbf{s})$, but also $x_j > \alpha_j^*(\mathbf{s})$ for all $j \neq i$.*

Because the total amount of resources is fixed, overspending on service j comes at the cost of subtracting resources from private consumption, which the government undervalues, or from other services like i, which it values on par with j. But when service i is already capped, the second cost decreases, making the government overspend even more on j.

Despite these drawbacks, specific caps can benefit society relative to granting the government full discretion.

Proposition 3. *Fix $i \neq 0$ and consider policies $C_{\mathbf{0},\mathbf{c}}$ with $c_j \geq 1$ for $j \neq i$. There exists $c_i < \overline{\alpha}_i^*$ such that society strictly benefits from it, i.e., $\mathcal{U}(C_{\mathbf{0},\mathbf{c}}) > \mathcal{U}(B)$.*

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[27] This situation is of course inconsistent with our assumption on G. It is, however, easy to see how to modify that assumption to describe this case.

[28] Indeed, $\alpha(\theta, \mathbf{r}) \in C_{\mathbf{0},\mathbf{c}} \subset C_{f_0}$ and $\alpha(\theta, \mathbf{r})$ is the unique optimal allocation in C_{f_0}.

[29] If we interpret the model as capturing an individual commitment problem involving consumption and savings (Section 3.1), this result says that binding caps on some goods exacerbate the overconsumption of others. Heath and Soll (1996) provide evidence consistent with this prediction.

To gain intuition, start from $c_i = \overline{\alpha}_i^*$ and lower it a bit. On the one hand, when binding, the cap distorts the allocation across public services. This reduces society's expected payoff, but this loss is initially of second-order importance. The reason is that, under full discretion, the government's allocation across $\mathbf{x}$ is always efficient; moreover, both parties have the same preference regarding $\mathbf{x}$. Hence, marginal distortions in $\mathbf{x}$ do not change society's payoffs. On the other hand, the cap induces the government to allocate more to private consumption with strictly positive probability. Since society cares *discretely* more about this dimension, this reallocation causes a first-order gain in its payoff. Overall the cap must then benefit society, provided that resources are reallocated to consumption and to the unrestricted services at comparable rates, which is not obvious. This key property is guaranteed by the additive structure of preferences. It should continue to hold if all public services are complements: Capping one of them renders all the others less valuable and hence should incentivize the government to reallocate even more resources to private consumption.

In light of Proposition 3, we might think that society always benefits by combining a floor on private consumption (or an aggregate cap on public spending) with service-specific caps. Perhaps surprisingly, this depends on the strength of the government's bias and the nature of its information: Whether it only affects the trade-off between private consumption and public spending (θ) or also those across services ($\mathbf{r}$).

Before deriving these results, the next proposition shows—as should be expected at this point—that binding caps on private consumption or floors on public services are never part of society's optimal policy.

Proposition 4. *For any $C_{\mathbf{f},\mathbf{c}} \in \mathcal{R}$, let $C_{f_0,\mathbf{c}_{-0}}$ be the policy obtained by removing the cap on x_0 and all floors on $\mathbf{x}$. Then $\mathcal{U}(C_{f_0,\mathbf{c}_{-0}}) \geq \mathcal{U}(C_{\mathbf{f},\mathbf{c}})$, where the inequality is strict if under $C_{\mathbf{f},\mathbf{c}}$ either c_0 or f_i for some $i \neq 0$ binds with strictly positive probability.*

Case of $b > 1$. The fact that the worker overvalues the dimension of conflict with the manager changes the results in this section as follows. Specific floors on the time he has to assign to each task can implement the same allocations as a binding cap on unproductive activities if and only if he observes information only on θ. A floor on task i induces the worker to allocate less time to unproductive activities, but also less time to the other tasks. Nonetheless, the manager would strictly benefit from a single binding floor on any task relative to granting the worker full discretion. This result is particularly interesting for this application because, in practice, the manager may easily restrict and monitor how much time a worker spends on a task, but not how much time he spends on unproductive activities. Finally, binding task-specific caps or a floor on x_0 are never part of an optimal delegation policy.

5 Multidimensional Information: Restrictions on both Agreement and Disagreement Dimensions

This section examines the case in which information also affects the trade-offs between public services (i.e., it is multidimensional). It provides sufficient conditions for the optimal delegation policy within $\mathcal{R}$ to *also* involve service-specific caps and to *only* involve an aggregate cap on public spending. It also considers how reducing the government's idiosyncratic information on public services renders the second policy more likely to be optimal. Note that this section's results hold under the very weak assumptions on the distribution of information introduced in Section 3.

Lemma 6 below first shows that, for any level of the government's bias, every optimal policy sets an effective lower bound on private consumption which is at least as high as society's lowest optimal level $\underline{\pi}_0^*$. Recall that by Proposition 4 we can focus on policies $C \in \mathcal{R}$ with $c_0 = 1$ and $f_i = 0$ for all $i = 1, \ldots, n$. Let $\underline{x}_0$ be the effective lower bound on private consumption implied by C:

$$\underline{x}_0 = \max\{f_0, 1 - \sum_{i=1}^{n} c_i\}$$

Given C, the government always uses all resources and therefore allocates to x_0 at least the amount $\underline{x}_0$: For all $\mathbf{s} \in S$, $\sum_{i=0}^{n} \alpha_i(\mathbf{s}) = 1$ and hence $\alpha_0(\mathbf{s}) \geq \underline{x}_0$. Without loss of generality, we can let $\underline{x}_0 = \min_S \alpha_0(\mathbf{s}) = \underline{\alpha}_0$.[30]

Lemma 6. *For every $b \in (0, 1)$, if $C \in \mathcal{R}$ is optimal, then $\underline{x}_0 \geq \underline{\pi}_0^*$.*

If C lets the government choose x_0 below $\underline{\pi}_0^*$, society realizes that no state justifies such a low x_0. By increasing the floor f_0 up to $\underline{\pi}_0^*$, society uniformly improves its payoff with regard to private consumption. As a consequence of the lower aggregate spending, caps (if any) are less likely to bind—recall that public services are normal goods—and hence less likely to distort the allocation across services. Thus, society cannot lose on this front either.

Proposition 5 shows that, everything else equal, there always exists a degree of the government's bias against private consumption *below* which an optimal $C \in \mathcal{R}$ must involve service-specific caps. On the other hand, there exists a degree of disagreement *above* which, to be optimal, a policy only needs to impose an aggregate cap on public spending.

Proposition 5. *There exist b_* and b^* that satisfy $0 < b_* \leq b^* < 1$ and the following properties:*[31]

(1) if $b > b^*$, then every optimal policy in $\mathcal{R}$ must involve service-specific caps;

(2) if $b < b_*$, then every optimal policy in $\mathcal{R}$ involves only a private-consumption floor.

Moreover, let $\underline{\mathbf{r}}' \geq \underline{\mathbf{r}}$ and $\overline{\mathbf{r}}' \leq \overline{\mathbf{r}}$ with $\underline{\mathbf{r}}' \neq \underline{\mathbf{r}}$ and $\overline{\mathbf{r}}' \neq \overline{\mathbf{r}}$. Then, the corresponding b'_* and b_* satisfy $b'_* > b_*$.

Proposition 5 relies on the following lemma, which summarizes some useful properties of society's and the government's optimal allocation under full-discretion (π^* and α^*). These properties follow by observing that, under additivity of $\hat{u}$, given r_{-i} an increase in r_i makes π_i^* (resp. α_i^*) rise and π_j^* (resp. α_j^*) fall for all $j \neq i$.

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[30]If $\underline{\alpha}_0 > \underline{x}_0$, we could simply raise f_0 to $\underline{\alpha}_0$ and nothing would change in the government's allocation.

[31]The Appendix shows how to calculate the thresholds b_* and b^*; importantly, b_* does not depend on the distribution G.

Lemma 7. *The allocations π^* and α^* satisfy*

- $\overline{\pi}_i^* = \pi_i^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}) > \pi_i^*(\overline{\theta}, \overline{\mathbf{r}})$ *and* $\overline{\alpha}_i^* = \alpha_i^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}) > \alpha_i^*(\overline{\theta}, \overline{\mathbf{r}})$ *for any* $i \neq 0$,
- $\underline{\pi}_0^* = \pi_0^*(\overline{\theta}, \overline{\mathbf{r}}) < \pi_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i})$ *and* $\underline{\alpha}_0^* = \alpha_0^*(\overline{\theta}, \overline{\mathbf{r}}) < \alpha_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i})$.

To gain intuition for part (1) of Proposition 5, suppose there are only two public services. Both society and the government want to allocate more resources to service i as its value relative to service j or private consumption rises. Hence, the states in which their optimal allocation to service 1 (respectively 2) is highest are *not* the states in which their optimal allocation to private consumption is lowest. A consumption floor primarily targets the government's decisions in the latter states, but may have no effect in the former states. Yet, in these states the government continues to overspend and society would like to intervene. Now recall that, if policies can involve only a consumption floor, society sets it lower and lower as the government's bias weakens, which makes it less and less likely to affect the states in which, say, service 1 is very valuable but 2 is not. Society does not want to raise the floor to address these states, but it can add a cap on service 1 that binds only when the floor does not. Our previous results show that such a cap will mitigate overspending when the allocation to service 1 is high, and despite its distorting effects, it strictly benefits society.

Perhaps surprisingly, part (2) of Proposition 5 says that a simple delegation policy involving only an aggregate cap on public spending is optimal when the government's bias is strong enough. To see why in this case adding service-specific caps to an aggregate one does not benefit society, recall that by Lemma 6 it never allows private consumption to fall below the level $\underline{\pi}_0^* > 0$. When b is very small, however, the government wants to allocate much less than $\underline{\pi}_0^*$ to private consumption, no matter what information it observes. Hence, when it has to spend less on service i because of a cap, it reallocates all the savings across the other services, but not to private consumption. Since binding caps distort the allocation across services, society cannot benefit from adding them if they do not improve aggregate spending. The threshold b_* is actually lower than $\underline{b}$ in Proposition 2 (see the proof of Proposition 5). Thus part (2) also implies that for strong enough biases, it is optimal for society to commit ex ante to a single level of public spending, and to grant the government full discretion in allocating it across services.

This reasoning leads to the following simple observation.

Corollary 1. *Suppose that $C \in \mathcal{R}$ involves binding caps but always induces the same level of private consumption $\underline{x}_0$. Then C cannot be optimal.*

Society can strictly improve on such a C by imposing only a floor $f_0 = \underline{x}_0$.

The weakest bias for which optimal policies include service-specific caps is hard to characterize and depends on the details of the setting at hand. Intuitively, as b falls below b^*, for any policy C it increases the probability that the government ends up in a state where C's effective floor $\underline{x}_0$ binds. Since in these states binding caps only create inefficiencies, their appeal for society falls accordingly. How society balances the inefficiencies in those states with the benefits that a cap can yield in other states ultimately depends on their distribution G. Nonetheless, since society can always set $f_0 = \underline{x}_0$, for biases below some level $\hat{b} \geq b_*$ every optimal policy involves only a floor on private consumption.

Overall Proposition 5 suggests that richer fiscal constitutions involving many restrictions may in fact prevail in environments with *weaker* conflict of interest, whereas simple constitutions may prevail when the conflict is *stronger*. At first glance, one may think that when the conflict is weak more rules are actually *less* valuable than discretion. As the government bias weakens, society cares relatively more about allowing it to act on its superior information, especially along the dimensions for which their preferences agree.

Finally, Proposition 5 gives an idea of how reducing the government's idiosyncratic information on each service may affect the optimal delegation policy. By shrinking the range of such information—without changing that on the overall trade-off between private consumption and public spending (θ)—it becomes more likely that the simple policy with only an aggregate cap is optimal for a fixed degree of the government's bias. We may expect that such a simple policy is always optimal in the limit—when information is only about θ. The next section analyzes this situation.

Case of $b > 1$. In this case the previous results change as follows. First, any optimal delegation policy sets an effective upper bound on the time the worker can allocate to unproductive activities, which is at least as low as the manager's highest optimal level ($\overline{\pi}_0^*$). Second, an optimal policy must involve binding task-specific floors if the worker's tendency to indulge in unproductive activities is weak enough. If this tendency is strong enough, however, imposing only a cap on unproductive activities is optimal.

