Adverse Selection in Team Formation under Discrimination

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Abstract

A model of adverse selection when teams form endogenously highlights the relationship between discrimination and occupational choice. It shows discrimination can persist even when management allocates credit fairly, if worker beliefs about discrimination influence their decision to engage in teamwork. Furthermore, among those choosing to work as entrepreneurs or in other individualistic occupations, discrimination victims outperform beneficiaries.

1 Introduction

A young black man finishes law school. He wonders, "Should I join a large firm or hang out my own shingle?" A newly minted Latina engineer ponders whether she should join a tiny startup or accept the job offer of a giant technology multinational. The decision to be an entrepreneur or an employee is among the most consequential any individual will ever face. Does race or gender influence that choice? Could discrimination affect occupational performance?

Several empirical studies on occupational segregation suggest (1) minorities are more likely to choose (or be chosen for) occupations in which teamwork plays a minimal role and (2) that minorities excel in these individualistic positions. For example, Clark and Drinkwater (2000) document minority over-representation in the British self-employment sector, even though most new firms fail within four years (Shane, 2008). Lempert, Chambers and Adams (2000) report that among Michigan Law School graduates choosing private sector employment, minorities are far more likely to open their own practice or join very small practices. Strikingly, they also find that minority graduates enjoy higher income than their white peers, despite the fact that salaries of small firm attorneys are significantly lower than those of large firms (NALP 2009).[1]

Popular media has recently focused attention on heightism, i.e. discrimination over physical stature.[2] Judge and Cable (2004) calculate that a worker, on average, earns $789 (1991 USD) more annually for each inch of physical height. Djankov, *et al.* (2005) conducted a survey of Russian entrepreneurs. Among individual characteristics, two of the top three strongest predictors of entrepreneurship were found to be cognitive ability (positively correlated) and physical height (negatively correlated). Both proved more robust than such stereotypical characteristics such as risk-taking. Although Djankov, *et al.* do not explain why smart, short individuals choose entrepreneurship more often, the model presented here can.

Even in team sports, minorities disproportionately occupy positions where contribution is individualistic and more measurable. Loy and Elvogue (1970) categorize the various positions in baseball and football as either central or peripheral based on their interaction level with other players. They observe that minorities occupy the majority of peripheral, or so-called "[measurable] skill," positions while whites dominate central ones. Similarly, in hockey, Lavoie, Grenier and Coulombe (1987) observe that Francophones are overrepresented at goalie and underrepresented on defense. By objective measures, like points scored, minority players outperform non-minorities in all of these sports (see Kahn 1991 for a review).

How are teamwork and discrimination linked? Teamwork facilitates discrimination because it obscures team members' individual contributions: managers can try to infer unobservable individual contribution from observable characteristics like race or sex. A minority worker should choose entrepreneurship, or other occupation where his individual accom-

[1] Though Lempert *et a*l. (2000) find a positive correlation between non-white ethnicities or minority status and log income, it is not always significant.

[2] Discrimination in the model presented here can be over any attribute. I generally use minority as a synonym for victim of discrimination and non-minority for beneficiary of discrimination.

plishments cannot easily be attributed to others, *if doing so will make him better off*. And indeed, Clark and Drinkwater (2000) verify that self-employed minorities earn more than their traditionally employed counterparts. But clearly, individuals who are most talented relative to their employers' perceptions have the greatest incentive to eschew teamwork—if high ability minorities believe they will face discrimination on teams, they will avoid them. Could this self-selection sustain discrimination even if managers paid workers proportional to their expected ability; that is, according to their merit?

This paper shows that it can. In the model, discrimination arises because workers endogenously choose either to work as part of a team, where individual merit is hard to measure, or as an individual, where it is easy to measure. Workers differ in unobservable ability as well as *ex ante* uncorrelated, observable characteristics, like race. If high ability minority workers mostly elect to work outside of teams, then meritocratic managers will rationally discriminate when rewarding team members. This effect reinforces itself, since high ability minorities will be better off working individually than facing discrimination on a team. How does discrimination impact those who choose individualistic work?

