Economic and Political Equilibrium for a Renewable Natural Resource with International Trade

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Abstract

International resources such as water are typically subject to conflict as individual countries perceive individual gains from increased use of the resource. This inherent conflict is also reflected in analytical studies which are typically partial equilibrium and hence naturally assume that welfare functions are increasing in the resource allocation. In this setting, the question arises if there are ever circumstances such that it is in the joint self-interest of political entities to share the resource.

This question is addressed here in the context of two countries which trade goods and services but also have joint access to a scarce resource (e.g. an international river basin). The analysis is based on a two-stage equilibrium model. Economic equilibrium with Ricardian trade is solved given a specific resource allocation. The trade model is then used to generate country welfare functions as a function of the allocation. These welfare functions then enter into a game-theoretic model which determines political equilibrium.

The results are striking. In the autarchic case, country welfare is increasing in water allocation as expected. However, when trade is allowed, then in some instances we find that the welfare functions can be non-monotone; that is, starting from some initial allocation, it can actually be in the self-interest of one country to give up water to another country. Furthermore, there can be instances in which the highest level of welfare for one country is achieved with joint use of the resource as opposed to having a full allocation of the resource. At a minimum, where productivity coefficients imply a comparative advantage such that trade occurs, then the level of conflict as measured by the gains from an additional allocation of the resource, will be reduced.

In general, the analysis indicates that some of the perceived conflict may be due to a narrow (partial equilibrium) focus on the natural resource. When the analytical and policy framework is broadened out to a more comprehensive general equilibrium framework, then the level of the conflict (gains from an increased allocation of the resource) may very well be reduced or even - in some cases - alleviated.

1 Introduction

Water is a critical natural resource for economic activity, and is increasingly scarce due to population and economic growth, and to increasing demand for environmental amenities stemming from water in its natural state. As difficult as water allocation is within countries and other political units, allocation is even more challenging when the water source is international. Wolf et al. (1999) documents that there are "261 international rivers, which covers 45.3 % of the land-surface of the earth (excluding Antarctica)". Since no supra-national government exists, international water management carries an even higher burden of cooperative self-interest than might exist in other settings subject to governmental allocation channels.

An international water allocation plan needs to be self-enforcing so that the riparian countries are better off under cooperation than non-cooperation. Ideally it will also be socially optimal so that the collective well-being of the river basin is improved. The construction of such a plan relies on the welfare functions of the riparian countries. The current literature of self-enforcing international environmental agreements (IEAs) and river basin management normally assumes a particular welfare function without deriving it from the microeconomic foundations. In particular, Ambec and Sprumont (2002) assumes the benefit function is strictly increasing and strictly concave in units of water. In another paper, Ambec and Ehlers (2008) assumes that the agents' benefit function exhibits a satiation point. Both of the papers are applications of cooperative game theory. In a noncooperative setting, Ansink (2009) analyzes self-enforcing agreements on water allocation based on the outcome of a bargaining game.

This paper considers two countries which have joint access to an international river and which are also involved in trading two produced goods/services. The configuration of the countries and the river is generally not a factor here: the countries might be upstream-downstream, or they might have a joint boundary along the river such as the Rio Grande between Mexico and the United States. Conceptually there are at least three motivations for trade between countries: productivity differences (Ricardian model), primary factor endowments (Hechsler-Ohlin model), or economies of scale (Krugman, 1979). While all of these are relevant to the problem at hand, here we concentrate on the Ricardian case as a reasonable starting point for understanding international natural resource allocation when countries are engaged in trade.

For a given technological parameter specification, the trade model is used to calculate world prices for the two goods as dependent on water allocation between the two countries. This is then used to specify country welfare as functions of the water allocation. The results are striking: under some circumstances country welfare can be declining in water allocation meaning that countries could potentially gain by giving up some water. This occurs by comparative advantage when the natural resource is necessary for production and productivity differences imply that a good can be produced more effectively elsewhere. Of course, there is a limit to this process as countries have to have sufficient resources to generate income to pay for imported goods/services.

The paper next turns to a consideration of political equilibrium, a topic addressed in previous literature. Here, however, the analysis starts from welfare functions derived from a formal trade model and not just assumed. Furthermore, besides a formal justification provided by the trade analysis, the welfare functions differ in functional form from previous work, primarily via the non-monotone behavior.

We first consider a bargaining model. The analysis demonstrates that bargaining outcomes are not unique as they depend on the initial water allocation level. In some instances countries might be willing to voluntarily give up water, but not necessarily in other instances, and only up to some point. A discrete-strategy game-theory model is also considered as in previous studies, with two possible water allocation strategies under the control of one country, and autarky/free trade as the two discrete strategies for the other country. Perhaps the main result here is that this may be a limited framework for analysis as the outcome depends on the discrete strategies selected, and in addition it is typically in the self-interest of countries to pursue free trade, so autarchic threats may well lack credibility.

Overall the results suggest that moving from a partial equilibrium framework to general equilibrium implies a considerably different perspective. In particular, there can be circumstances such that it is in the self-interest of political units to support allocations with less of the natural resource than they might otherwise have. Finally, while we focus on international river water allocation, clearly the results are applicable to natural resources in general. Examples might be access to a joint groundwater aquifer, a common property resource such as fisheries or forests, or waste assimilative capacity of an environmental resource.

2 Model

We consider an international river basin where there are two countries ($i = 1, 2$) with joint access to the river and which potentially engage in Ricardian trade in produced goods. Annual water flow is W, of which country i takes $W_i = \theta_i W$. The river is assumed to be fully allocated with $\theta_1 + \theta_2 = 1$. There are also two goods, each of which is produced utilizing water with production coefficients specific to each country. Subsequent analysis considers autarky and free trade with exogenous water allocations, and social welfare optimum and political equilibrium with endogenous water allocations and trade policy.

For simplicity, identical household preferences

$$U_i = c_{i1}^{\alpha} c_{i2}^{1-\alpha} \tag{1}$$

are assumed for both countries. Here U_i is utility and c_{ij} is good j consumption in country i, with the preference parameter satisfying $0 < \alpha < 1$.

The two goods ($j = 1, 2$) are homogeneous across countries, but technologies in the two countries differ. Linear production functions are

$$y_{ij} = \beta_{ij} w_{ij} \tag{2}$$

where β_{ij} is country i's output coefficient to produce good j, w_{ij} is water allocated to the production of good j in country i, $i \in \{1, 2\}$ and $j \in \{1, 2\}$. Without loss of generality, we assume that

$$\frac{\beta_{12}}{\beta_{11}} < \frac{\beta_{22}}{\beta_{21}} \tag{3}$$

implying that country 1 has a comparative advantage in good 1 and country 2 has a comparative advantage in good 2. This comparative advantage assumption will prevail in the rest of the paper, even if one country has absolute advantage in both goods.

The resource constraint is

$$\sum_{j=1}^{2} w_{ij} = W_i \tag{4}$$

for each country. As both utility and production are increasing functions, this constraint will always be binding.

The two countries can choose to stay in autarky and produce both goods to meet the domestic demand, or they can specialize in the good that they have comparative advantage in and trade with each other in order to increase welfare. The question we want to answer is: if the gap between the two countries' productivities is substantially large, would the countries benefit by giving up water and enjoy low-cost goods imported from the other country? Hence, the following sections will derive the welfare functions for each country under both autarky and free trade as a function of the water allocation parameter. Welfare in this context is measured by consumer utility in each country.