6 Unidimensional Information: Restrictions only on Disagreement Dimensions

Suppose now that information is only about the trade-off between public spending and private consumption (θ). We will show that, under a regularity condition on its distribution, the optimal fiscal constitution within $\mathcal{R}$ will involve an aggregate cap on spending but *no* service-specific caps, for any degree of the government's bias. To show this, we will argue that Propositions 3 and 4 in AWA can be applied to the present environment with multidimensional public spending. The assumption that the government's information is only about θ can be interpreted as saying that this is the only part of information that is not contractible, or that is still uncertain when society commits to a policy. To formalize this, let $\underline{r}_i = \overline{r}_i$ for all $i = 1, \ldots, n$; given this, we will suppress the dependence of $\hat{u}$ on $\mathbf{r}$. In this section, we also assume that the distribution G has a density function g that is strictly positive and continuous on $[\underline{\theta}, \overline{\theta}]$. As will become clear, this section's analysis does not change if we allow $\hat{u}$ to be non-separable across public services.

Even if now information has only one dimension, given a level y of public spending, its division across services remains a multidimensional decision. Changing the allocation of y yields different utilities from public spending $\hat{u}(\mathbf{x})$, a fact that can be exploited to curb the government's tendency to overspend. For instance, if a policy requires that y be allocated in a distorted way which does not provide much more utility than $y' < y$, it reduces the government's willingness to choose y in states in which society prefers y'.

Since we assumed free disposal—the resource constraint B is defined by an inequality—society has another tool to curb the government's tendency to overspend: "money burning." In theory—but perhaps not in practice—it could force the government to "throw away" part of what it does *not* allocate to private consumption. For instance, using low enough caps, society can force the government to choose allocations strictly inside B. It is easy to see that, given any level $\hat{u}$ of utility obtained by allocating $y = 1 - x_0$ inefficiently across $\mathbf{x}$, we can always achieve $\hat{u}$ by burning part of $1 - x_0$ and allocating the rest across $\mathbf{x}$ *efficiently*. Indeed, for any $y \in [0, 1]$ and $\mathbf{x} \in \mathbb{R}^n_+$ that satisfy $\sum_{i=1}^n x_i = y$, the utility $\hat{u}(\mathbf{x})$ belongs to the interval $[\hat{u}(\mathbf{0}), u^*(y)]$, where

$$u^*(y) = \max_{\left\{\mathbf{x}' \in \mathbb{R}^n_+ : \sum_{i=1}^n x'_i \leq y\right\}} \hat{u}(\mathbf{x}'). \tag{7}$$

Since u^* is strictly increasing and continuous and $u^*(0) = \hat{u}(\mathbf{0})$, there always exists $y' \leq y$ such that $u^*(y') = \hat{u}(\mathbf{x})$.

Building on these observations, we can show that in settings with unidimensional information it is without loss of generality to restrict attention to *all* delegation policies that regulate only private consumption and aggregate public spending. Formally, any such policy corresponds to a subset C^{as} of the resource constraint defined only in terms of private consumption and public spending, namely

$$B^{\text{as}} = \{(y, x_0) \in \mathbb{R}^2_+ : y + x_0 \leq 1\}.$$

Given $C^{\text{as}} \subset B^{\text{as}}$, in each state θ the government's problem becomes to maximize $\theta\hat{u}(\mathbf{x}) + bv(x_0)$ subject to $\sum_{i=1}^n x_i \leq y$ and $(y, x_0) \in C^{\text{as}}$.

Lemma 8. *Suppose that information affects only the trade-off between private consumption and public spending. Then, there exists an optimal $C \subset B$ with $\mathcal{U}(C) = \mathcal{U}^*$ if and only if there exists an optimal $C^{\text{as}} \subset B^{\text{as}}$ with $\mathcal{U}(C^{\text{as}}) = \mathcal{U}^*$.*

Remark 1. Besides information's being unidimensional, the important assumption for Lemma 8 is that money burning is always feasible. Without money burning, constraints on private consumption x_0 translate one-to-one into constraints on public spending y. Yet, society can still regulate how the government is allowed allocate y across services (see also Proposition 7 below).

Lemma 8 allows us to recast our problem into AWA's framework. Since the function $\hat{u}$ is strictly increasing, the constraint $\sum_{i=1}^n x_i \leq y$ will always bind when the government faces C^{as}. Therefore, using the function u^* in (7), we can express society's problem as choosing $C^{\text{as}} \subset B^{\text{as}}$ so as to maximize

$$\int_{\underline{\theta}}^{\overline{\theta}} [\theta u^*(\alpha_y(\theta)) + v(\alpha_0(\theta))] g(\theta) d\theta$$

subject to

$$(\alpha_y(\theta), \alpha_0(\theta)) \in \arg\max_{C^{\text{as}}} \theta u^*(y) + bv(x_0) \quad \text{for all } \theta \in [\underline{\theta}, \overline{\theta}]. \tag{8}$$

AWA show that the properties of the solution to this problem depend on the function $H : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ given by

$$H(\theta) = (1-b)\theta g(\theta) + G(\theta),$$

and the threshold

$$\theta^* = \inf \left\{ \theta \in [\underline{\theta}, \overline{\theta}] : \textstyle\int_{\theta'}^{\overline{\theta}} (1 - H(\hat{\theta}))d\hat{\theta} \le 0 \text{ for all } \theta' \ge \theta \right\}. \tag{9}$$

To understand what $H(\theta)$ captures, ignore feasibility for the moment. Suppose that society forces the government to adjust its allocation in state θ in favor of private consumption, without changing its total payoff—so that it does not select other allocations. Doing so requires inducing the government to allocate less to public spending. Overall this adjustment benefits society when θ occurs, because it cares discretely more about private consumption. This explains the term $(1-b)\theta g(\theta)$. The adjustment, however, also renders the allocation chosen in θ more attractive for the government in states where it values public spending less: in all $\theta' < \theta$, which have mass $G(\theta)$. Thus, society also has to induce the government to increase private consumption in these states, which is exactly what it wants. This explains the term $G(\theta)$.

Following AWA, we introduce the following condition.

Condition 1. The function H is non-decreasing over $[\underline{\theta}, \theta^*]$.

Also, given θ^*, define

$$C^{\text{as}}(\theta^*) = \{(y, x_0) \in B^{\text{as}} : x_0 \ge \alpha_0^*(\theta^*)\},$$

where α^* is the government's optimal allocation under full discretion (i.e., $C^{\text{as}} = B^{\text{as}}$).

Proposition 6 (Amador et al. (2006))**.** *The delegation policy $C^{\text{as}}(\theta^*)$ is optimal among all subsets of B^{as} if and only if Condition 1 holds.*[32]

As AWA noted, many distributions commonly used in applications satisfy Condition 1. Nonetheless, even if this condition holds, Proposition 5 implies that AWA's result does not extend to settings in which information also affects the trade-offs across public services.[33] Note also that since G is strictly increasing, Condition 1 is more likely to hold when the government's bias is weak (i.e., b is close to 1). A weak bias, however, characterizes exactly those settings with richer information structures where imposing only a consumption floor can be suboptimal.

Together with Lemma 8, Proposition 6 provides a sufficient condition for a delegation policy C_{f_0} to be optimal within $\mathcal{R}$.

Corollary 2. *Define θ^* as in (9). If Condition 1 holds, then $C_{f_0^*}$ with $f_0^* = \alpha_0^*(\theta^*)$ is optimal within $\mathcal{R}$.*

---

[32]It can be easily checked that, when the function $\hat{u}$ is strictly increasing, concave, and continuously differentiable, then the function u^* in (7) satisfies the same properties, as assumed in AWA.

[33]We can imagine a setting in which θ and $\mathbf{r}$ are independent so that $G(\theta, \mathbf{r}) = G_1(\theta)G_2(\mathbf{r})$ and G_1 satisfies Condition 1.

In general, Condition 1 is not necessary for the conclusion of Corollary 2. This is because policies that improve on $C_{f_0^*}$ may lie outside $\mathcal{R}$.[34]

AWA argue that, if Condition 1 fails, an optimal C^{as} may have to rely on money burning. In this case, society can again benefit from the multidimensionality of public spending. By forcing the government to implement distorted allocations across services based on the level of public spending, society can achieve the same curbing effect on the government's tendency to overspend with strictly less (possibly no) money burning. This highlights a possible limitation of treating public spending as a monolithic entity, even if information is only about θ. To state the result, define $u_* : [0,1] \to \mathbb{R}$ as

$$u_*(y) = \min_{\{\mathbf{x} \in \mathbb{R}_+^n : \sum_{i=1}^n x_i = y\}} \hat{u}(\mathbf{x}).$$

Note that, since $\hat{u}$ is strictly concave, $u_*(y) < u^*(y)$ for all $y > 0$.

Proposition 7. *Suppose that $C^{\text{as}} \subset B^{\text{as}}$ is optimal and induces an allocation α in (8) which satisfies $\alpha_y(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and $\alpha_y(\theta) < 1 - \alpha_0(\theta)$ over some set $\Theta \subset [\underline{\theta}, \overline{\theta}]$.*
(1) There exists $C' \subset B$ that satisfies $\mathcal{U}(C') = \mathcal{U}(C^{\text{as}})$ and involves less money burning: The ensuing allocation α' in (5) satisfies $\alpha'_0(\theta) = \alpha_0(\theta)$ and $\sum_{i=1}^n \alpha'_i(\theta) \geq \alpha_y(\theta)$ for all θ, with strict inequality over Θ.
(2) If $u_*(1 - \alpha_0(\theta)) \leq u^*(\alpha_y(\theta))$ for all $\theta \in \Theta$, then C' can be chosen so that money burning never occurs: $\sum_{i=0}^n \alpha'_i(\theta) = 1$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

The condition in part (2) means that, in every state in which society would like to burn some money, it can alternatively let the government spend all the resources subtracted from consumption but in such an inefficient way that the extra money does not improve the payoff from public spending. This condition is more likely to hold if, for instance, allocating no resources to some public service would be extremely inefficient and lead to a very low payoff for society. Examples of such services seem abundant: national security, law enforcement, criminal detention, or public medical treatment. Reducing the share of $1 - x_0$ allocated to these services can replicate the effect of money burning at the aggregate level. In these settings, if money burning is infeasible (or forbidden), society may achieve strictly higher expected payoffs by again imposing distortions along dimension which cause *no* conflict of interest with the government. Finally, note that one easy way to implement these distortions is to use service-specific caps and floors that vary based on how much the government allocates to private consumption.

Case of $b > 1$. In this case, the results in this section change as follows. A similar monotonicity condition on the distribution of θ can be obtained which is necessary and sufficient for a cap on unproductive activity alone to be fully optimal when the manager can choose among all subsets of B. This condition is sufficient for the single cap to be optimal within $\mathcal{R}$. Finally, failures of this condition may require an optimal $C^{\text{as}} \subset B^{\text{as}}$ to force the worker to sit idle for part of the time he does not spend on unproductive activities. In this case, there is a policy $C \subset B$ which involves strictly less (possibly no) waste of time and gives the manager the same expected payoff. This policy, however, has to induce the worker to allocate inefficiently his time across tasks.

[34]See AWA for an example of a superior policy outside $\mathcal{R}$.

7 Implications for Other Applications

This section discusses the implications of the analysis for the other applications outlined in Section 3.1. The application on workers' time management has already been discussed throughout the paper.

Intrapersonal Commitment Problems. Our analysis shows how the Planner can commit to simple caps and floors limiting what Doer can allocate to each consumption good and savings, so as to mitigate the impact of Doer's present bias while granting him some flexibility to act on information. On the one hand, if the bias is strong, Planner's optimal commitment strategy is simply to impose only a minimum level of savings. On the other hand, if the bias is weak, Planner can do strictly better by using a richer strategy which also imposes some good-specific caps. Such caps will distort consumption from Planner's viewpoint, but are still beneficial because they lead Doer to save more when the minimum-savings rule has no bite.

It has been often observed that some individuals earmark their monthly pay according to multiple categories of spending—sometimes as explicitly as dividing it into use-specific envelopes or setting up category-specific budgets via online services like Mint or Quicken. The literature has informally suggested that such earmarking and budgeting—called "mental accounting"—represents how individuals deal with self-control problems (Thaler (1985), Heath and Soll (1996), Thaler (1999)). This paper sheds light on this conjecture by offering an explicit foundation based on a precise and well-known cause of self-control problems: present bias. It also suggests some qualifications by showing that in fact only individuals whose present bias is weak may set up category-specific budgets (see Antonides et al. (2011) for some consistent evidence). Of course, these budgets cause several anomalies in consumption behavior, most notably non-fungibility of money with all its far-reaching consequences, anomalies for which again our theory provides an explanation.

Finally, the analysis has implications for the demand of commitment devices. AWA argued that illiquid assets may suffice to allow present-biased individuals to implement their best minimum-savings strategy. This paper shows, however, that this strategy may be dominated by simply adding category-specific budgets. This can explain why, in addition to illiquid assets, some individuals also rely on services that allow them to budget their expenses by categories (again, for example, using Mint or Quicken).