Race becomes an *ex post* indicator of ability not just for team workers but for those who work alone—among individualistic workers, minorities will outperform non-minorities on average. Thus, if the fraction of team credit allocated to minority team members quantifies the level of discrimination, then this measure always overestimates the relative disadvantage minorities face overall. In fact, under certain circumstances, minorities receive higher average compensation than non-minorities when both team and solo workers are considered.

Discrimination may induce teamwork that would not occur under egalitarianism. For example, if credit is to be split evenly, a highly skilled non-minority worker (with no taste for discrimination) may be unwilling to collaborate with anyone less competent than himself, but under discrimination he may be willing to team up with a minority worker who happens to be less competent than he is, since he will get most of the credit anyway. If synergy between these teammates is high enough, discrimination is socially preferable in that total societal output is higher than under egalitarianism; in fact, under certain circumstances, both non-minorities and minorities alike (as groups) earn more under discrimination.

The model presented here distinguishes itself in a rich literature on statistical discrimination, both by explaining empirically observed behavior that has not yet been addressed and by enabling the analysis of additional forms of discrimination. First, the existing class of models is silent on career choice. Second, existing models require that victims differ from beneficiaries at the time of employment either in (a) discourse or (b) (endogenously) acquired human capital. Thus, extant theory implies powerful, pervasive mechanisms that transform individuals from birth. But when teams form endogenously, selection can lead to statistical discrimination, which leaves all victim and beneficiary attributes, except the one over which discrimination occurs, *statistically identical across the population at all times.* In addition to explaining traditional forms of discrimination such as racism and sexism, the model here also covers forms of discrimination that alter individuals less forcefully or may exist in only certain occupations.

The next section highlights related literature. The third presents a basic model of endogenous team formation, production and credit allocation. The fourth section illustrates the basic intuitions when workers are only of two abilities. The fifth tests the robustness of these intuitions in more general settings, and the sixth concludes. Technical proofs are contained in the appendix.

2 Related Literature

The economics literature stresses that workplace discrimination either stems from employer tastes (Becker 1957) or from conditioning expected worker contribution on observable characteristics, like race, to (partially) resolve imperfect observability of employee productivity. Phelps (1972) and Arrow (1973) pioneered work on this latter type, known as statistical discrimination.[3]

Arrow and later Coate and Loury (1993) showed that because minorities respond strategically to lower incentives to invest in unobservable skill sets prior to employment, employers' discriminatory beliefs can be confirmed ex post, even when minorities and non-minorities

[3] The body of literature after Phelps' tradition relies on employers being able to extract a less noisy signal of ability from employees they most closely resemble (e.g. in race or sex). It does not inform the model presented here.

have identical ability ex ante. The discriminatory equilibrium resembles the one presented here, but "ex ante" in their models means "at birth" Since minorities and non-minorities acquire different human capital, they no longer possess identical ability when they take jobs.

In this paper discrimination is classically statistical, arising from imperfect observability of worker productivity. Team synergy both incents workers to collaborate and obscures their individual contributions. Although minorities who select into teams differ from those who do not, the overall populations of minorities and non-minorities always have statistically identical ability. The signal in this sorting model is slightly more complex than in others. In Arrow's and Coate and Loury's models, race alone (eventually) correlates to ability, but in this model, it is the interaction of race and *observable* self-selection, which informs the manager.

The model relates in a secondary way to the literature on team productivity. Since Alchian and Demsetz' (1972) discussion of the free-rider problem inherent in teams, the team literature has focused on moral hazard in effort.[4] Hamilton, Nickerson and Owan (2003) empirically suggest that this focus on free-riding in teams is too narrow, because even when individual incentives are feasible, many firms implement group incentives to increase productivity.[5] Accounting for such synergies, my model examines the selection in team formation that occurs before any moral hazard can—imperfect observability of individual contributions in teams drives both. Because teamwork is always synergistic in the model this selection into individualistic work is adverse for society and firm alike—incomplete information causes both to suffer a productive loss.