3 Autarky

We first consider autarky over the entire range of exogenous water allocations. The subsequent welfare functions will be used later to demonstrate that allowing for free trade can substantially change the nature of individual countries valuation of water allocations. Welfare functions under autarky will also be necessary for the later political equilibrium analysis.

Under autarky, each country maximizes utility subject to the technology and resource constraints with consumption just equal to production $c_{ij} = y_{ij}$. The optimization problem is then

$$\max\ U_i = c_{i1}^{\alpha} c_{i2}^{1-\alpha} \tag{5}$$

$$\text{s.t. } c_{ij} = \beta_{ij} w_{ij} \quad j \in \{1, 2\}$$

$$w_{i1} + w_{i2} = \theta_i W$$

for country $i \in \{1, 2\}$ and given the allocation parameter θ_i.

The utility maximization problem is illustrated in Figure 1. Solving this problem gives consumption and output:

$$\bar{c}_{i1} = \bar{y}_{i1} = \alpha\beta_{i1}\theta_1 W \tag{6}$$

$$\bar{c}_{i2} = \bar{y}_{i2} = (1-\alpha)\beta_{i2}\theta_1 W \tag{7}$$

for $i \in \{1, 2\}$. Substituting the optimal consumption levels into the utility function gives the maximized utility for country i under autarky.

$$U_i^A = (\alpha\beta_{i1})^{\alpha}((1-\alpha)\beta_{i2})^{1-\alpha}\theta_i W \tag{8}$$

Autarky prices are just the slope of the production possibility frontier in Figure 1. Hence, autarky relative prices are

$$\bar{p}_{i1}/\bar{p}_{i2} = \beta_{i2}/\beta_{i1} \tag{9}$$

under autarky. Furthermore, as illustrated in Figure 2, autarky implies that both countries' welfare functions are linear and monotonically increasing in the water allocation parameter, θ_i . Both countries would be better off as they get more water to produce more goods that are only consumed domestically. Neither country would voluntarily concede to less water without any other conditions.

4 Free Trade

Now suppose the two countries engage in free trade. We want to see if free trade will give countries some leverage in negotiating the water allocation. Would the gains from free trade let the countries give up some water out of its self-interest? First, we derive the free trade welfare as a function of θ_1. The free trade equilibrium has three cases. The most general case is that the two countries each specializes in the good that they have comparative advantage in. The other two cases arise when one country is relatively large compared to the other. In those cases, if the two countries still specialize in one good, then production in the small country would not be able to meet the demand of both countries. The large country has to produce both goods while the small country still specializes in the good that it has comparative advantage in. Hence the world relative price would be the autarky relative price in the large country.

In general, the free trade utility maximization problem for country i is

$$\max\ U_i = c_{i1}^{\alpha} c_{i2}^{1-\alpha} \tag{10}$$

$$\text{s.t. } \tilde{p}_1 c_{i1} + \tilde{p}_2 c_{i2} = \tilde{p}_1 y_{i1}^* + \tilde{p}_2 y_{i2}^*$$

where $\tilde{p}_j$ is the free trade equilibrium world price for commodity j, and y_{ij}^* are the corresponding optimal production levels.

4.1 Intermediate water allocation

Here we consider an intermediate water allocation such that the world equilibrium price ratio $\tilde{p}_1/\tilde{p}_2$ falls between autarky prices

$$\beta_{12}/\beta_{11} = \bar{p}_{11}/\bar{p}_{12} < \tilde{p}_1/\tilde{p}_2 < \bar{p}_{21}/\bar{p}_{22} = \beta_{22}/\beta_{21} \tag{11}$$

with the parametric condition for this to occur to be derived later. Country 1 then specializes in good 1

$$y_{11}^* = \beta_{11}\theta_1 W, \quad y_{12}^* = 0 \tag{12}$$

while Country 2 specializes in good 2

$$y_{21}^* = 0, \quad y_{22}^* = \beta_{22}(1-\theta_1)W \tag{13}$$

implying that each country uses all the water assigned to it to produce the good in which it has a comparative advantage.

Each country solves the consumer optimization problem (10), resulting in the optimal consumption levels

$$c_{11}^* = \alpha\beta_{11}\theta_1 W, \quad c_{12}^* = \frac{\tilde{p}_1}{\tilde{p}_2}\beta_{11}(1-\alpha)\theta_1 W \tag{14}$$

$$c_{21}^* = \frac{\tilde{p}_2}{\tilde{p}_1}\beta_{22}\alpha(1-\theta_1)W, \quad c_{22}^* = (1-\alpha)\beta_{22}(1-\theta_1)W \tag{15}$$

for the two respective countries. Market clearing for the first good is

$$y^*_{11} = c^*_{11} + c^*_{21} \tag{16}$$

with good 2 market clearing implied by Walras Law. This yields

$$\frac{\tilde{p_1}}{\tilde{p_2}} = \frac{\beta_{22}\alpha(1-\theta_1)}{\beta_{11}(1-\alpha)\theta_1} \tag{17}$$

as the world equilibrium price ratio in this particular case.

Figure 3 illustrates the free trade equilibrium for the two countries. Country 1 specializes in good 1, exports $(y^*_{11} - c^*_{11})$ of good 1 to country 2 and imports (c^*_{12}) of good 2 from country 2. Country 2 specializes in good 2, exports $(y^*_{22} - c^*_{22})$ of good 2 and imports (c^*_{21}) of good 1. This is the standard free trade case.

As previously noted, for this case to occur the world equilibrium price must be bounded by the autarky prices. Substituting (17) into (11) yields

$$\frac{\beta_{12}}{\beta_{11}} < \frac{\beta_{22}\alpha(1-\theta_1)}{\beta_{11}(1-\alpha)\theta_1} < \frac{\beta_{22}}{\beta_{21}} \tag{18}$$

and solving for θ_1 gives

$$\frac{\alpha\beta_{21}}{\alpha\beta_{21} + (1-\alpha)\beta_{11}} < \theta_1 < \frac{\alpha\beta_{22}}{\alpha\beta_{22} + (1-\alpha)\beta_{12}} \tag{19}$$

as the parametric condition for this free trade pattern. This requires that the water allocation θ_1 must not be too large, in which instance country 2 could not meet the world demand, nor can it be too small, implying that country 1 production would be insufficient to meet good 1 world demand.

Under these conditions, we can then substitute the world equilibrium price ratio (17) into the optimal consumption equations (14-15), and then optimal consumption into the utility functions (1) to find the country welfare functions. This yields

$$(U_1^{FT})_1 = (\beta_{11}\theta_1)^{\alpha}(\beta_{22}(1-\theta_1))^{1-\alpha}\alpha W \tag{20}$$

and

$$(U_2^{FT})_1 = (\beta_{11}\theta_1)^\alpha(\beta_{22}(1-\theta_1))^{1-\alpha}(1-\alpha)W \tag{21}$$

as utilities for the respective countries in free trade case 1.