Corporate Governance. Our results suggest that to best incentivize a CEO who undervalues R&D, the shareholders of a multi-product company may have to impose caps on how much can be spent each year on marketing specific products, possibly in addition to requiring a minimum investment in R&D. Due to the caps, the CEO may end up spending too little for some products and too much for others from the shareholders' viewpoint. This, however, is a risk they should accept, as it is more than compensated in expectation by better allocations to R&D. A detailed budget plan with rules applying to specific products is more likely to benefit the shareholders when they do not disagree too much with the CEO on how important R&D is for the company. This may be true, for instance, if the CEO has significant stakes in the company as well. Otherwise, the shareholders can simply demand a minimum investment in R&D, or even impose a fix one if they think that the CEO is seriously biased. Of course, in this case they may consider changing how he gets paid or hiring a new manager.

Research vs. Teaching in Academia. Research universities usually specify a minimum amount of time that professors have to allocate each week to their teaching duties (classes, office hours, etc.), granting them discretion on, say, course-preparation time. However, we do not observe universities restricting how much time professors should spend on each of their research projects. On the one hand, this is consistent with our theory if we believe that, from the universities' viewpoint, professors' bias against teaching is very severe, which is not completely implausible. On the other hand, the theory raises the possibility that universities' common practices may actually leave room for welfare gains. Finally, monitoring how much time a professor spends on a research project may simply be infeasible, so imposing any cap is pointless.

Public Finance. When deciding on the fiscal rules for the coming year, a government may realize that due to its present bias it will tend to borrow excessively against future tax revenues. It is then not surprising that the government benefits, as we saw, from committing to a cap on how much it will be allowed to borrow, a provision often observed in reality in the form of budget-deficit ceilings. The paper shows, however, that in some settings the government can easily improve on a policy that imposes only a borrowing cap. Although present bias never interferes with how the government trades off the value of different public services within a time period, introducing specific caps on how much it will be allowed to spend on some services can lead to a superior policy. Such caps also appear often in reality as part of fiscal budgets. This observation suggests, according to the theory, that governments' present bias tends to be mild. Another point highlighted in the paper is that service-specific caps are not a free lunch: Even though they are useful to curb excessive borrowing, they also lead to inefficient compositions of public spending.

8 Conclusions

This paper examines a broad, new class of principal-agent delegation problems which arise in many economic settings, from the design of fiscal rules to individual commitment problems, corporate governance, and workforce management. In such problems, the agent controls how to allocate a fixed amount of resources (money, time, etc.) across multiple categories, having better information on their returns than does the principal but pursuing different goals from hers.

The paper characterizes how optimal delegation policies trade off rules and discretion and how they depend on the degree of conflict between parties as well as the nature of the agent's information. Perhaps counterintuitively, it can be optimal for the principal to impose distorting restrictions on categories for which there is no conflict of interest with the agent, so as to curb how the conflict along other categories affects his overall resource allocation. Moreover, such restrictions are more likely to be optimal when the conflict of interest is weaker and when the agent's information is about the specific value of categories causing no conflict. The paper also shows that requiring distorted allocations across these categories can reduce or even eliminate the need for money burning as a way to manage the agent's incentives. By considering a tractable class of simple delegation policies, this paper offers insights that can be easily applied to many concrete problems, which existing models cannot handle.

A Appendix: Proofs

A.1 Proof of Lemma 2

Each $C \in \mathcal{R}$ is defined by a vector $(\mathbf{f}, \mathbf{c}) \in \mathbb{R}_+^{2(n+1)}$. Given the normalization $I = 1$, without loss we can restrict attention to the following compact subset of $\mathbb{R}_+^{2(n+1)}$:

$$\mathcal{FC} = \{(\mathbf{f}, \mathbf{c}) \in [0,1]^{2(n+1)} : \mathbf{f} \leq \mathbf{c}, \sum_{i=0}^{n} f_i \leq 1\}.$$

So, we can think that P chooses $(\mathbf{f}, \mathbf{c}) \in \mathcal{FC}$.

Given any such $(\mathbf{f}, \mathbf{c})$, let $\alpha(\mathbf{s}|\mathbf{f}, \mathbf{c})$ be A's optimal allocation in state $\mathbf{s}$ from the compact set $C_{\mathbf{f},\mathbf{c}}$. Since $C_{\mathbf{f},\mathbf{c}}$ is convex (Theorem 2.1 in Rockafellar (1997)), $\alpha(\mathbf{s}|\mathbf{f}, \mathbf{c})$ is unique for every $\mathbf{s} \in S$ by strict concavity of $V(\cdot; \mathbf{s})$. Clearly, the correspondence that for each $(\mathbf{f}, \mathbf{c}) \in \mathcal{FC}$ maps to $C_{\mathbf{f},\mathbf{c}}$ is non-empty, compact valued, and continuous. It follows from the Maximum Theorem that $\alpha(\mathbf{s}; \cdot, \cdot)$ is continuous for every $\mathbf{s} \in S$.

We can now show that P's payoff is continuous in $(\mathbf{f}, \mathbf{c})$. For each $(\mathbf{f}, \mathbf{c}) \in \mathcal{FC}$, let

$$\mathcal{U}(\mathbf{f}, \mathbf{c}) = \int_S U(\alpha(\mathbf{s}|\mathbf{f}, \mathbf{c}); \mathbf{s}) dG.$$

Since $U(\alpha(\mathbf{s}|\mathbf{f}, \mathbf{c}); \mathbf{s})$ is continuous in $(\mathbf{f}, \mathbf{c})$ for every $\mathbf{s} \in S$ and is uniformly bounded over B, Lebesgue's Dominated Convergence Theorem implies the claimed property of $\mathcal{U}(\cdot, \cdot)$.

A second application of the Maximum Theorem gives the result.

A.2 Proof of Lemma 3

For simplicity, drop the subscript 0 from f_0 and let $\Psi(f) = \mathcal{U}(C_f)$. Also, we will consider only $f \in [\underline{\alpha}_0^*, \overline{\pi}_0^*]$ without specifying this every time. Given f and any $\mathbf{s}$, define

$$\tilde{u}(f; \mathbf{s}) \equiv \hat{u}(\mathbf{x}^f(\mathbf{s}); \mathbf{s}) = \max_{\{\mathbf{x} \in \mathbb{R}_+^n : \sum_{i=1}^n x_i \leq 1 - f\}} \hat{u}(\mathbf{x}; \mathbf{s}). \tag{10}$$

and $\tilde{U}(f; \mathbf{s}) = \tilde{u}(f; \mathbf{s}) + v(f)$. We first want to show that $\tilde{U}(f; \mathbf{s})$ is strictly concave in f for every $\mathbf{s}$. Take any f , f', and $\zeta \in (0,1)$. We have

$$\begin{aligned} \tilde{u}(\zeta f + (1-\zeta) f'; \mathbf{s}) + v(\zeta f + (1-\zeta) f') &\geq \hat{u}(\zeta \mathbf{x}^f(\mathbf{s}) + (1-\zeta)\mathbf{x}^{f'}(\mathbf{s}); \mathbf{s}) \\ &\quad + v(\zeta f + (1-\zeta) f') \\ &> \zeta \left[\hat{u}(\mathbf{x}^f(\mathbf{s}); \mathbf{s}) + v(f)\right] \\ &\quad + (1-\zeta)\left[\hat{u}(\mathbf{x}^{f'}(\mathbf{s}); \mathbf{s}) + v(f')\right] \end{aligned} \tag{11}$$

$$
\begin{aligned}
&= \zeta\left[\tilde{u}(f;\mathbf{s})+v(f)\right] \\
&\quad +(1-\zeta)\left[\tilde{u}(f'(\mathbf{s});\mathbf{s})+v(f')\right],
\end{aligned}
$$

where the weak inequality follows because $\sum_i x_i^f(\mathbf{s}) \leq f$ and $\sum_i x_i^{f'}(\mathbf{s}) \leq f'$ implies

$$
\sum_i \left[\zeta x_i^f(\mathbf{s}) + (1-\zeta)x_i^{f'}(\mathbf{s})\right] \leq \zeta f + (1-\zeta)f',
$$

and the strict inequality follows from strict concavity of $\hat{u}(\cdot,\mathbf{s})$ and $v(\cdot)$.

Now consider the derivative of $\tilde{U}(f;\mathbf{s})$ with respect to f. Whenever it is defined, we have

$$
\tilde{U}'(f;\mathbf{s}) = \tilde{u}'(f;\mathbf{s}) + v'(f).
$$

By considering the FOC of the Lagrangian defining $\tilde{u}(f;\mathbf{s})$, we see that $\frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}^f(\mathbf{s});\mathbf{s}) = \lambda(\mathbf{s};f)$ for any $i = 1,\ldots,n$, where $\lambda(\mathbf{s};f)$ is the Lagrange multiplier of the constraint $\sum_{i=1}^n x_i \leq 1-f$. Since $\mathbf{x}^f(\mathbf{s})$ is continuous in f for every $\mathbf{s}$, so it $\lambda(\mathbf{s};f)$ given our assumptions on $\hat{u}$. By Theorem 1, p. 222, of Luenberger (1969), for every $f', f'' \in [0,1]$ we have

$$
\lambda(\mathbf{s};f')(f''-f') \leq \tilde{u}(f';\mathbf{s}) - \tilde{u}(f'';\mathbf{s}) \leq \lambda(\mathbf{s};f'')(f''-f').
$$

Continuity of $\lambda(\mathbf{s};\cdot)$ then implies that $\tilde{u}'(f;\mathbf{s})$ exists for every f and equals $-\lambda(\mathbf{s};f)$. Therefore,

$$
\tilde{U}'(f;\mathbf{s}) = v'(f) - \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}^f(\mathbf{s});\mathbf{s}) \quad \text{for all } \mathbf{s}. \tag{12}
$$

For any f, denote by α^f the behavior of A as a function of $\mathbf{s}$ under f. Note that $\alpha^f(\mathbf{s})$ is continuous in both f and $\mathbf{s}$ by the Maximum Theorem. Since, given any choice of x_0, P and A choose the same bundle $\mathbf{x}$ in every state, by definition we have

$$
\Psi(f) = \int_S \tilde{U}(\alpha_0^f(\mathbf{s});\mathbf{s})dG.
$$

Consider any $f > \hat{f}$. Recall that $\overline{S}(f) = \{\mathbf{s} : \alpha_0^*(\mathbf{s}) \leq f\}$. Then,

$$
\begin{aligned}
\Psi(f) - \Psi(\hat{f}) &= \int_S \left[\tilde{U}(\alpha_0^f(\mathbf{s});\mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s});\mathbf{s})\right] dG \\
&= \int_{\overline{S}(f)} \left[\tilde{U}(f;\mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s});\mathbf{s})\right] dG \\
&= \int_{\overline{S}(f)\cap\left(\overline{S}(\hat{f})\right)^c} \left[\tilde{U}(f;\mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s});\mathbf{s})\right] dG \\
&+ \int_{\overline{S}(\hat{f})} \left[\tilde{U}(f;\mathbf{s}) - \tilde{U}(\hat{f};\mathbf{s})\right] dG.
\end{aligned}
$$

where the second equality follows because $\alpha_0^f(\mathbf{s}) = \alpha_0^{\hat{f}}(\mathbf{s})$ for $\mathbf{s} \notin \overline{S}(f)$ and $\alpha_0^f(\mathbf{s}) = f$ for $\mathbf{s} \in \overline{S}(f)$. Dividing both sides by $f - \hat{f}$, we get

$$
\lim_{f\downarrow\hat{f}} \frac{\Psi(f)-\Psi(\hat{f})}{f-\hat{f}} = \lim_{f\downarrow\hat{f}} \int_{\overline{S}(\hat{f})} \frac{\tilde{U}(f;\mathbf{s}) - \tilde{U}(\hat{f};\mathbf{s})}{f-\hat{f}} dG \tag{13}
$$

$$+ \lim_{f \downarrow \hat{f}} \int_{\overline{S}(f) \cap \left(\overline{S}(\hat{f})\right)^c} \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s}); \mathbf{s})}{f - \hat{f}} dG.$$

Consider the first limit. For all $\mathbf{s}$, we have

$$\lim_{f \downarrow \hat{f}} \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\hat{f}; \mathbf{s})}{f - \hat{f}} = \tilde{U}'(\hat{f}; \mathbf{s}).$$

Since $\tilde{U}(\cdot; \mathbf{s})$ is concave,

$$\left| \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\hat{f}; \mathbf{s})}{f - \hat{f}} \right| \leq \max \left\{ \left| \tilde{U}'(f; \mathbf{s}) \right|, \left| \tilde{U}'(\hat{f}; \mathbf{s}) \right| \right\}.$$