This adverse selection creates a link between discrimination and occupational choice, especially with respect to the entrepreneurial decision, that the literature has not discussed. But adverse selection in team formation also happens in traditional firms, even though most

[4] See Holmström 1982, McAfee and McMillan 1991, Legros and Matthews 1993 among many others.

[5] First, Hamilton et al (2003) capture an 18% productivity increase in a garment factory after a change from individual piece rate compensation to group piece rate pay. Second, they attribute about a fifth of that increase to the fact that high ability workers were more likely to join teams but the remaining 14% is attributable to the synergistic team effect. High ability workers were no more likely to leave the company than low ability ones after joining a team. These findings counter the free-riding theories, which have dominated economic analysis of teams.

employees cannot choose their coworkers. When a manager hires all employees, the teamwork decision and, hence, adverse selection often remain. Presented with a menu of tasks, an employee may choose an individual task with high management visibility over another, in which his contribution may be lost because of the number and rank of other contributors.[6] The common managerial performance critique, "(not) a team player" indicates that even when tasks are completely assigned, how they are performed can be an expression of the teamwork decision. The model applies to any situation in which workers can trade off synergy for visibility.

3 Basic Model

Two workers, A and B, decide to produce individually, say as entrepreneurs, or together, say as employees at a large firm. A and B have positive abilities, α and β respectively, drawn randomly from identical independent distributions. Abilities are known to both A and B but unobservable to management. Worker names (i.e. A or B) are the only available attribute of discrimination.

A and B produce their respective abilities when working individually (i.e. individualistic technology is linear in ability with slope 1). Teams produce $g(\alpha, \beta)$. Assume team production is

$$\text{Symmetric:} \qquad g\left(x,y\right) = g\left(y,x\right)$$

$$\text{Synergistic:} \qquad g\left(\alpha,0\right) \geq \alpha \text{ and } g(\alpha,\beta) > \alpha + \beta \ \forall \alpha, \beta > 0,$$

A manager determines the portion of team credit (or production) due each worker. This credit may be compensation or a manager identifying her next promotee. The fraction of team credit assigned to A is denoted by γ, where γ lies in the unit interval. B receives the remainder. Thus, γ represents the strength of discrimination: $\gamma = 1$ represents a world in which A gets all credit from group work and $\gamma = 0$ one in which he gets none.

[6] Even such characteristics such as job title or seniority can be attributes of discrimination to the extent they are imperfectly correlated to private ability. A gifted junior employee may rather work alone than share a majority of credit with a mediocre senior one.

Without loss of generality, I restrict $\gamma \in \left[\frac{1}{2}, 1\right]$, so that B always denotes the minority. For every discriminatory equilibrium belief, $\gamma > \frac{1}{2}$, favoring A, model symmetry induces another, $1-\gamma < \frac{1}{2}$, favoring B. Analysis, though, is limited to $\gamma \geq \frac{1}{2}$. Importantly, a solo worker always receives all credit for his work. A and B have common beliefs on γ.[7]

Each worker chooses teamwork if and only it would make him (weakly) better off than working individualistically. A team forms if and only both workers choose teamwork (Table 1 describes the game in normal form). Thus, each worker has a clear dominant strategy. Worker A chooses $Team$ if and only if

$$g(\alpha, \beta)\,\gamma > \alpha \tag{C_A}$$

Otherwise he chooses $Solo$. Similarly, worker B chooses $Team$ if and only if

$$g(\alpha, \beta)\,(1-\gamma) > \beta \tag{C_B}$$

and $Solo$ otherwise.

		Worker B: $Team$	Worker B: $Solo$
Worker A	$Team$	$g(\alpha,\beta)\,\gamma$, $g(\alpha,\beta)\,(1-\gamma)$	α, β
	$Solo$	α, β	α, β

Table 1. Normal form of the simultaneous game.