4.2 Large country 1 water allocation

If θ_1 violates condition (19), then the world equilibrium price won't fall between the two autarky prices, and full specialization will not occur. Consider first a large θ_1

$$\theta_1 \geq \frac{\alpha\beta_{22}}{\alpha\beta_{22} + (1-\alpha)\beta_{12}} \tag{22}$$

implying that Country 1 gets a relatively large share of the water resource. Full specialization as in Case 1 won't occur for two reasons. First, the production of good 2 in country 2 would not meet the total demand in both countries. Second, if free trade was similar to case 1, then country 1 free trade utility would be lower than its autarky utility, $(U_1^{FT})_1 \leq U_1^A$ as implied by condition (22). Hence, Country 1 would not have an incentive to participate in such trade activity.

In this case, the world equilibrium price will be determined by the autarky price in country 1, since country 2 is not influential in world prices. Hence

$$\frac{\tilde{p}_1}{\tilde{p}_2} = \frac{\beta_{12}}{\beta_{11}} \tag{23}$$

defines the relative world equilibrium price. Country 2 still has a comparative advantage in good 2 in the sense that the relative price for good 2 is lower than the world price (price in country 1), so it still specializes in good 2 with production $y_{22}^* = \beta_{22}(1-\theta_1)W$.

As country 2 is small, it's production cannot meet total demand, hence country 1 will produce both goods. Its consumption is equal to the autarky consumption, while country 2 consumption is

$$c_{21}^* = \frac{\tilde{p}_2}{\tilde{p}_1}\beta_{22}\alpha(1-\theta_1)W, \quad c_{22}^* = (1-\alpha)\beta_{22}(1-\theta_1)W \tag{24}$$

which follows from (10) after substituting the world price (23). This is illustrated as bundle A in Figure 4a.

Country 1 production

$$y_{11}^* = \beta_{11}(\alpha\theta_1 W + \frac{\beta_{22}}{\beta_{12}}\alpha(1-\theta_1)W) \tag{25}$$

$$y_{12}^* = \beta_{12}((1-\alpha)\theta_1 W - \alpha\frac{\beta_{22}}{\beta_{12}}(1-\theta_1)W) \tag{26}$$

follows from the market clearing conditions. Furthermore, these imply that

$$w_{11}^* = \alpha\theta_1 W + \frac{\beta_{22}}{\beta_{12}}\alpha(1-\theta_1)W \tag{27}$$

$$w_{12}^* = (1-\alpha)\theta_1 W - \alpha\frac{\beta_{22}}{\beta_{12}}(1-\theta_1)W \tag{28}$$

define Country 1 water allocation. It can be verified that the total amount of water used by the two sectors equals the total amount allocated to country 1, i.e. $w_{11}^* + w_{12}^* = \theta_1 W$.

Country 1 utility in this case is just equal to the autarky level U_1^A. Given that the world price is β_{12}/β_{11}, we can find country 2 utility by substituting the world price into the optimal consumption level (24). Thus

$$(U_2^{FT})_2 = (\frac{\beta_{11}}{\beta_{12}}\alpha)^\alpha(1-\alpha)^{1-\alpha}\beta_{22}(1-\theta_1)W \tag{29}$$

gives free trade utility for country 2 under case 2. We can verify that $(U_2^{FT})_2 > U_2^A$ based on the comparative advantage assumption $\beta_{12}/\beta_{11} < \beta_{22}/\beta_{21}$. Therefore, when two countries with large enough disparities in size (water allocation) are involved in free trade, the small country (country 2 in this case) gains from free trade while the large country still gets its autarky utility.

4.3 Small country 1 water allocation

The case of a relatively small θ_1

$$\theta_1 \leq \frac{\alpha\beta_{21}}{\alpha\beta_{21} + (1-\alpha)\beta_{11}} \tag{30}$$

is symmetric to case 2. Country 2 now becomes the large country, produces both goods and the consumption level and utility is equal to the autarky level. The world equilibrium price

$$\tilde{p}_1/\tilde{p}_2 = \beta_{22}/\beta_{21} \tag{31}$$

equals the country 2 autarky price.

Country 1 specializes in producing good 1 $y^*_{11} = \beta_{11}\theta_1 W$, and it's consumption is

$$c^*_{11} = \alpha\beta_{11}\theta_1 W \quad c^*_{12} = \frac{p_1}{p_2}\beta_{11}(1-\alpha)\theta_1 W \tag{32}$$

from the utility maximization problem (10) with the world price ratio equal to β_{22}/β_{21}. The expression

$$(U_1^{FT})_3 = \alpha^\alpha(\frac{\beta_2 2}{\beta_2 1}(1-\alpha))^{1-\alpha}\beta_{11}\theta_1 W \tag{33}$$

gives free trade utility for country 1 in this case.

Country 2's consumption equals autarky consumption and its production is

$$y^*_{21} = \beta_{21}(\alpha(1-\theta_1)W - \frac{\beta_{11}}{\beta_{21}}(1-\alpha)\theta_1 W) \tag{34}$$

$$y^*_{22} = \beta_{22}((1-\alpha)(1-\theta_1)W + \frac{\beta_{11}}{\beta_{21}}(1-\alpha)\theta_1 W) \tag{35}$$

from market clearing and the production and consumption levels of country 1. As before, a consistency check indicates that the country 2 water resource constraint is satisfied by these relations.

5 Welfare Analysis

This section synthesizes the above three cases which are conditional on the water allocation parameter θ_1 to analyze the qualitative properties of the welfare functions. As noted above, these welfare functions give country utility as a function of the water allocation parameter θ_1. The specific questions of interest include monotonicity of the welfare functions in water allocation, a comparison of the welfare gains from additional water allocation to a given country under autarky and free trade, and conditions under which it might be in the self-interest of countries to share water.

5.1 Welfare functions

Country 1 welfare is

$$U_1^A = (\alpha\beta_{11})^\alpha((1-\alpha)\beta_{12})^{1-\alpha}\theta_1 W$$

under autarky, and

$$U_1^{FT} = \begin{cases} (U_1^{FT})_3 = \alpha^\alpha(\frac{\beta_22}{\beta_21}(1-\alpha))^{1-\alpha}\beta_{11}\theta_1 W & \text{if } 0 \leq \theta_1 \leq \frac{\alpha\beta_{21}}{\alpha\beta_{21}+(1-\alpha)\beta_{11}} \\ (U_1^{FT})_1 = (\beta_{11}\theta_1)^\alpha(\beta_{22}(1-\theta_1))^{1-\alpha}\alpha W & \text{if } \frac{\alpha\beta_{21}}{\alpha\beta_{21}+(1-\alpha)\beta_{11}} < \theta_1 < \frac{\alpha\beta_{22}}{\alpha\beta_{22}+(1-\alpha)\beta_{12}} \\ (U_1^{FT})_2 = (\alpha\beta_{11})^\alpha((1-\alpha)\beta_{12})^{1-\alpha}\theta_1 W & \text{if } 1 \geq \theta_1 \geq \frac{\alpha\beta_{22}}{\alpha\beta_{22}+(1-\alpha)\beta_{12}} \end{cases}$$

in free trade equilibrium. Likewise, country 2 welfare is

$$U_2^A = (\alpha\beta_{21})^\alpha((1-\alpha)\beta_{22})^{1-\alpha}(1-\theta_1)W$$