Since $\tilde{U}'(f; \mathbf{s})$ is continuous in $\mathbf{s}$ and f as illustrated by (12),

$$\max_{\{(f, \mathbf{s}) \in [\underline{\alpha}_0^*, \overline{\pi}_0^*] \times S\}} \left| \tilde{U}'(f; \mathbf{s}) \right| = M < +\infty.$$

Therefore, by Lebesgue's Bounded Convergence Theorem, we have

$$\lim_{f \downarrow \hat{f}} \int_{\overline{S}(\hat{f})} \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\hat{f}; \mathbf{s})}{f - \hat{f}} dG = \int_{\overline{S}(\hat{f})} \tilde{U}'(\hat{f}; \mathbf{s}) dG.$$

Consider now the second limit in (13). Again, by concavity of $\tilde{U}(\cdot; \mathbf{s})$ and since $\alpha_0^f(\mathbf{s}) \in [\underline{\alpha}_0^*, \overline{\pi}_0^*]$ for $f \in [\underline{\alpha}_0^*, \overline{\pi}_0^*]$, we have that

$$\left| \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s}); \mathbf{s})}{f - \alpha_0^{\hat{f}}(\mathbf{s})} \right| \leq M.$$

Therefore,

$$\begin{aligned}
\left| \int_{\overline{S}(f) \cap \left(\overline{S}(\hat{f})\right)^c} \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s}); \mathbf{s})}{f - \hat{f}} dG \right| &\leq \int_{\overline{S}(f) \cap \left(\overline{S}(\hat{f})\right)^c} \left| \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s}); \mathbf{s})}{f - \hat{f}} \right| dG \\
&\leq \int_{\overline{S}(f) \cap \left(\overline{S}(\hat{f})\right)^c} \left| \frac{\tilde{U}(f; \mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s}); \mathbf{s})}{f - \alpha_0^{\hat{f}}(\mathbf{s})} \right| dG \\
&\leq M \int_{\overline{S}(f) \cap \left(\overline{S}(\hat{f})\right)^c} dG.
\end{aligned}$$

Now, observe that $\overline{S}(f) \cap \left(\overline{S}(\hat{f})\right)^c = \{\mathbf{s} : \hat{f} < \alpha_0^{\hat{f}}(\mathbf{s}) \leq f\}$ which converges to an empty set as $f \downarrow \hat{f}$. It follows that the second limit in (13) converges to zero as $f \downarrow \hat{f}$. We conclude that for every $\hat{f} \in [\underline{\alpha}_0^*, \overline{\pi}_0^*)$, we have

$$\Psi'(\hat{f}+) = \int_{\overline{S}(\hat{f})} \tilde{U}'(\hat{f}; \mathbf{s}) dG.$$

Now consider any $f < \hat{f}$. Then,

$$\Psi(f) - \Psi(\hat{f}) = \int_S \left[\tilde{U}(\alpha_0^f(\mathbf{s}); \mathbf{s}) - \tilde{U}(\alpha_0^{\hat{f}}(\mathbf{s}); \mathbf{s}) \right] dG$$

$$= \int_{\overline{S}(\hat{f})} \left[\tilde{U}(\alpha_0^f(\mathbf{s});\mathbf{s}) - \tilde{U}(\hat{f};\mathbf{s})\right] dG$$
$$= \int_{\overline{S}(\hat{f})} \left[\tilde{U}(f;\mathbf{s}) - \tilde{U}(\hat{f};\mathbf{s})\right] dG + \int_{\overline{S}(\hat{f})} \left[\tilde{U}(\alpha_0^f(\mathbf{s});\mathbf{s}) - \tilde{U}(f;\mathbf{s})\right] dG$$
$$= \int_{\overline{S}(\hat{f})} \left[\tilde{U}(f;\mathbf{s}) - \tilde{U}(\hat{f};\mathbf{s})\right] dG + \int_{\overline{S}(\hat{f})\cap\left(\overline{S}(f)\right)^c} \left[\tilde{U}(\alpha_0^f(\mathbf{s});\mathbf{s}) - \tilde{U}(f;\mathbf{s})\right] dG,$$

where the second equality follows because $\alpha_0^f(\mathbf{s}) = \alpha_0^{\hat{f}}(\mathbf{s})$ for $\mathbf{s} \notin \overline{S}(\hat{f})$ and $\alpha_0^{\hat{f}}(\mathbf{s}) = \hat{f}$ for $\mathbf{s} \in \overline{S}(\hat{f})$, and the last equality follows because $\alpha_0^f(\mathbf{s}) = f$ for $\mathbf{s} \in \overline{S}(f)$. By the same argument as before,

$$\lim_{f\uparrow\hat{f}} \int_{\overline{S}(\hat{f})} \frac{\tilde{U}(f;\mathbf{s}) - \tilde{U}(\hat{f};\mathbf{s})}{f-\hat{f}} dG = \int_{\overline{S}(\hat{f})} \tilde{U}'(\hat{f};\mathbf{s}) dG,$$
$$\lim_{f\uparrow\hat{f}} \int_{\overline{S}(\hat{f})\cap\left(\overline{S}(f)\right)^c} \frac{\tilde{U}(\alpha_0^f(\mathbf{s});\mathbf{s}) - \tilde{U}(f;\mathbf{s})}{f-\hat{f}} dG = 0.$$

We conclude that for every $\hat{f} \in (\underline{\alpha}_0^*, \overline{\pi}_0^*]$, we have

$$\Psi'(\hat{f}-) = \int_{\overline{S}(\hat{f})} \tilde{U}'(\hat{f};\mathbf{s}) dG.$$

Hence, $\Psi(f)$ is differentiable over the restricted domain $[\underline{\alpha}_0^*, \overline{\pi}_0^*]$.

A.3 Proof of Proposition 1

For simplicity, drop the subscript from f_0. We shall show that $\Psi'(f) > 0$ for all $f \in (\underline{\alpha}_0^*, \underline{\pi}_0^*]$ and $\Psi'(f-) < 0$ for $f = \overline{\pi}_0^*$. Recall that $\alpha^f(\mathbf{s})$ is continuous in f for every $\mathbf{s}$ and therefore $\Psi(f)$ is continuous in f. These observations imply that the optimal $f^* \in (\underline{\pi}_0^*, \overline{\pi}_0^*)$.

For any $f \in (\underline{\alpha}_0^*, \overline{\pi}_0^*]$, define

$$S^+(f) = \{\mathbf{s} : \pi_0^*(\mathbf{s}) > f\} \quad \text{and} \quad S^-(f) = \{\mathbf{s} : \pi_0^*(\mathbf{s}) \leq f\}.$$

For $\mathbf{s} \in S^+(f)$, consider the the following problem:

$$\max \hat{u}(\mathbf{x};\mathbf{s}) + v(x_0)$$

subject to $\sum_i x_i \leq 1$ and $x_0 \leq f$. The associated Lagrangian is

$$\hat{u}(\mathbf{x};\mathbf{s}) + v(x_0) + \mu(\mathbf{s}) \left[1 - \sum_{i=0}^{n} x_i\right] + \phi^+(\mathbf{s})[f - x_0].$$

Hence, the FOC are[35]

$$v'(x_0) = \mu(\mathbf{s}) + \phi^+(\mathbf{s}) \quad \text{and} \quad \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x};\mathbf{s}) = \mu(\mathbf{s}) \quad \text{for all } i.$$

[35] Here, as well as in the other proofs, the complementary slackness conditions are omitted for simplicity.

Clearly, the constraint $x_0 \leq f$ must be binding for $\mathbf{s} \in S^+(f)$, which implies that $x_0 = f$ and $\phi^+(\mathbf{s}) > 0$. Also, conditional on choosing $x_0 = f$, both P and A choose the same $\mathbf{x}$ in state $\mathbf{s}$, which therefore equals $\mathbf{x}^f(\mathbf{s})$. Using (12), it follows that, for any i,

$$\phi^+(\mathbf{s}) = v'(f) - \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}^f(\mathbf{s}); \mathbf{s}) = \tilde{U}'(f; \mathbf{s})$$

when $\mathbf{s} \in S^+(f)$.

For $\mathbf{s} \in S^-(f)$, consider the following problem:

$$\max \hat{u}(\mathbf{x}; \mathbf{s}) + v(x_0)$$

subject to $\sum_i x_i \leq 1$ and $x_0 \geq f$. The associated Lagrangian is

$$\hat{u}(\mathbf{x}; \mathbf{s}) + v(x_0) + \mu(\mathbf{s}) \left[1 - \sum_{i=0}^{n} x_i\right] + \phi^-(\mathbf{s})[x_0 - f].$$

Hence, the FOC are

$$v'(x_0) = \mu(\mathbf{s}) - \phi^-(\mathbf{s}) \quad \text{and} \quad \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}; \mathbf{s}) = \mu(\mathbf{s}) \quad \text{for all } i,$$

Clearly, the constraint $x_0 \geq f$ must be binding for $\mathbf{s} \in S^-(f)$ except when $\pi_0^*(\mathbf{s}) = f$, which implies that $x_0 = f$ and $\phi^-(\mathbf{s}) \geq 0$. Also, conditional on choosing $x_0 = f$, both P and A choose the same $\mathbf{x}$ in state $\mathbf{s}$, which therefore equals $\mathbf{x}^f(\mathbf{s})$. Using (12), it follows that, for any i,

$$\phi^-(\mathbf{s}) = \frac{\partial}{\partial x_i}\hat{u}(\mathbf{x}^f(\mathbf{s}); \mathbf{s}) - v'(f) = -\tilde{U}'(f; \mathbf{s})$$

when $\mathbf{s} \in S^-(f)$.

Consider any $f \in (\underline{\alpha}_0^*, \underline{\pi}_0^*]$. Recall that $\overline{S}(f) = \{\mathbf{s} : \alpha_0^*(\mathbf{s}) \leq f\}$. Using Lemma 3, we have

$$\begin{aligned}
\Psi'(f) &= \int_{\overline{S}(f)} \tilde{U}'(f; \mathbf{s}) dG \\
&= \int_{\overline{S}(f) \cap S^+(f)} \tilde{U}'(f; \mathbf{s}) dG + \int_{\overline{S}(f) \cap S^-(f)} \tilde{U}'(f; \mathbf{s}) dG \\
&= \int_{\overline{S}(f) \cap S^+(f)} \phi^+(\mathbf{s}) dG,
\end{aligned}$$

where the last equality follows because either $S^-(f) = \varnothing$ or $\phi^-(\mathbf{s}) = 0$ for $\mathbf{s} \in S^-(f)$. The function $\phi^+(\mathbf{s})$ is strictly positive over $\overline{S}(f) \cap S^+(f)$. We need to show that this set has strictly positive measure, which implies $\Psi'(f) > 0$. This is immediate if $f \in (\underline{\alpha}_0^*, \underline{\pi}_0^*)$, because in this case $S^+(f) = S$. Consider $f = \underline{\pi}_0^*$. Clearly, $\overline{S}(\underline{\pi}_0^*) \cap S^+(\underline{\pi}_0^*)$ contains the open set

$$\overline{S}^\circ(\underline{\pi}_0^*) \cap S^+(\underline{\pi}_0^*) = \{\mathbf{s} : \alpha_0^*(\mathbf{s}) < \underline{\pi}_0^* < \pi_0^*(\mathbf{s})\}.$$

If we can show that this set is non-empty, we are done because G assigns strictly positive probability to it. Both $\overline{S}^\circ(\underline{\pi}_0^*)$ and $S^+(\underline{\pi}_0^*)$ are nonempty. Suppose that there is no $\mathbf{s} \in S^+(\underline{\pi}_0^*)$ such that we also have $\mathbf{s} \in \overline{S}^\circ(\underline{\pi}_0^*)$. Then, it means that for every $\mathbf{s} \in S^+(\underline{\pi}_0^*)$, we have $\alpha_0^*(\mathbf{s}) \geq \underline{\pi}_0^*$ and that $\overline{S}^\circ(\underline{\pi}_0^*) \subset S^-(\underline{\pi}_0^*) = \{\mathbf{s} : \pi_0^*(\mathbf{s}) = \underline{\pi}_0^*\}$. Now, consider $\hat{\mathbf{s}} \in \overline{S}^\circ(\underline{\pi}_0^*)$ and any sequence $\{\mathbf{s}_n\}$ is $S^+(\underline{\pi}_0^*)$ converging to $\hat{\mathbf{s}}$. We have that

$$\lim_{\mathbf{s}_n \to \hat{\mathbf{s}}} \inf \alpha_0^*(\mathbf{s}_n) \geq \underline{\pi}_0^* > \alpha_0^*(\hat{\mathbf{s}}).$$

But this contradicts the continuity of α^* and hence leads to a contradiction.