under autarky, and

$$U_2^{FT} = \begin{cases} (U_2^{FT})_3 = (\alpha\beta_{21})^\alpha((1-\alpha)\beta_{22})^{1-\alpha}(1-\theta_1)W & \text{if } 0 \leq \theta_1 \leq \frac{\alpha\beta_{21}}{\alpha\beta_{21}+(1-\alpha)\beta_{11}} \\ (U_2^{FT})_1 = (\beta_{11}\theta_1)^\alpha(\beta_{22}(1-\theta_1))^{1-\alpha}(1-\alpha)W & \text{if } \frac{\alpha\beta_{21}}{\alpha\beta_{21}+(1-\alpha)\beta_{11}} < \theta_1 < \frac{\alpha\beta_{22}}{\alpha\beta_{22}+(1-\alpha)\beta_{12}} \\ (U_2^{FT})_2 = (\frac{\beta_{11}}{\beta_{12}}\alpha)^\alpha(1-\alpha)^{1-\alpha}\beta_{22}(1-\theta_1)W & \text{if } 1 \geq \theta_1 \geq \frac{\alpha\beta_{22}}{\alpha\beta_{22}+(1-\alpha)\beta_{12}} \end{cases}$$

in free trade.

5.2 Qualitative properties

We now analyze the qualitative properties of the welfare functions, in particular monotonicity. As delineated below, there are three cases to consider dependent on the production parameters. Note that in all these cases, country 1 is assumed to have a comparative advantage in good 1 (Equation 3). We also define the bounds

$$m_1 = \frac{\alpha\beta_{21}}{\alpha\beta_{21} + (1-\alpha)\beta_{11}} \qquad m_2 = \frac{\alpha\beta_{22}}{\alpha\beta_{22} + (1-\alpha)\beta_{12}}. \tag{36}$$

for convenience in partitioning the water allocation space. Equation (3) implies that $m_1 < m_2$.

Case P1. $\beta_{11} > \beta_{21}$ and $\beta_{12} > \beta_{22}$.

In this case, country 1 not only has a comparative advantage in good 1, but also an absolute advantage in both goods. It can be shown that when $\beta_{11} > \beta_{21}$, $m_1 < \alpha$, and when $\beta_{12} > \beta_{22}$, $m_2 < \alpha$. Hence, $m_1 < m_2 < \alpha$.

The welfare functions in this case are shown in Figure 6. The welfare function for country 1 is monotonically increasing as the water allocated to it increases. As country 2 gets increased water allocation, welfare first increases, then decreases for $\theta_1 \in (m_1, m_2)$, and then increases again.

Intuitively, since country 1 has an absolute advantage in both goods, it does not have an incentive to share water with the other country. The loss from sharing water cannot be offset by the gains from trade, hence the more water the better. However, for country 2, when it's water allocations reaches the level of $(1 - m_2)$, then it would be better off not getting additional water. If initially, $\theta_1 \in (m_1, m_2)$, country 2 would even be better off by giving up some water. This can happen because the more water country 2 gets, the more goods it has to produce by itself. But country 2 has lower productivity coefficients, thus they could give up water, let country 1 produce with lower costs, and then gain via trade.

Case P2. $\beta_{11} < \beta_{21}$ and $\beta_{12} < \beta_{22}$.

In this case, country 2 has absolute advantage in both goods. Given that $\beta_{11} < \beta_{21}$ and

$\beta_{12} < \beta_{22}$, $\alpha < m_1 < m_2$. This case is symmetric to P1.

Figure 7 shows that country 2's welfare function will be monotonically increasing with water allocated to it, θ_2, while for country 1, welfare starts to decrease when $\theta_1 > m_1$, and increases again when θ_1 exceeds m_2. There's a region where its welfare will be decreasing with the water allocation parameter θ_1. This result once again demonstrates that increased water allocations may not be welfare-enhancing in the presence of low productivity. This occurs because the gain from more water cannot offset the loss in trade.

Case P3. $\beta_{11} > \beta_{21}$ and $\beta_{12} < \beta_{22}$.

In this case, neither country has absolute advantage in both goods. Country 1 has comparative advantage in good 1 and country 2 has comparative advantage in good 2. As a result, $m_1 < \alpha < m_2$.

In Figure 8, when $\theta_1 \in (m_1, m_2)$, both countries' welfare functions are concave with a local maximum at $\theta_1 = \alpha$. It would be in both countries' mutual interest to set the water allocation parameter θ_1 equal to α if the welfare at the two boundary points ($\theta_1 = 0$ or $\theta_1 = 1$) does not exceed the welfare at $\theta_1 = \alpha$. Even if the extreme welfare is higher, there would be many obstacles to reach that point.

5.3 Water valuation under autarky and free trade

With intermediate water allocation, even if a country's welfare function under free trade is not declining, it is less steeply sloped under free trade than autarky, as illustrated by Figures 6-7. This means that in the presence of trade, the marginal valuation of water can be lower with trade than without. This has two implications. First, even if it is in the country's self-interest to obtain more water, the gains are less than they would otherwise be. Thus, even if trade does not eliminate conflict over water, it can serve to reduce the level of conflict. Second, these results show that partial equilibrium studies could mis-estimate welfare gains if there are strong general equilibrium effects such as trade impacts.

5.4 Conflict and cooperation

To summarize, when one of the countries has absolute advantage in both goods, then it can gain more from trade. As it gets a substantial portion of water, the loss from trade is larger than the gain from more water. Because when the other country only gets a small portion of water, it won't be able to meet the demand of the large country (in terms of water) in trade. As a result, its welfare function starts to turn down when θ is substantially large. This can be seen from Case P1 and Case P2, where country 2's welfare function turns down when it has absolute disadvantage and country 1's welfare function turns down when it has absolute disadvantage.

The other country which has absolute advantage has monotonically increasing welfare function. However, the middle part of the graph has flatter slope than the other two parts, illustrating that the gain from more water is somewhat, though not completely offset by the loss from trade when water allocated to that country is too much.

In the last case, when each country only has comparative advantage in one good, the gains from trade are more obvious. Both country's welfare function will turn down as it gets too much water. Hence, countries would agree to share the water at $\theta_1 = \alpha$. Both countries' welfare function will reach a local maximum. Also, notice that the welfare when one country gets all the water may be greater than the sharing strategy. However, for the upstream country to block the other country's access to the river water is not quite realistic. First, it requires financial and technological support to build dams to divert and save water from the river. Second, it requires the upstream country to be in a superior position in negotiation with the downstream country. However, in reality, countries often have many complicated interactions with each other. The advantageous position in water issue does not guarantee advantageous position in other issues. Diverting all the water would put the two countries in hostile positions.

6 Bargaining

The welfare functions from the trade model are used in this and subsequent sections to analyze political strategies and equilibrium. Here, we first consider bargaining starting from a given initial distribution of water rights denoted as θ_1^0. This distribution might be determined by rainfall and runoff occurring in each country, or it might be based on historical usage. The precise circumstances leading to this initial distribution aren't relevant here, we simply take this distribution as given. In this analysis, coercion is not possible; countries only agree to move from their initial distribution out of self-interest. Later sections will consider a game theoretic model with joint water and trade policies.