Now consider $f = \overline{\pi}_0^*$. Using again Lemma 3, we have

$$\Psi'(\overline{\pi}_0^*-) \;=\; \int_{\overline{S}(\overline{\pi}_0^*)} \tilde{U}'(\overline{\pi}_0^*; \mathbf{s}) dG = \int_S \tilde{U}'(\overline{\pi}_0^*; \mathbf{s}) dG = -\int_S \phi^-(\mathbf{s}) dG,$$

where $\phi^-(\mathbf{s}) > 0$ for all $\mathbf{s}$ such that $\pi_0^*(\mathbf{s}) < \overline{\pi}_0^*$. Therefore, $\Psi'(\overline{\pi}_0^*-) < 0$.[36]

A.4 Proof of Proposition 2

Fix $f_0 \in [\underline{\alpha}_0^*, \overline{\pi}_0^*]$. Changes in b affect $\overline{S}(f_0)$ through the change in α^*. By standard arguments, if $b < b' < 1$, then $\alpha_0^*(\mathbf{s}; b) < \alpha_0^*(\mathbf{s}; b')$ for every $\mathbf{s}$ and hence $\overline{S}(f_0; b') \subset \overline{S}(f_0; b)$. On the other hand, for every $b < 1$, we have $S_l(f_0) \subset \overline{S}(f_0; b)$ because $\alpha_0^*(\mathbf{s}; b) < \pi_0^*(\mathbf{s})$ for every $\mathbf{s}$. So, if $b < b' < 1$, we have

$$\Psi'(f_0; b) - \Psi'(f_0; b') = \int_{\left(\overline{S}(f_0;b)\setminus\overline{S}(f_0;b')\right)\cap S_m(f_0)} \phi(\mathbf{s}) dG \geq 0,$$

where the inequality follows from (6). Standard monotone-comparative-static results then imply that $F(b)$ is increasing in the strong set order.

Define $\overline{f}_0(b) = \max F(b)$. Since $\overline{f}_0(b) \geq \underline{\pi}_0^*$ for all b and $\overline{f}_0(\cdot)$ is decreasing, $\lim_{b\uparrow 1} \overline{f}_0(b)$ exists; denote it by $\overline{f}_0(1-) \geq \underline{\pi}_0^*$. Clearly, $\overline{f}_0(1) = \underline{\pi}_0^*$. Now suppose that $\overline{f}_0(1-) > \overline{f}_0(1)$. By a similar argument, for any $f_0 > \underline{\pi}_0^*$, $\lim_{b\uparrow 1} \Psi'(f_0; b)$ exists and satisfies

$$\lim_{b\uparrow 1} \Psi'(f_0; b) = -\int_{S_l(f_0)} \phi(\mathbf{s}) dG < 0.$$

This implies that for b close enough to 1, $\overline{f}_0(b) \geq \overline{f}_0(1-)$ cannot be optimal. A contradiction which implies that $\overline{f}_0(1-) = \overline{f}_0(1)$.

It is easy to see that, for all $\mathbf{s} \in S$, $\alpha_0^*(\mathbf{s}; b) \to 0$ as $b \downarrow 0$. Therefore, $\overline{\alpha}_0^*(b) = \max_S \alpha_0^*(\mathbf{s}; b)$ also decreases monotonically to 0 as $b \downarrow 0$. Let $\underline{b} = \max\{b \in [0,1] : \overline{\alpha}_0^*(b) \leq \underline{\pi}_0^*\}$ which is strictly positive because $\underline{\pi}_0^* > 0$. Then, $\overline{S}(f_0) = S$ for all $b \leq \underline{b}$ and $f_0 \in [\underline{\pi}_0^*, \overline{\pi}_0^*]$, which implies that

$$\Psi(f_0; b) = v(f_0) + \int_S \hat{u}(\mathbf{x}^{f_0}(\mathbf{s}); \mathbf{s}) dG. \tag{14}$$

From the proof of Lemma 3, we have that $\hat{u}(\mathbf{x}^{f_0}(\mathbf{s}); \mathbf{s}) = \tilde{u}(f_0; \mathbf{s})$ is strictly concave in f_0 for all $\mathbf{s} \in S$. This implies that the maximizer of (14) is unique. From the proof of Proposition 4.1, we know that the derivative of (14) is negative at $\overline{\pi}_0^*$ and hence $\overline{f}_0 < \overline{\pi}_0^*$.

[36] It is easy to see that the optimal f satisfies $f \leq \overline{\pi}_0^*$. Suppose $f \in (\overline{\pi}_0^*, 1)$. Then, for all $\mathbf{s}$, A chooses $x_0(\mathbf{s}) = f$ and $\mathbf{x}(\mathbf{s}) = \mathbf{x}^f(\mathbf{s})$. Take any $f' \in (\overline{\pi}_0^*, f)$. Then, for every $\mathbf{s}$, $f' = \zeta(\mathbf{s}) f + (1 - \zeta(\mathbf{s}))\pi_0^*(\mathbf{s})$ for some $\zeta(\mathbf{s}) \in (0, 1)$. Therefore, for every $\mathbf{s}$, $\tilde{U}(f'; \mathbf{s}) > \tilde{U}(f; \mathbf{s})$ because $\tilde{U}(\pi_0^*(\mathbf{s}); \mathbf{s}) > \tilde{U}(f; \mathbf{s})$ and $\tilde{U}(\cdot; \mathbf{s})$ is strictly concave. It follows that P's payoff is strictly larger under f' than under f.

A.5 Proof of Lemma 4

Part 1: Fix $f_0 > \underline{\alpha}_0^*$. Suppose $\mathbf{r}$ is constant. For every $\theta \in [\underline{\theta}, \overline{\theta}]$, A maximizes $\theta \sum_{i=1}^n u^i(x_i; r_i) + bv(x_0)$ subject to $(\mathbf{x}, x_0) \in B$ and $x_0 \geq f_0$, which leads to the unique optimal allocation $\alpha(\theta)$. By strong monotonicity of preferences, $\sum_{i=1}^n \alpha_i(\theta) = 1 - \alpha_0(\theta)$ for every θ. By standard arguments, each $\alpha_i(\cdot)$ is a strictly increasing, continuous function of θ for $i = 1, \ldots, n$. Let $c_i = \max_{\theta \in [\underline{\theta}, \overline{\theta}]} \alpha_i(\theta)$. Clearly, $1 - \sum_{i=1}^n c_i = \min_{\theta \in [\underline{\theta}, \overline{\theta}]} \alpha_0(\theta)$. Since f_0 has to be binding for some θ, $\min_{\theta \in [\underline{\theta}, \overline{\theta}]} \alpha_0(\theta) = f_0$. It is clear that if we replace the floor f_0 with the caps $\{c_i\}_{i=1}^n$, A's choices across θ's do not change.

Part 2: Fix $f_0 > \underline{\alpha}_0^*$. Suppose $\mathbf{r}$ is not constant, i.e., $\underline{r}_i < \overline{r}_i$ for some $i = 1, \ldots, n$. Let $\overline{\mathbf{r}} = (\overline{r}_1, \ldots, \overline{r}_n)$. It is easy to see that $\alpha_0^*(\overline{\theta}, \overline{\mathbf{r}}) = \underline{\alpha}_0^*$. Therefore, f_0 must be binding in state $\overline{\mathbf{s}} = (\overline{\theta}, \overline{\mathbf{r}})$. Since α^* is continuous in $\mathbf{s}$, there exists $\varepsilon > 0$ such that, if $|\mathbf{r} - \overline{\mathbf{r}}| < \varepsilon$, then $\alpha_0^*(\overline{\theta}, \mathbf{r}) > f_0$ and hence the floor is still binding. When f_0 binds, A's allocation $\hat{\alpha}_{-0}$ must maximize $\theta \sum_{i=1}^n u^i(x_i; r_i)$ subject to $\sum_{i=1}^n x_i \leq 1 - f_0$. So, for all $\mathbf{r}$ with $|\mathbf{r} - \overline{\mathbf{r}}| < \varepsilon$, we must have

$$u_1^i(\hat{\alpha}_i(\overline{\theta}, \mathbf{r}); r_i) = u_1^j(\hat{\alpha}_j(\overline{\theta}, \mathbf{r}); r_j) \quad \text{for all } i, j.$$

It follows that there exists $\mathbf{r}'$ with $|\mathbf{r}' - \overline{\mathbf{r}}| < \varepsilon$ such that $\hat{\alpha}_{-0}(\overline{\theta}, \mathbf{r}') \neq \hat{\alpha}_{-0}(\overline{\theta}, \overline{\mathbf{r}})$. Since $\sum_{i=1}^n \hat{\alpha}_i(\overline{\theta}, \mathbf{r}') = \sum_{i=1}^n \hat{\alpha}_i(\overline{\theta}, \overline{\mathbf{r}}) = 1 - f_0$, there exists $i \neq j$ such that $\hat{\alpha}_i(\overline{\theta}, \mathbf{r}') > \hat{\alpha}_i(\overline{\theta}, \overline{\mathbf{r}})$ and $\hat{\alpha}_j(\overline{\theta}, \mathbf{r}') < \hat{\alpha}_j(\overline{\theta}, \overline{\mathbf{r}})$. Now let $S(f_0)$ be the set of states for which $\hat{\alpha}_0(\mathbf{s}) = f_0$. By the previous argument, $\hat{\alpha}_i$ and $\hat{\alpha}_j$ cannot be constant over $S(f_0)$.

For each $k = 1, \ldots, n$, let $\hat{c}_k = \max_S \hat{\alpha}_k(\mathbf{s})$. When $\hat{\alpha}_i(\mathbf{s}) = \hat{c}_i$, we must have $\hat{\alpha}_j(\mathbf{s}) < \hat{c}_j$, and when $\hat{\alpha}_j(\mathbf{s}) = \hat{c}_j$, we must have $\hat{\alpha}_i(\mathbf{s}) < \hat{c}_i$. Therefore, $\sum_{i=1}^n \hat{c}_i > 1 - f_0$. It follows that any collection of caps $\mathbf{c}_{-0} = \{c_i\}_{i=1}^n$ satisfying $\sum_{i=1}^n c_i = 1 - f_0$ must involve $c_i < \hat{c}_i$ for some $i = 1, \ldots, n$. So, when A faces $\mathbf{c}_{-0}$, for some i and state $\mathbf{s}$, $\alpha_i(\mathbf{s}) \leq c_i$ for all states $\mathbf{s}$ such that A chooses $x_i > c_i$ under f_0. Since $\hat{\alpha}$ is continuous in $\mathbf{s}$, the set $S(\mathbf{c}_{-0}) = \{\mathbf{s} : \hat{\alpha}_i(\mathbf{s}) > c_i\}$ is open and hence it has strictly positive probability under G.

A.6 Proof of Lemma 5

Without loss, let $i = 1$ and take any $c_1 \in (0, \alpha_1^*(\mathbf{s}))$. Consider A's problem in state $\mathbf{s}$ subject to c_1:

$$\max_{\{(\mathbf{x}, x_0) \in B : x_1 \leq c_1\}} \hat{u}(\mathbf{x}, \mathbf{s}) + bv(x_0).$$

The first-order conditions of the associated Lagrangian are

$$\begin{aligned} bv'(\alpha_0(\mathbf{s})) &= \mu(\mathbf{s}), \\ \theta u_1^1(\alpha_1(\mathbf{s}); r_1) &= \mu(\mathbf{s}) + \lambda_1(\mathbf{s}), \\ \theta u_1^i(\alpha_i(\mathbf{s}); r_i) &= \mu(\mathbf{s}) \quad \text{for all } j \neq 0, 1, \end{aligned}$$

where $\mu(\mathbf{s}) \geq 0$ and $\lambda_1(\mathbf{s}) \geq 0$ are the Lagrange multipliers for constraints $\sum_{i=1}^n x_i \leq 1$ and $x_1 \leq c_1$.