In the specific instance of case P1 illustrated in Figure 6, country 1 utility is increasing in it's water allocation, therefore it will never voluntarily give up water. For country 2 in this specific instance, if $\theta_1^0 < m_2$, then it will voluntarily give up water, ending at the equilibrium point $\theta_1^* = m_2$. Thus the bargaining equilibrium in the Figure 6 case will always lie in the interval $\theta_1^* \in \{m_2, 1\}$.

In the figure 7 specific instance of Case P2, country 2 will never agree to an expansion of θ_1 beyond the initial allocation θ_1^0. However, if $\theta_1^0 \in \{m_1, 1\}$, then bargaining will result in an equilibrium allocation of $\theta_1^* = m_1$. A similar analysis can be conducted for the Figure 8 specific instance of Case P3. If the initial θ_1^0 is sufficiently large, then this will be the resulting equilibrium; otherwise the equilibrium allocation will be $\theta_1^* = \alpha$.

There are several general conclusions from this analysis. First, there is no necessary unique bargaining solution for a given model parameterization. The self-interest outcome from bargaining can be dependent on the initial allocation. Second, bargaining can result in self-interested, mutually beneficial reallocation due to the presence of trade. While this can occur in each of the three Cases P1-P3, it is most pronounced for the specific instance illustrated in Case P3 in which no country has an absolute advantage in production.

Finally, we note that some of the outcomes noted above may be specific to the particular parameterization used. While Figures 6-8 accurately convey monotonicity properties of the welfare functions for the respective Cases P1-P3, they may not be completely general with respect to the height of the endpoints relative to interior points. This can potentially affect the equilibrium outcomes in some instances.

7 The Water and Trade Game with Discrete Strategies

The international cooperation literature notes the possibility of issue linkage with trade as a prominent example. For example, Kolstad (2010) discusses various forms of issue linkage with respect to transboundary pollution, while Bennett et al. (1998) and Pham-Do et al. (2012) consider issue linkage in the context of international river basins. Accordingly, we now consider political equilibrium formulated as a water and trade game. Suppose country 1 is the upstream country and country 2 is a downstream country. Country 1 controls the water allocation by deciding how much to allocate to itself and leaves the rest to the other country, while Country 2 decides on trade policy.

Following the literature, we first formulate this as a two strategy discrete game (Bennett et al., 1998). As illustrated in table 1, Country 1 can choose between two levels of water $\theta_1 = \theta_{high}$ and $\theta_1 = \theta_{low}$. This discrete strategy can be conceptualized as a water diverting program. If the program is launched, then the water diverted by country 1 increases from θ_{low} to θ_{high}. Country 2 decides between free trade or autarky.

From the welfare functions, Country 2 free trade utility is always greater than or equal to autarky utility (See Figure 9), hence trade is a weakly dominant strategy for country 2 regardless of country 1 water allocation. Since country 2 always pursues free trade, country 1's water allocation decision depends on it's free trade welfare function (different cases of its welfare functions are shown in Figures 10-14).

Table 2 considers production coefficients and water allocation discrete strategies such that Country 1 welfare increases as θ_1 increases from low to high. In this instance, country 1 will choose the large allocation, and the Nash equilibrium is $\{\theta_{high}, \text{Trade}\}$. This solution will hold for Case P1 under all water allocation strategy spaces. It can also hold for Cases P2 and P3 conditional on the selected allocation strategies.

Table 3 considers the case where Country 1 welfare is decreasing as θ_1 increases from low to high. In this instance, country 1 selects the low water allocation to share water with the downstream country, and the Nash equilibrium is $\{\theta_{low}, \text{Trade}\}$. This can occur when country 2 has an absolute advantage in both goods, or when country 1 only has comparative advantage in good 1 and it's welfare function has a decreasing segment. In both instances, however, the qualitative game in Table 3 may be conditional on the selected strategy set.

The Table 3 equilibrium in which countries share water and engage in free trade is facilitated by the fact that the upstream country 1 with water property rights is disadvantaged in production (either absolute disadvantage in both goods or comparative advantage in just one good). Clearly cooperation is easier to achieve when each country has leverage in some dimension.

The literature emphasizes issue linkage as a way to solve international cooperation problems (Bennett et al., 1998; Pham-Do et al., 2012), with trade policy as a specific example. The analysis in this section offers a somewhat different perspective, in that introducing trade does not give the second country any explicit leverage over the actions of the first country holding the water rights. Trade does influence the welfare function of the water-rights holding country, and in some circumstances it will be of self-interest for that country to jointly allocate water as noted previously. However, a threat by the second country to impose autarky is not credible since it is never better off from doing this.

Thus, while trade can influence the political outcome, it is not through the channel of political bargaining power as in the issue linkage literature. Rather it is through the evaluation process of individual country welfare. The outcome is determined solely by the self-interest of the water-rights country, the other country does not have any credible bargaining power, at least within the context of the game-theoretic model here under standard rationality assumptions. Of course, richer game-theoretic models with asymmetric information or perhaps repeated play might yield a different outcome.

A methodological conclusion from these results is that the discrete strategy game is a limited analytical engine for this problem. The difficulty is that for a given trade model parameterization, the choice of discrete strategies is arbitrary but can influence the qualitative properties of the Nash equilibrium, i.e. whether or not joint water allocation is self-interest. This is evident from the bargaining analysis of the previous section. Similar conclusions hold for trade policy as the game in this section only considers the extremes of autarky and free trade.

8 Conclusions

The paper models water allocation for two countries which share a river and also engage in trade. Trade is a two-country/two-good Ricardian model, with water as the only factor of production and country variation in productivity as the conceptual motivation for trade. The analysis considers behavioral regimes of autarky, free trade, and trade with import quotas. In each instance, equilibrium prices are derived for a given water allocation, and these in turn are used to derive country welfare as a function of water allocation. Game-theoretic models for political equilibrium are then formulated and analyzed utilizing the welfare functions from the economic model. Game-theoretic analysis of international water allocation has been studied in the previous literature. However, to our knowledge, the economic analysis of the welfare functions under trade and the subsequent game-theory models derived from those functions is novel.

Country welfare depends on the water allocation and the subsequent welfare functions exhibit some regularity. (1) First, consistent with standard trade theory, countries gain from free trade in the sense that the free trade welfare is larger or equal to autarky welfare. However, import quotas can be welfare-enhancing from the perspective of an individual country. (2) As long as a country does not have an absolute advantage in both goods, the benefit of getting more water can be offset by a trade loss as it gets more water, which is then reflected as a decrease in the welfare function. (3) Even if a country has an absolute advantage in both goods, the benefit of getting more water will still be somewhat, though not completely, offset by the loss from trade, reflected by a flatter growth in the welfare function when $\theta_1 \in (m_1, m_2)$.

Thus, when riparian countries are engaged in free trade, and for certain parameter specifications, there are circumstances in which country welfare can actually be decreasing in water allocation. Hence, it would be in the countries' self-interest to share water. Furthermore, even if the welfare function is increasing in water allocation, trade means that the gains from additional water can be smaller than that under autarky. This observation then serves to reduce conflict over the resource, although not necessarily eliminating it.