Suppose $\alpha_0(\mathbf{s}) \leq \alpha_0^*(\mathbf{s})$. Since $\alpha_1(\mathbf{s}) = c_1 < \alpha_1^*(\mathbf{s})$ and $\sum_j \alpha_j(\mathbf{s}) = \sum_j \alpha_j^*(\mathbf{s}) = 1$ by strong monotonicity of preferences, $\alpha_j(\mathbf{s}) > \alpha_j^*(\mathbf{s})$ for some $j \neq 0, 1$. By strict concavity of u_j and v,

$$\theta u_1^j(\alpha_j(\mathbf{s}); r_j) < \theta u_1^j(\alpha_j^*(\mathbf{s}); r_j) = bv'(\alpha_0^*(\mathbf{s})) \leq bv'(\alpha_0(\mathbf{s})).$$

This violates the first-order conditions for $\alpha(\mathbf{s})$. So we must have $\alpha_0(\mathbf{s}) > \alpha_0^*(\mathbf{s})$. This in turn implies that for $j \neq i$

$$\theta u_1^j(\alpha_j(\mathbf{s}); r_j) = bv'(\alpha_0(\mathbf{s})) < bv'(\alpha_0^*(\mathbf{s})) = \theta u_1^j(\alpha_j^*(\mathbf{s}); r_j).$$

By concavity, we have $\alpha_j(\mathbf{s}) > \alpha_j^*(\mathbf{s})$ for $j \neq 0, 1$.

A.7 Proof of Proposition 3

Fix $i = 1$ and consider any $c_1 \leq \overline{\alpha}_1^*$. Let α^{c_1} describe A's choices under cap c_1. Then, let

$$\Phi(c_1) = \int_S U(\alpha^{c_1}(\mathbf{s}); \mathbf{s}) dG.$$

Let $S(c_1) = \{\mathbf{s} : \alpha_1^*(\mathbf{s}) > c_1\}$. Note that for any $c_1 < \overline{\alpha}_1^*$, since α_1^* is continuous, $S(c_1)$ is non-empty and open and hence has strictly positive probability under G. We have

$$\begin{aligned}
\Phi(c_1) - \Phi(\overline{\alpha}_1^*) &= \int_{S(c_1)} \left[U(\alpha^{c_1}(\mathbf{s}); \mathbf{s}) - U(\alpha^*(\mathbf{s}); \mathbf{s})\right] dG \\
&= (1-b) \int_{S(c_1)} \left[v(\alpha_0^{c_1}(\mathbf{s})) - v(\alpha_0^*(\mathbf{s}))\right] dG \\
&\quad + \int_{S(c_1)} \left[\tilde{V}(\alpha_1^{c_1}(\mathbf{s}); \mathbf{s}) - \tilde{V}(\alpha_1^*(\mathbf{s}); \mathbf{s})\right] dG
\end{aligned}$$

where

$$\tilde{V}(\hat{c}_1; \mathbf{s}) = V(\alpha^{\hat{c}_1}(\mathbf{s}); \mathbf{s}) = \max_{\{(\mathbf{x}, x_0) \in \mathbb{R}_+^{n+1} : \sum_{j=1}^n x_j \leq 1, x_1 \leq \hat{c}_1\}} \{\hat{u}(\mathbf{x}; \mathbf{s}) + bv(x_0)\}.$$

Clearly, $\tilde{V}(\alpha_1^*(\mathbf{s}); \mathbf{s}) \geq \tilde{V}(c_1; \mathbf{s})$ for every $\mathbf{s}$. From the first-order conditions of the Lagrangian defining $V(\alpha^{\hat{c}_1}(\mathbf{s}); \mathbf{s})$, we have $\lambda_1(\mathbf{s}; \hat{c}_1) = \theta u_1^1(\alpha_1^{\hat{c}_1}(\mathbf{s}); \mathbf{s}) - bv'(\alpha_0^{\hat{c}_1}(\mathbf{s}))$, where $\lambda_1(\mathbf{s}; \hat{c}_1)$ is the Lagrange multiplier on the constraint $x_1 \leq \hat{c}_1$. Since $\alpha^{\hat{c}_1}(\mathbf{s})$ is continuous in $\hat{c}_1$ as well as $\mathbf{s}$, so is $\lambda_1(\mathbf{s}; \hat{c}_1)$. Relying again on Theorem 1, p. 222, of Luenberger (1969), we conclude that $\tilde{V}'(\hat{c}_1; \mathbf{s})$ exists for every $\hat{c}_1$ and equals $\lambda_1(\mathbf{s}; \hat{c}_1)$. It follows that $\tilde{V}'(\alpha_1^*(\mathbf{s}); \mathbf{s}) = 0$ for every $\mathbf{s}$ by the definition of α^*. Therefore, by the Mean Value Theorem,

$$\tilde{V}(\alpha_1^{c_1}(\mathbf{s}); \mathbf{s}) - \tilde{V}(\alpha_1^*(\mathbf{s}); \mathbf{s}) = \tilde{V}'(\chi(\mathbf{s}); \mathbf{s})(\alpha_1^{c_1}(\mathbf{s}) - \alpha_1^*(\mathbf{s})),$$

$$v(\alpha_0^{c_1}(\mathbf{s})) - v(\alpha_0^*(\mathbf{s})) = v'(\xi(\mathbf{s}))(\alpha_0^{c_1}(\mathbf{s}) - \alpha_0^*(\mathbf{s})),$$

where $\chi(\mathbf{s}) \in [\alpha_1^{c_1}(\mathbf{s}), \alpha_1^*(\mathbf{s})]$ and $\xi(\mathbf{s}) \in [\alpha_0^*(\mathbf{s}), \alpha_0^{c_1}(\mathbf{s})]$.

Let $c_1^\varepsilon = \overline{\alpha}_1^* - \varepsilon$ for some small $\varepsilon > 0$. Fix any $\mathbf{s} \in S(c_1^\varepsilon)$ and, for now, suppress the dependence on $\mathbf{s}$ for simplicity. Recall that $\sum_i \alpha_i^{c_1^\varepsilon} = \sum_i \alpha_i^* = 1$. Since $\alpha_0^{c_1^\varepsilon} > \alpha_0^*$ for any $\varepsilon > 0$, we can write

$$-\frac{\alpha_1^{c_1^\varepsilon} - \alpha_1^*}{\alpha_0^{c_1^\varepsilon} - \alpha_0^*} = 1 + \sum_{j \neq 0,1} \frac{\alpha_j^{c_1^\varepsilon} - \alpha_j^*}{\alpha_0^{c_1^\varepsilon} - \alpha_0^*}.$$

Now, for any c_1^ε, the following first order condition must hold for every $j \neq 1$:

$$bv'(\alpha_0) - \theta u_1^j(\alpha_j; r_j) = 0.$$

This defines an implicit function $\alpha_j(\alpha_0)$ and, by the Implicit Function Theorem,

$$\frac{d}{d\alpha_0}\alpha_j(\alpha_0) = \frac{bv''(\alpha_0)}{\theta u^j_{11}(\alpha_j(\alpha_0); r_j)}.$$

Since $u^j_{11} < 0$, $v'' < 0$, $\underline{\theta} > 0$, we have $\frac{d}{d\alpha_0}\alpha_j > 0$ everywhere. Moreover, u^j_{11} and v'' are continuous and we can restrict attention to α_0 and α_j that take values in the compact set $[\underline{\alpha}^*_0, 1] \times [\underline{\alpha}^*_j, 1]$ where $\underline{\alpha}^*_0 > 0$ and $\underline{\alpha}^*_j > 0$. Therefore, $\frac{d}{d\alpha_0}\alpha_j$ is bounded above by some finite $k_j > 0$ for all $\mathbf{s} \in S(c^\varepsilon_1)$. Hence, for any $\varepsilon > 0$, $\alpha^{c^\varepsilon_1}_j - \alpha^*_j \leq k_j(\alpha^{c^\varepsilon_1}_0 - \alpha^*_0)$. Letting $K = \sum_{j\neq 0,1} k_j$, we then have

$$-\frac{\alpha^{c^\varepsilon_1}_1 - \alpha^*_1}{\alpha^{c^\varepsilon_1}_0 - \alpha^*_0} \leq 1 + K \quad \Rightarrow \quad \alpha^{c^\varepsilon_1}_0 - \alpha^*_0 \geq \frac{\alpha^*_1 - \alpha^{c^\varepsilon_1}_1}{1+K}.$$

Using these observations, we have that $\Phi(c^\varepsilon_1) - \Phi(\overline{\alpha}^*_1)$ is bounded below by

$$\int_{S(c^\varepsilon_1)} \left[\frac{1-b}{1+K} v'(\xi(\mathbf{s})) - \tilde{V}'(\chi(\mathbf{s}); \mathbf{s})\right] (\alpha^*_1(\mathbf{s}) - c^\varepsilon_1) dG. \tag{15}$$

Since v' is continuous and strictly positive everywhere and $\xi(\mathbf{s}) \in [\underline{\alpha}^*_0, 1]$ with $\underline{\alpha}^*_0 > 0$ for all $\mathbf{s} \in S(c^\varepsilon_1)$, there exists a finite $\kappa > 0$ such that $v'(\xi(\mathbf{s})) \geq \kappa$ for all $\mathbf{s} \in S(c^\varepsilon_1)$.

Next let $\overline{S}(c^\varepsilon_1) = \{\mathbf{s} : \alpha^*_1(\mathbf{s}) \geq c^\varepsilon_1\}$ which is a closed and bounded set by continuity of α^*_1 and hence is compact. As a function of c^ε_1, the correspondence $\overline{S}(\cdot)$ is continuous by continuity of α^*_1. Note that, if $\alpha^*_1(\mathbf{s}) = c^\varepsilon_1$, then $\tilde{V}'(\chi(\mathbf{s}); \mathbf{s}) = \tilde{V}'(\alpha^\varepsilon_1(\mathbf{s}); \mathbf{s}) = 0$. We have

$$\sup_{\mathbf{s}\in S(c^\varepsilon_1)} \tilde{V}'(\chi(\mathbf{s}); \mathbf{s}) \quad = \quad \sup_{\mathbf{s}\in \overline{S}(c^\varepsilon_1)} \tilde{V}'(\chi(\mathbf{s}); \mathbf{s}) \leq \max_{c^\varepsilon_1 \leq \zeta \leq \overline{\alpha}^*_1, \mathbf{s}\in\overline{S}(c^\varepsilon_1)} \tilde{V}'(\zeta; \mathbf{s}) \equiv \kappa(c^\varepsilon_1).$$

Clearly, $\kappa(c^\varepsilon_1) \geq 0$ for any $\varepsilon > 0$, $\varepsilon' > \varepsilon > 0$ implies that $\kappa(c^\varepsilon_1) \leq \kappa(c^{\varepsilon'}_1)$, and $\lim_{\varepsilon\to 0} \kappa(c^\varepsilon_1) = 0$ because $\kappa(\cdot)$ is also continuous. Therefore, there exists $\varepsilon^* > 0$ such that

$$\kappa(c^{\varepsilon^*}_1) < \kappa \frac{1-b}{1+K}.$$

It follows that for ε^*, expression (15) is strictly positive and hence $\Phi(c^{\varepsilon^*}_1) > \Phi(\overline{\alpha}^*_1)$. This also holds for all $\varepsilon \in (0, \varepsilon^*)$.

A.8 Proof of Proposition 4

Let $\hat{\alpha}$ and $\hat{\alpha}'$ describe A's choices across states under $C_{\mathbf{f},\mathbf{c}}$ and $C_{f_0,\mathbf{c}_{-0}}$. Then, $\mathcal{U}(C_{f_0,\mathbf{c}_{-0}}) - \mathcal{U}(C_{\mathbf{f},\mathbf{c}})$ equals

$$\begin{aligned}\int_S [U(\hat{\alpha}'(\mathbf{s}); \mathbf{s}) - U(\hat{\alpha}(\mathbf{s}); \mathbf{s})]\, dG \quad &= \quad \int_S (1-b)\,[v(\hat{\alpha}'_0(\mathbf{s})) - v(\hat{\alpha}_0(\mathbf{s}))]\, dG \\ &\quad + \int_S [V(\hat{\alpha}'(\mathbf{s}); \mathbf{s}) - V(\hat{\alpha}(\mathbf{s}); \mathbf{s})]\, dG \\ &= \quad \int_S (1-b)\,[v(\hat{\alpha}'_0(\mathbf{s})) - v(\hat{\alpha}_0(\mathbf{s}))]\, dG \qquad (16)\end{aligned}$$

$$+\int_S \left[\widehat{V}(C_{f_0,\mathbf{c}_{-0}};\mathbf{s}) - \widehat{V}(C_{\mathbf{f},\mathbf{c}};\mathbf{s})\right] dG,$$

where for any $(\tilde{\mathbf{f}}, \tilde{\mathbf{c}})$

$$\widehat{V}(C_{\tilde{\mathbf{f}},\tilde{\mathbf{c}}};\mathbf{s}) = \max_{\{(\mathbf{x},x_0)\in B:\tilde{\mathbf{f}}\leq(\mathbf{x},c_0)\leq\tilde{\mathbf{c}}\}} V(\mathbf{x}, x_0;\mathbf{s}).$$

Clearly, for every $\mathbf{s}$, $\widehat{V}(C_{f_0,\mathbf{c}_{-0}};\mathbf{s}) \geq \widehat{V}(C_{\mathbf{f},\mathbf{c}};\mathbf{s})$. Moreover, the inequality is strict in states in which either c_0 or f_i are binding for A, given the strict concavity of A's payoff function and convexity of the feasible set for A under both $C_{f_0,\mathbf{c}_{-0}}$ and $C_{\mathbf{f},\mathbf{c}}$. Therefore, if any of them binds with strictly positive probability, the second integral in (16) is strictly positive.