Political equilibrium is analyzed as a game as in previous literature. Under bargaining, we find that there is no unique equilibrium, the outcome is highly dependent on the initial water rights allocation. Also considered is a discrete strategy game of two water allocations for one country, and autarky/free trade for the other country. The primary conclusion here is that the trade policy of the second country may not be credible as a means of getting additional water since free trade is generally advantageous to that country over autarky. In this setting, then, the primary role of trade is not as a bargaining tool, but rather it affects country's evaluation of their welfare function and self-interest in water allocation.

In general then, moving to a general equilibrium setting can potentially be conflict-reducing, although not necessarily conflict-eliminating. This is due to the fact that in general equilibrium, there can be additional channels through which water allocation affects an entity, and some of these may be adverse. Furthermore, in some circumstances policies not directly related to water may be used to leverage additional resource allocation.

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Tables

Table 1: General Payoff Matrix

		Country 2	
		Autarky	Free Trade
Country 1	$\theta_1 = \theta_{low}$	$(U_1^A \theta_{low}, U_2^A(\theta_{low}))$	$(U_1^{FT}(\theta_{low}), U_2^{FT}(\theta_{low}))$
	$\theta_1 = \theta_{high}$	$(U_1^A \theta_{high}, U_2^A(\theta_{high}))$	$(U_1^{FT}(\theta_{high}), U_2^{FT}(\theta_{high}))$

Table 2: Payoff Matrix when Country 1's welfare increases with θ_1
(with $\beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$)

		Country 2	
		Autarky	Free Trade
Country 1	$\theta_1 = \theta_{low} = 0.2$	(0.8556,1.5780)	(1.2969,1.9454)
	$\theta_1 = \theta_{high} = 0.3$	(1.2835,1.3808)	(1.4079,2.1118)

Table 3: Payoff Matrix when Country 1's welfare decreases with θ_1
(with $\beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$)

		Country 2	
		Autarky	Free Trade
Country 1	$\theta_1 = \theta_{low} = 0.6$	(0.5330,1.6610)	(1.2244,1.8366)
	$\theta_1 = \theta_{high} = 0.8$	(0.7106,0.8305)	(0.9063,1.3595)

Figures

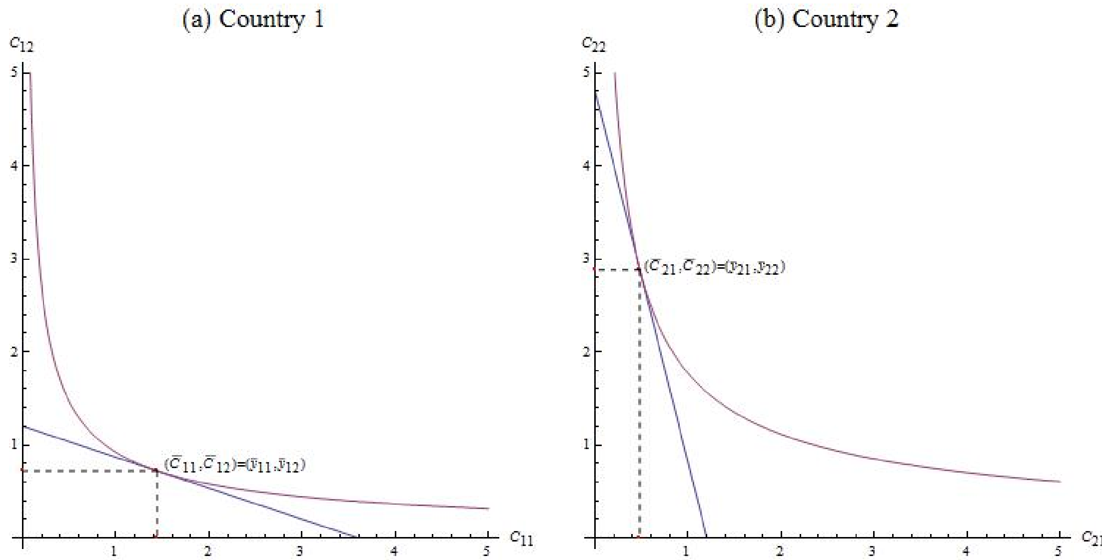

Figure 1: Utility Maximization Under Autarky For Both Countries

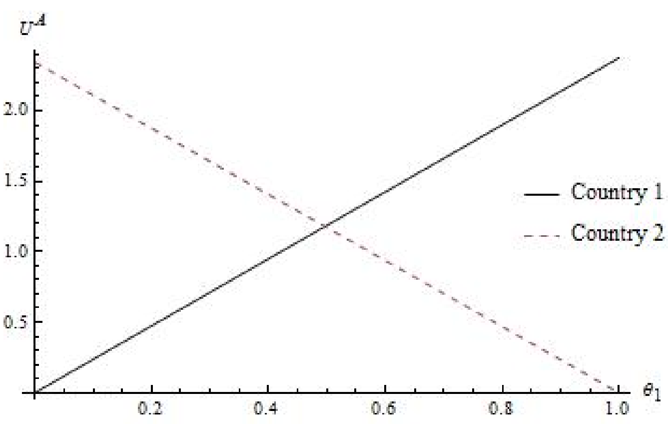

Figure 2: Welfare Functions Under Autarky

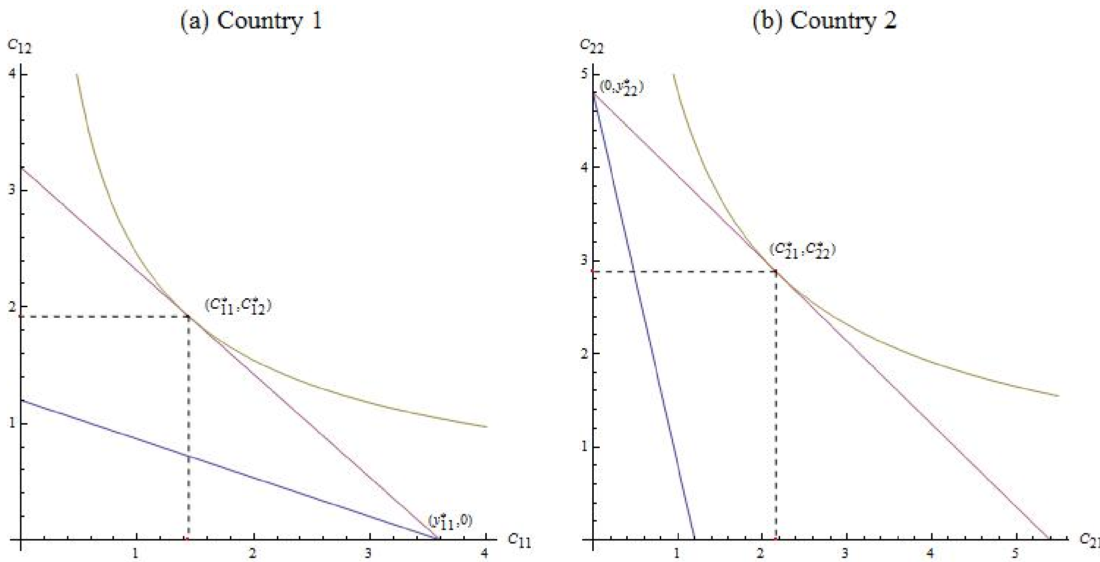

Figure 3: Case 1: Utility Maximization Under Free Trade For Both Countries

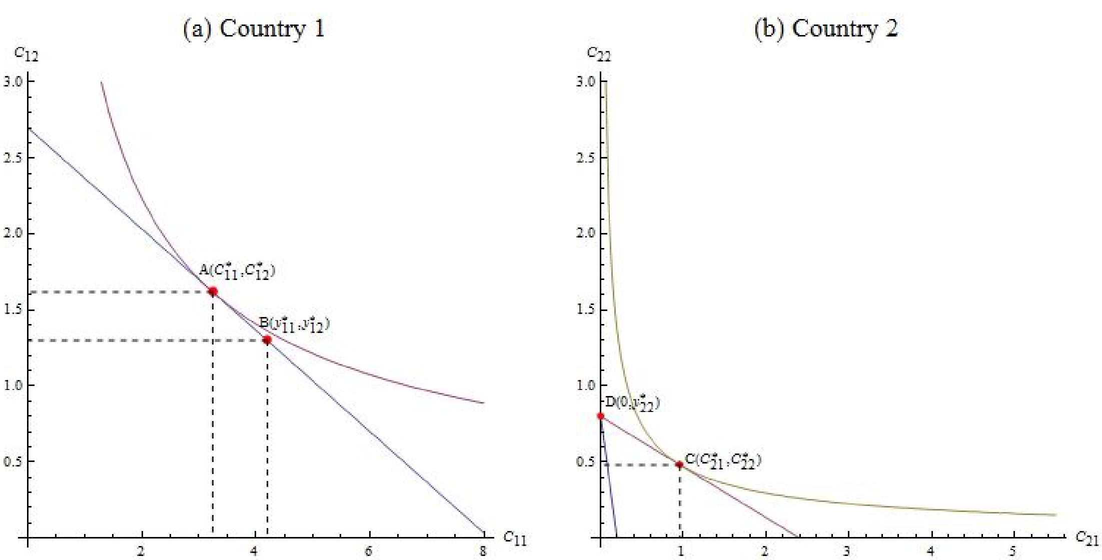

Figure 4: Case 2: Utility Maximization Under Free Trade for Both Countries

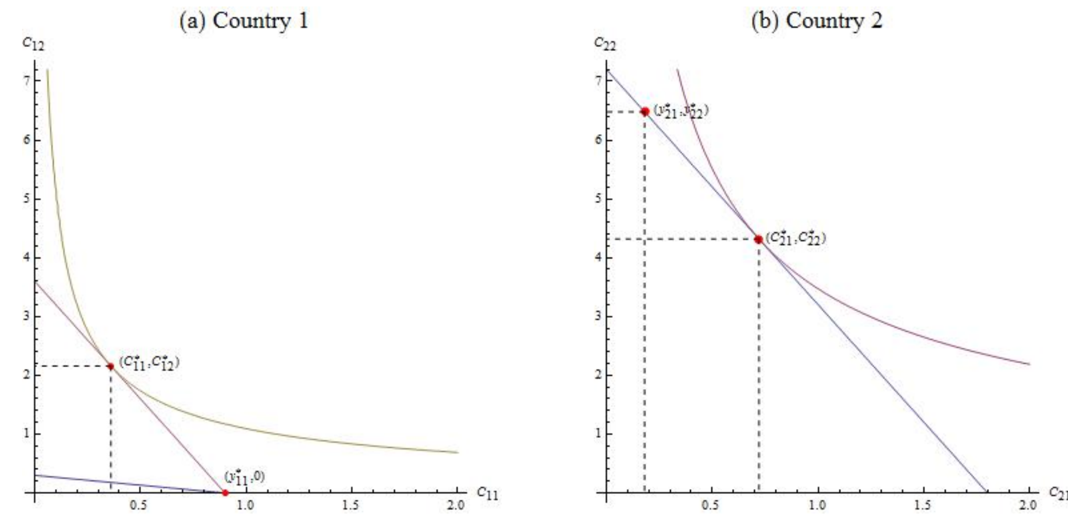

Figure 5: Case 3: Utility Maximization Under Free Trade for Both Countries

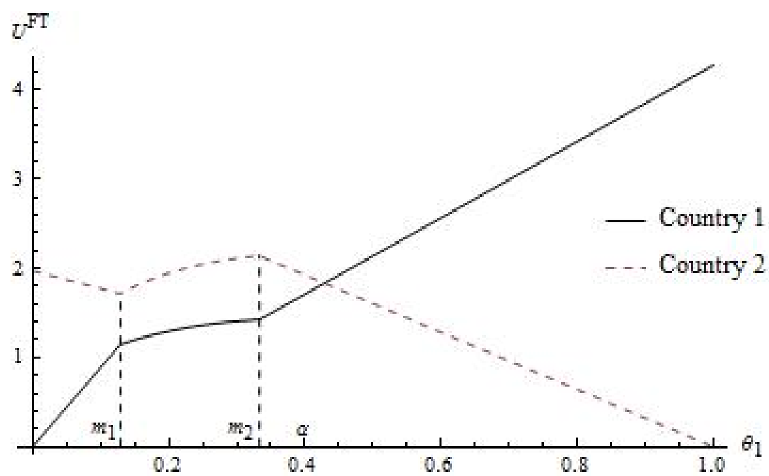

Figure 6: Free Trade Utilities When Country 1 has Absolute Advantage in Both Goods, with $\alpha = 0.4, \beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$.

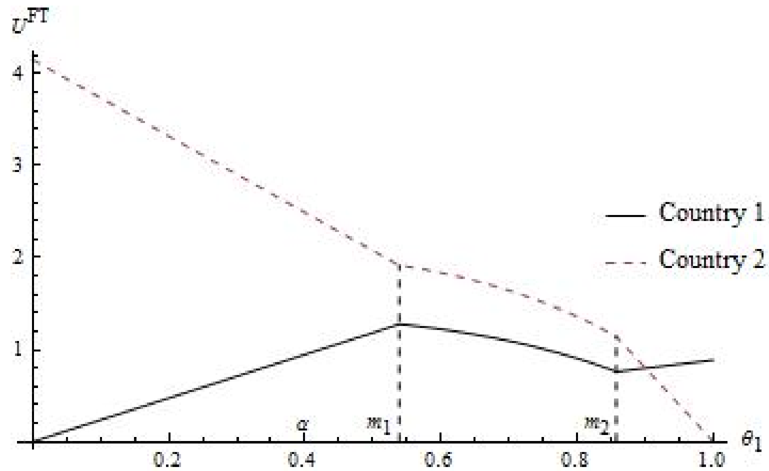

Figure 7: Free Trade Utilities When Country 2 has Absolute Advantage in Both Goods, with $\alpha = 0.4, \beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$.