Now consider the first integral, if we can show that $\hat{\alpha}'_0(\mathbf{s}) \geq \hat{\alpha}_0(\mathbf{s})$ for every $\mathbf{s}$, we are done. To show this, we proceed in steps, removing one constraint from $C_{\mathbf{f},\mathbf{c}}$ at a time. Consider first removing only c_0 which leads to an intermediate behavior of A described by the function α^0. If c_0 is never binding for A, then it does not affect his choices and hence $\alpha^0_0(\mathbf{s}) = \hat{\alpha}_0(\mathbf{s})$ for every $\mathbf{s}$. In any state $\mathbf{s}$ in which c_0 is binding, removing only this cap cannot decrease $\hat{\alpha}_0(\mathbf{s})$ because A could have decreased it when the cap was in place. So, $\alpha^0_0(\mathbf{s}) \geq \hat{\alpha}_0(\mathbf{s})$ for every $\mathbf{s}$. Note that, once we remove the cap on x_0, for all $\mathbf{s}$ we must have $\sum_i x_i = 1$ because v is strictly increasing.

Now consider removing one floor f_i for $i \neq 0$ at a time. Fix any state $\mathbf{s}$ and suppress the dependence on it for simplicity. The Lagrangian of A's problem after we remove only c_0 is

$$\theta \sum_{i=1}^{n} u^i(x_i; r_i) + bv(x_0) + \mu \left[1 - \sum_{i=0}^{n} x_i\right] + \sum_{i=1}^{n} \gamma_i[c_i - x_i] + \sum_{i=0}^{n} \phi_i[x_i - f_i].$$

Hence, the first-order necessary and sufficient conditions are

$$\theta u^i_1(\alpha^0_i; r_i) - \mu^0 + \phi^0_i - \gamma^0_i = 0 \quad \text{for } i = 1, \ldots, n,$$
$$bv'(\alpha^0_0) - \mu^0 + \phi^0_0 = 0,$$

with the usual complementary-slackness conditions. Without loss, start by removing f_1, thus obtaining α^1. First, if $\alpha^0_0 = f_0$, then $\alpha^1_0 \geq \alpha^0_0$. So suppose that $\alpha^0_0 > f_0$ so that $\phi^0_0 = 0$. If $\phi^0_1 = 0$, then removing f_1 has no effect and hence again $\alpha^1_0 \geq \alpha^0_0$. So suppose that $\phi^0_1 > 0$; since $c_1 \geq f_1 = \alpha^0_1$, it follows that $\gamma^0_1 = 0$ without loss of generality.[37] After removing f_1 only, the new conditions are

$$\theta u^i_1({\alpha^1}_i; r_i) - \mu^1 + \phi^1_i - \gamma^1_i = 0 \quad \text{for } i = 1, \ldots, n,$$
$$bv'(\alpha^1_0) - \mu^1 + \phi^1_0 = 0.$$

Clearly, at the resulting α^1, we must have $\alpha^1_1 < \alpha^0_1$ because the opposite choice was feasible for A before removing f_1. Suppose $\alpha^1_0 < \alpha^0_0$. Then, we must have $\alpha^1_j > \alpha^0_j$ for some $j \neq 0, 1$, because $\sum_{i=0}^n \alpha^0_i = \sum_{i=0}^n \alpha^1_i = 1$, and hence $u^j_1(\alpha^1_j; r_j) < u^j_1(\alpha^0_j; r_j)$ by

[37]Recall that, by Lagrange Duality, γ^0_1 is the result of a minimization of the Lagrangian at α^0.

strict concavity. To see that this leads to a contradiction, first observe that we must have $\gamma_j^0 = 0$, because if $\gamma_j^0 > 0$, then $\alpha_j^0 = c_j \geq \alpha_j^1$. Given this, then

$$bv'(\alpha_0^1) + \gamma_j^1 = \theta u_1^j(\alpha_j^1; r_j) < \theta u_1^j(\alpha_j^0; r_j) = bv'(\alpha_0^0) - \phi_j^0,$$

but this condition cannot hold because $v'(\alpha_0^1) > v'(\alpha_0^0)$ for $\alpha_0^1 < \alpha_0^0$ by our starting assumption. We conclude that $\alpha_0^1 \geq \alpha_0^0$.

Continuing in this way, we can remove every f_i for $i = 2, \ldots, n$, obtaining at each step that $\alpha_0^i \geq \alpha_0^{i-1}$. Since $\alpha_0^n = \hat{\alpha}_0'$, by transitivity we get $\hat{\alpha}_0' \geq \hat{\alpha}_0$. Since this steps assumed an arbitrary $\mathbf{s}$, we have that $\hat{\alpha}_0'(\mathbf{s}) \geq \hat{\alpha}_0(\mathbf{s})$ for every $\mathbf{s}$ as desired.

A.9 Proof of Lemma 6

Fix $b \in (0, 1)$. Suppose C' is optimal, but $\underline{x}_0' < \underline{\pi}_0^*$. Consider $C'' \in \mathcal{R}$ identical to C', except that $f_0'' = \underline{\pi}_0^*$. We claim that $\mathcal{U}(C'') > \mathcal{U}(C')$, which contradicts the optimality of C'. Since C' is convex and compact, the ensuing allocation α' is a continuous function of $\mathbf{s}$. Hence, the set $S(\underline{\pi}_0^*) = \{\mathbf{s} \in S : \alpha_0'(\mathbf{s}) < \underline{\pi}_0^*\}$ contains an open subset and hence has strictly positive probability under G.

Consider any $\mathbf{s} \in S(\underline{\pi}_0^*)$. Suppose P faces the following problem:

$$\max\{\hat{u}(\mathbf{x}; \mathbf{s}) + v(x_0)\}$$

subject to $(\mathbf{x}, x_0) \in \mathbb{R}_+^{n+1}$, $x_i \leq c_i'$, and $x_0 \leq f_0$. For any $f_0 < \underline{\pi}_0^*$, the latter constraint must bind for P because, by the same logic of Lemma 5, P would choose $\pi_0(\mathbf{s}) \geq \pi_0^*(\mathbf{s}) \geq \underline{\pi}_0^*$ if facing only the constraints $x_i \leq c_i'$ for $i = 1, \ldots, n$. Therefore, P's payoff from this fictitious problem is strictly increasing in f_0 for $f_0 \leq \underline{\pi}_0^*$. When A faces C'', the constraint $x_0 \geq \underline{\pi}_0^*$ must bind. Hence, his allocation $\alpha''(\mathbf{s}) = (\alpha_{-0}''(\mathbf{s}), \underline{\pi}_0^*)$ solves $\max \hat{u}(\mathbf{x}; \mathbf{s})$ subject to $\mathbf{x} \in \mathbb{R}_+^n$, $x_i \leq c_i'$, and $\sum_{i=1}^n x_i \leq 1 - \underline{\pi}_0^*$. This allocation coincides with P's allocation under the fictitious problem with $f_0 = \underline{\pi}_0^*$. Hence, in $\mathbf{s}$, $\alpha''(\mathbf{s})$ is strictly better for P than $\alpha'(\mathbf{s})$.

We conclude that, for all $\mathbf{s} \in S(\underline{\pi}_0^*)$, P's payoff is strictly larger under C'' than under C'. Since for $\mathbf{s} \notin S(\underline{\pi}_0^*)$ A's allocation is unchanged, we must have $\mathcal{U}(C'') > \mathcal{U}(C')$.

A.10 Proof of Proposition 5

Part (1): By Proposition 2, $\overline{f}_0(b) = \max F(b)$ decreases monotonically to $\underline{\pi}_0^*$ when $b \uparrow 1$. Also, for every $i = 1, \ldots, n$, we have that $\alpha_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}; b)$ increases monotonically to $\pi_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i})$ as $b \uparrow 1$. By Lemma 7, $\pi_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}) > \underline{\pi}_0^*$. Given this, define

$$b^* = \inf\{b \in (0,1) : \overline{f}_0(b) < \max_i \alpha_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}; b)\}.$$

Clearly, $b^* < 1$ and for every $b > b^*$ we have $\alpha_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}; b) > \overline{f}_0(b)$ for at least some $i = 1, \ldots, n$. Hereafter, fix $b > b^*$ and any i that satisfies this last condition.

For $\varepsilon \geq 0$, consider $c_i^\varepsilon = \overline{\alpha}_i^* - \varepsilon$ as in Proposition 3 where $\overline{\alpha}_i^* = \alpha_i^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i})$ by Lemma 7. Let $\Phi(c_i^\varepsilon, \overline{f}_0)$ be P's expected payoff from adding c_i^ε to the existing optimal floor $\overline{f}_0$. We will show that there exists $\varepsilon > 0$ such that $\Phi(c_i^\varepsilon, \overline{f}_0) > \Phi(c_i^0, \overline{f}_0)$ where $\Phi(c_i^0, \overline{f}_0) = \Psi(\overline{f}_0)$ in Section (4.1). To do so, for any $\varepsilon \geq 0$, let α^ε be A's allocation function under $(c_i^\varepsilon, \overline{f}_0)$ and $S(c_i^\varepsilon) = \{\mathbf{s} \in S : \alpha_i^0(\mathbf{s}) > c_i^\varepsilon\}$. Then,

$$\Phi(c_i^\varepsilon, \overline{f}_0) - \Phi(c_i^0, \overline{f}_0) = \int_{S(c_i^\varepsilon)} \left[U(\alpha^\varepsilon(\mathbf{s}); \mathbf{s}) - U(\alpha^0(\mathbf{s}); \mathbf{s})\right] dG.$$

Note that, if there exists $\overline{\varepsilon} > 0$ such that for all $0 < \varepsilon < \overline{\varepsilon}$ we have $\alpha^0(\mathbf{s}) = \alpha^*(\mathbf{s})$ for all $\mathbf{s} \in S(c_i^\varepsilon)$, then for such ε's the previous difference equals $\Phi(c_i^\varepsilon) - \Phi(\overline{\alpha}_i^*)$ in the proof of Proposition 3. The conclusion of that proof then implies that there exists $\varepsilon^{**} \in (0, \overline{\varepsilon})$ such that $\Phi(c_i^{\varepsilon^{**}}, \overline{f}_0) > \Phi(c_i^0, \overline{f}_0)$.

Thus we only need to prove the existence of $\overline{\varepsilon}$. Let $\overline{S}(\overline{f}_0) = \{\mathbf{s} \in S : \alpha_0^*(\mathbf{s}) \leq \overline{f}_0\}$, which is compact by continuity of α^*. Define $\tilde{\alpha}_i = \max_{\overline{S}(\overline{f}_0)} \alpha_i^0(\mathbf{s})$ which is well defined by continuity of α^0. Since $\alpha_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}) > \overline{f}_0$, it follows that $(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}) \notin \overline{S}(\overline{f}_0)$ and hence $\alpha_i^0(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}) = \alpha_i^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i})$ where $\alpha_i^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i}) = \overline{\alpha}_i^*$ by Lemma 7. We must also have $\tilde{\alpha}_i < \overline{\alpha}_i^*$: for all $\mathbf{s} \in \overline{S}(\overline{f}_0)$, optimality requires

$$\theta u_1^i(\alpha_i(\mathbf{s}); r_i) = v'(\overline{f}_0) + \lambda_0(\mathbf{s}) > v'(\alpha_0^*(\overline{\theta}, \overline{r}_i, \underline{r}_{-i})) = \overline{\theta} u_1^i(\overline{\alpha}_i^*; \overline{r}_i),$$

where $\lambda_0(\mathbf{s}) \geq 0$ is the Lagrange multiplier for constraint $x_0 \geq \overline{f}_0$. If $\mathbf{s} \in S$ is such that $\alpha_i^0(\mathbf{s}) > \tilde{\alpha}_i$, then $\mathbf{s} \notin \overline{S}(\overline{f}_0)$—otherwise it would contradict the definition of $\tilde{\alpha}_i$—and therefore $\alpha^0(\mathbf{s}) = \alpha^*(\mathbf{s})$. Now define $\overline{\varepsilon} = \overline{\alpha}_i^* - \tilde{\alpha}_i > 0$. By construction for any $\varepsilon \in (0, \overline{\varepsilon})$, $\alpha_i^0(\mathbf{s}) > c_i^\varepsilon$ implies that $\alpha^0(\mathbf{s}) = \alpha^*(\mathbf{s})$, as desired.