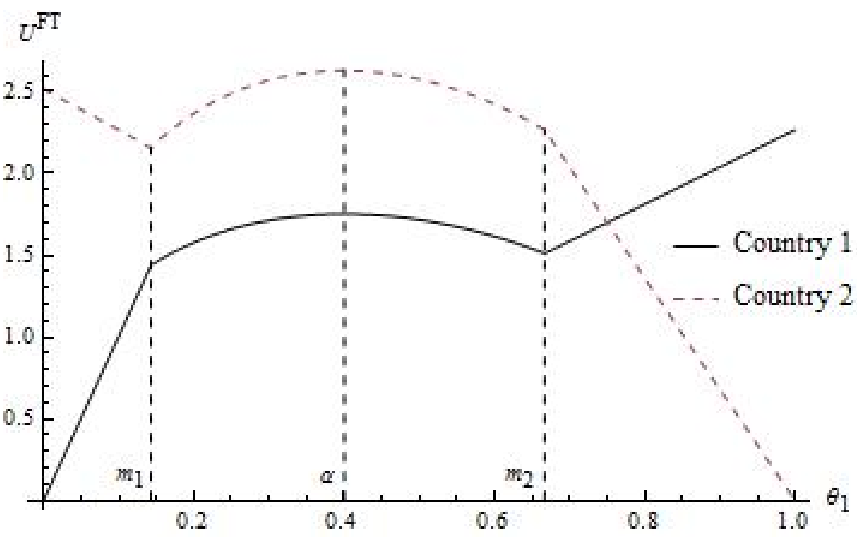

Figure 8: Free Trade Utilities When Each Country Only has Comparative Advantage in One Good, with $\alpha = 0.4, \beta_{11} = 8, \beta_{12} = 3, \beta_{21} = 2, \beta_{22} = 9$.

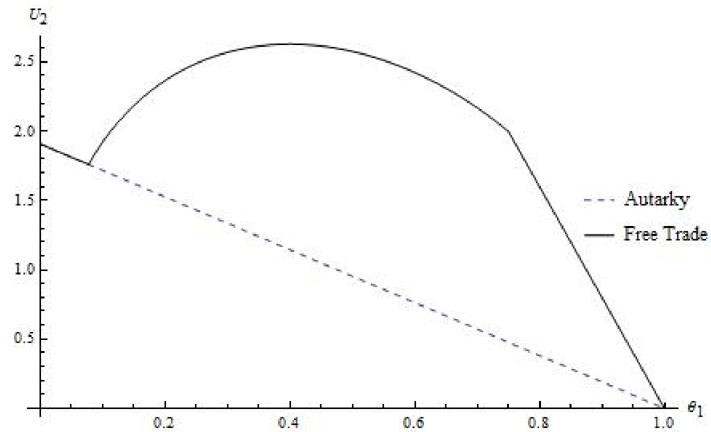

Figure 9: Country 2's Welfare Function Under Autarky and Free Trade

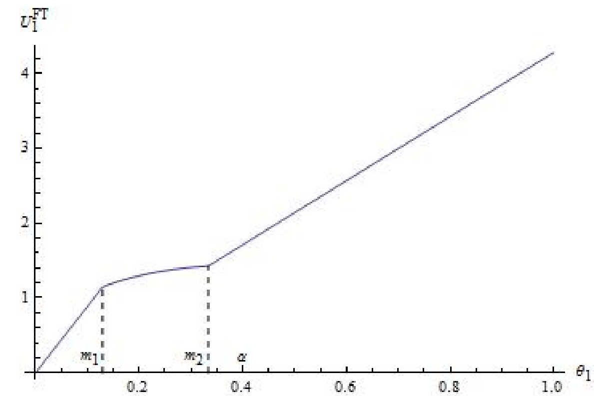

Figure 10: Country 1's Free Trade Utility When Country 1 has Absolute Advantage in Both Goods, with $\beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$

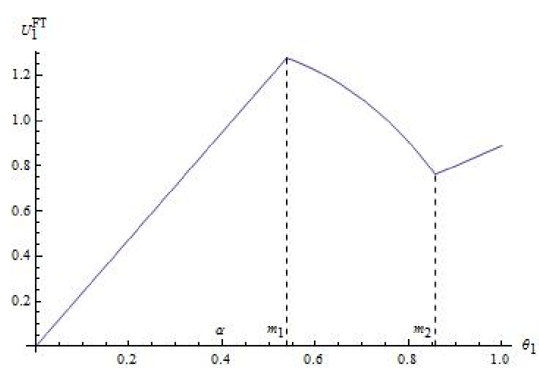

Figure 11: Country 1's Free Trade Utility When Country 2 has Absolute Advantage in Both Goods, Case 1:Utility at the End Smaller than the Middle Peak, with $\alpha = 0.4, \beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$.

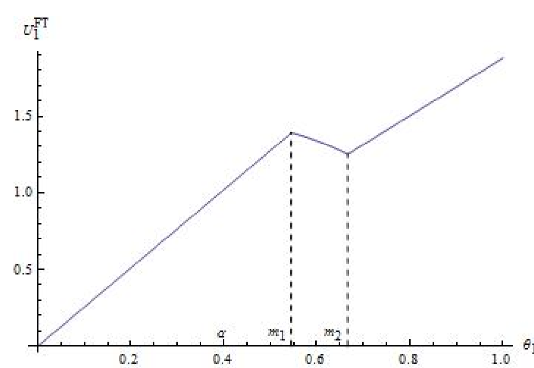

Figure 12: Country 1's Free Trade Utility When Country 2 has Absolute Advantage in Both Goods, Case 2:Utility at the End Larger than the Middle Peak, with $\alpha = 0.4, \beta_{11} = 5, \beta_{12} = 3, \beta_{21} = 9, \beta_{22} = 9$.

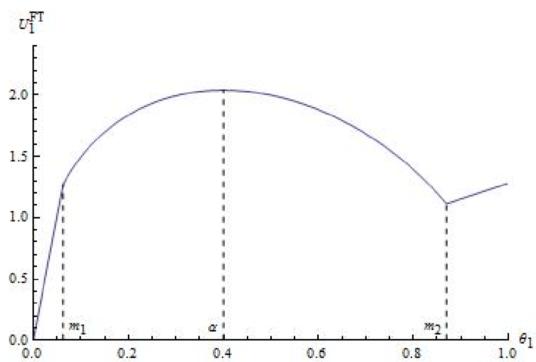

Figure 13: Country 1's Free Trade Utility When Each Country Has Comparative Advantage in One Good, Case 1, $\alpha = 0.4, \beta_{11} = 10, \beta_{12} = 1, \beta_{21} = 1, \beta_{22} = 10$.

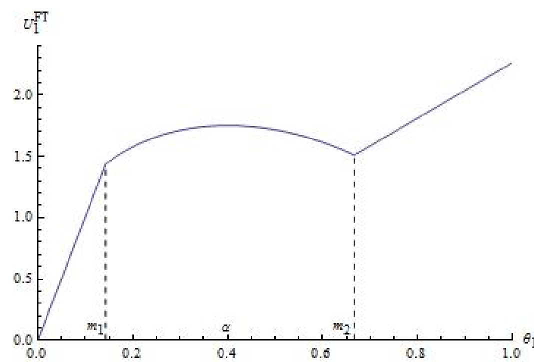

Figure 14: Country 1's Free Trade Utility When Each Country Has Comparative Advantage in One Good, Case2,with $\alpha = 0.4, \beta_{11} = 8, \beta_{12} = 3, \beta_{21} = 2, \beta_{22} = 9$.