Part (2): We first show that there exists $b_{**} > 0$ such that, if $b < b_{**}$, then for any $C \in \mathcal{R}$ with $\underline{x}_0 \geq \underline{\pi}_0^*$ the resulting α satisfies $\alpha_0(\mathbf{s}) = \underline{x}_0$ for all $\mathbf{s} \in S$. It is enough to show that $\alpha_0(\underline{\mathbf{s}}) = \overline{\alpha}_0 = \max_S \alpha_0(\mathbf{s})$ must equal $\underline{x}_0$. By strict concavity of v, $v'(\overline{\alpha}_0) \leq v'(\underline{\pi}_0^*) < +\infty$ because $\underline{\pi}_0^* > 0$. By considering the Lagrangian of A's problem in state $\underline{\mathbf{s}}$ (see Proposition 4's proof), we have that $\alpha(\underline{\mathbf{s}})$ must satisfy

$$bv'(\alpha_0(\underline{\mathbf{s}})) + \phi_0(\underline{\mathbf{s}}) + \gamma_i(\underline{\mathbf{s}}) = \underline{\theta} u_1^i(\alpha_i(\underline{\mathbf{s}}); \underline{r}_i) \quad \text{for all } i = 1, \ldots, n,$$

where $\phi_0(\underline{\mathbf{s}}) \geq 0$ and $\gamma_i(\underline{\mathbf{s}}) \geq 0$ are the Lagrange multipliers for constraints $x_0 \geq f_0$ and $x_i \leq c_i$. For every $i = 1, \ldots, n$, since $\alpha_i(\underline{\mathbf{s}}) \leq 1$ and $u^i(\cdot; \underline{r}_i)$ is strictly concave, $u_1^i(\alpha_i(\underline{\mathbf{s}}); \underline{r}_i) \geq u_1^i(1; \underline{r}_i) > 0$. Now let

$$b_{**} = \min_i \frac{\underline{\theta} u_1^i(1; \underline{r}_i)}{v'(\underline{\pi}_0^*)} > 0. \tag{17}$$

Then, for any $b < b_{**}$, we have $bv'(\alpha_0(\underline{\mathbf{s}})) < \underline{\theta} u_1^i(\alpha_i(\underline{\mathbf{s}}); \underline{r}_i)$ for all $i = 1, \ldots, n$. Therefore, $\phi_0(\underline{\mathbf{s}}) + \gamma_i(\underline{\mathbf{s}}) > 0$ for all $i = 1, \ldots, n$. Hence, either $\phi_0(\underline{\mathbf{s}}) > 0$, in which case $\overline{\alpha}_0 = f_0 = \underline{x}_0$; or $\gamma_i(\underline{\mathbf{s}}) > 0$ for all $i = 1, \ldots, n$, in which case $\overline{\alpha}_0 = 1 - \sum_{i=1}^n \alpha_i(\underline{\mathbf{s}}) = 1 - \sum_{i=1}^n c_i = \underline{x}_0$.

Finally, let $b < b_* = \min\{\underline{b}, b_{**}\}$ where $\underline{b} > 0$ was defined in Proposition 2. Let $C^b \in \mathcal{R}$ be an optimal policy for b. By Proposition 6, $\underline{x}_0^b \geq \underline{\pi}_0^*$. The previous result then implies that

$$\mathcal{U}(C^b) = v(\underline{x}_0^b) + \int_S \hat{u}(\alpha_{-0}(\mathbf{s}); \mathbf{s}) dG.$$

Hence,

$$\mathcal{U}(C^b) \leq v(\underline{x}_0^b) + \int_S \hat{u}(\mathbf{x}^{\underline{x}_0^b}(\mathbf{s}); \mathbf{s}) dG \leq v(\overline{f}_0) + \int_S \hat{u}(\mathbf{x}^{\overline{f}_0}(\mathbf{s}); \mathbf{s}) dG = \mathcal{U}(C_{\overline{f}_0}),$$

where the first inequality follows since $\hat{u}(\alpha_{-0}(\mathbf{s});\mathbf{s}) \leq \max_{\{\mathbf{x}\in\mathbb{R}^n_+:\sum_{i=1}^n x_i \leq \underline{x}_0^b\}} \hat{u}(\mathbf{x};\mathbf{s}) = \hat{u}(\mathbf{x}^{\underline{x}_0^b}(\mathbf{s});\mathbf{s})$ for all $\mathbf{s}\in S$ and from the definition of $\overline{f}_0$ in Proposition 4.1. It is immediate to see that if C^b involves caps that bind for a set of states S' whose probability is strictly positive, then $\hat{u}(\alpha_{-0}(\mathbf{s});\mathbf{s}) < \hat{u}(\mathbf{x}^{\underline{x}_0^b}(\mathbf{s});\mathbf{s})$ for all $\mathbf{s}\in S'$, and hence $\mathcal{U}(C^b) < \mathcal{U}(C_{\overline{f}_0})$. Therefore, optimal policies can only involve a private-consumption floor.

Finally, let $\underline{\mathbf{r}}'$, $\underline{\mathbf{r}}$, $\overline{\mathbf{r}}'$, and $\overline{\mathbf{r}}$ satisfy the properties in the statement of Proposition 5. The corresponding states $\underline{\mathbf{s}}'$, $\underline{\mathbf{s}}$, $\overline{\mathbf{s}}'$, and $\overline{\mathbf{s}}$ satisfy the same properties. It follows that $\underline{\pi}_0^{*\prime} = \pi_0^*(\overline{\mathbf{s}}') \geq \pi_0^*(\overline{\mathbf{s}}) = \underline{\pi}_0^*$ with strict inequality if $\overline{\mathbf{s}} \neq \overline{\mathbf{s}}'$ (Lemma 7). Similarly, for each $b\in(0,1)$, $\overline{\alpha}_0^{*\prime}(b) = \alpha_0^*(\underline{\mathbf{s}}';b) \leq \alpha_0^*(\underline{\mathbf{s}};b) = \overline{\alpha}_0^*(b)$ again with strict inequality if $\underline{\mathbf{s}}' \neq \underline{\mathbf{s}}$. Using the definition of b_{**} in (17), the strict concavity of the function v, and that $\underline{r}_i' \geq \underline{r}_i$, we have that $b_{**}' > b_{**}$. Using the definition of $\underline{b}$ in the proof of Proposition 2 and that $\overline{\alpha}_0^*$ is strictly increasing in b, we have that $\underline{b}' > \underline{b}$. Therefore $b_*' > b_*$.

A.11 Proof of Lemma 8

Recall the definition of $\mathcal{U}(C)$ and $\alpha(\theta|C)$ in (4) and (5). There exists $C\subset B$ such that $\mathcal{U}(C)\geq\mathcal{U}(C')$ for all $C'\subset B$ if and only if there exist functions $\chi:[\underline{\theta},\overline{\theta}]\to\mathbb{R}^n_+$ and $t:[\underline{\theta},\overline{\theta}]\to\mathbb{R}_+$ that satisfy two conditions:
(1) for all $\theta,\theta'\in[\underline{\theta},\overline{\theta}]$

$$\theta\hat{u}(\chi(\theta)) + bv(t(\theta)) \geq \theta\hat{u}(\chi(\theta')) + bv(t(\theta'))$$

and

$$\sum_{i=1}^n \chi_i(\theta) + t(\theta) \leq 1;$$

(2) the pair (χ,t) maximizes

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\hat{u}(\chi(\theta)) + v(t(\theta))\right] g(\theta)d\theta.$$

On the other hand, there exists $C^{\text{as}}\subset B^{\text{as}}$ such that $\mathcal{U}(C^{\text{as}})\geq\mathcal{U}(\hat{C}^{\text{as}})$ for all $\hat{C}^{\text{as}}\subset B^{\text{as}}$ if and only if there exist functions $\varphi:[\underline{\theta},\overline{\theta}]\to\mathbb{R}_+$ and $\tau:[\underline{\theta},\overline{\theta}]\to\mathbb{R}_+$ that satisfy two conditions:
(1') for all $\theta,\theta'\in[\underline{\theta},\overline{\theta}]$

$$\theta u^*(\varphi(\theta)) + bv(\tau(\theta)) \geq \theta u^*(\varphi(\theta')) + bv(\tau(\theta')),$$

where $u^*(y) = \max_{\left\{\mathbf{x}':\in\mathbb{R}^n_+ \sum_{i=1}^n x_i'\leq y\right\}} \hat{u}(\mathbf{x}')$, and

$$\varphi(\theta) + \tau(\theta) \leq 1;$$

(2') the pair (φ,τ) maximizes

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta u^*(\varphi(\theta)) + v(\tau(\theta))\right] g(\theta)d\theta.$$

Suppose (χ,t) that satisfies condition **(1)** and **(2)**. Then, by our discussion on money burning before the statement of Lemma 8, there exists a function $\varphi:[\underline{\theta},\overline{\theta}]\to\mathbb{R}_+$ such that $u^*(\varphi(\theta)) = \hat{u}(\chi(\theta))$ and $\varphi(\theta) \leq \sum_{i=1}^n \chi_i(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Hence, letting $\tau \equiv t$, we have that (φ, τ) satisfy both **(1')** and **(2')**.

Suppose (φ, τ) satisfy conditions **(1')** and **(2')**. For every $\theta \in [\underline{\theta}, \overline{\theta}]$, let $\chi(\theta) = \arg\max_{\{\mathbf{x} \in \mathbb{R}_+^n : \sum_{i=1}^n x_i \leq \varphi(\theta)\}} \hat{u}(\mathbf{x})$. Then, by definition, $\hat{u}(\chi(\theta)) = u^*(\varphi(\theta))$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Letting $t \equiv \tau$, we have that (χ, t) satisfy both **(1)** and **(2)**.

A.12 Proof of Proposition 7

Let $C^{\text{as}} \subset B^{\text{as}}$ satisfy the premise of Proposition 7. Then, as noted in the proof of Lemma 8, we can describe A's allocation from C^{as} with the functions (φ, τ) that satisfy condition **(1')** and such that $0 < \varphi(\theta) < 1 - \tau(\theta)$ for all $\theta \in \Theta$ and

$$\mathcal{U}(C^{\text{as}}) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta u^*(\varphi(\theta)) + v(\tau(\theta))\right] g(\theta) d\theta.$$

Now, since $\hat{u}$ is continuous and $E_y = \{\mathbf{x} \in \mathbb{R}_+^n : \sum_{i=1}^n x_i = y\}$ is connected, $\hat{u}(E_y) = [u_*(y), u^*(y)]$. Since $\hat{u}$ is strictly concave, $u_*(y) < u^*(y)$ for all $y > 0$. Since $\hat{u}$ is strictly increasing, so are u_* and u^*. Clearly, u_* is continuous.

These properties imply that, for every $\theta \in \Theta$, there exists $y(\theta) \in (\varphi(\theta), 1 - \tau(\theta)]$ and $\mathbf{x}(\theta) \in E_{y(\theta)}$ such that $\hat{u}(\mathbf{x}(\theta)) = u^*(\varphi(\theta))$. So, for every $\theta \in [\underline{\theta}, \overline{\theta}]$, define $t(\theta) = \tau(\theta)$ and

$$\chi(\theta) = \begin{cases} \mathbf{x}(\theta) & \text{if } \theta \in \Theta \\ \arg\max_{\{\mathbf{x} \in \mathbb{R}_+^n : \sum_{i=1}^n x_i \leq \varphi(\theta)\}} \hat{u}(\mathbf{x}) & \text{if } \theta \notin \Theta \end{cases}.$$

Then, by construction the pair (χ, t) satisfy conditions **(1)** and **(2)** in the proof of Lemma 8. Now, let $C' = \{(\mathbf{x}, x_0) \in \mathbb{R}_+^n : (\mathbf{x}, x_0) = (\chi(\theta), t(\theta)),$ for some $\theta \in [\underline{\theta}, \overline{\theta}]\}$. We have $C' \subset B, \mathcal{U}(C') = \mathcal{U}(C^{\text{as}})$, and A's allocation satisfies $\alpha'_{-0}(\theta) = \chi(\theta)$ and $\alpha'_0(\theta) = \tau(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. By construction, α' satisfies the stated relationship with α.

The last part is immediate because we can choose $y(\theta) = 1 - \tau(\theta)$ for all $\theta \in \Theta$ in the previous construction.

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