Employment law constrains the manager to allocate credit according to employees' relative contributions as well as she can with available information.[8] Each team member should receive rewards commensurate with his relative contribution to the team, but since teamwork obscures individual contribution, the manager has limited information about team members' individual contributions. However, the manager has a useful piece of information: her employees chose to work for her rather than for themselves—they chose teamwork. Thus, a

[7] Assuming that victims and beneficiaries believe that discrimination exists to a common degree is strong, but the assuming common beliefs is standard in BNE analysis. Likewise, one may assume that the abilities of *previous* employees working on teams have been revealed to the manager to give some basis for beliefs, but this is not strictly necessary for the analyisis.

[8] Statistical discrimination is illegal in the US; however, since the burden of proof that an employer is not paying equally for equal work belongs to the employee, this constraint of the model is practical.

central quantity of interest is the non-minority's average fraction of ability in all teams that form *under a given set of beliefs*, $E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]$.[9] If work was completed as a team, the manager credits A equal to his expected proportion of total team ability, given that the work was done as a team. In a Bayesian Nash Equilibrium (BNE), if the manager is assigning credit according to merit, this fraction must, in expectation, equal the credit split, γ, that A and B believed when they decided to form a team:

$$\gamma = E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right] \tag{1}$$

In other words, a particular set of teams will form in a society that believes discrimination of level γ exists; if γ is also non-minorities' expected relative contribution on those teams then the discrimination is fair ex post and the beliefs are self-reinforcing.

4 Analysis

In deciding whether or not to join the team, a worker compares his fraction of (realized) total team ability to the fraction of team credit he believes he will receive. I refer to the ratio of these two quantities as his *discriminatory alignment*. The discriminatory alignment of A is $\frac{\alpha}{\alpha+\beta}/\gamma$, while B's is $\frac{\beta}{\alpha+\beta}/\left(1-\gamma\right)$. If this discriminatory alignment is less than unity then team production must be sufficiently synergistic for that worker to choose teamwork.[10] For example, if team production were only slightly synergistic (i.e. $g\left(\alpha,\beta\right) = \alpha+\beta+\varepsilon$, where $\varepsilon > 0$ is very small) it is easy to see that (C_A) and (C_B) would only be satisfied by pairs of workers whose discriminatory alignment is approximately equal to 1 (i.e. $\frac{\alpha}{\alpha+\beta} = \gamma$ and $\frac{\beta}{\alpha+\beta} = (1-\gamma)$)—only worker pairs whose abilities mirror their beliefs about discrimination will team up. At the other extreme, suppose that team production were extremely synergistic, say $g\left(\alpha,\beta\right) = +\infty$. Then, regardless of the workers' discriminatory alignments, a team forms—both workers will choose teamwork regardless of each other's ability or the perceived level of discrimination.

[9] Although the denominator does not reflect the total output of the team in the above above definition of meritocracy, it is trivial to show that the alternative formulation, $\gamma = E\left[C\frac{\alpha}{g(\alpha,\beta)} \mid team, \gamma\right]$, where $C = \frac{g(\alpha,\beta)}{\alpha+\beta}$ is the unique normalizer such that $1-\gamma = E\left[C\frac{\beta}{g(\alpha,\beta)} \mid team, \gamma\right]$ is equivalent.

[10] If it is greater than unity for one worker then it is less than unity for the other.

It is easy to visualize the effects of synergy and beliefs about discrimination by graphing (C_A) and (C_B) over the workers' joint ability sample space. Figures 1 and 2 depict the team forming regions of the ability sample space (in $\mathbb{R}^+ \times \mathbb{R}^+$) under two different scenarios.[11] Figure 1 depicts highly synergistic team production under very discriminatory beliefs; the environment depicted in Figure 2 is both less synergistic and less discriminatory ($g_H(\alpha, \beta) > g_L(\alpha, \beta) \, \forall \alpha, \beta > 0$ and $\gamma_H > \gamma_L$). The meshed region of each figure bordered by a dashed line represents all of the realizations of α and β, which would jointly satisfy (C_A). Analogously the shaded region of each figure bordered by a dot-dashed line represents all of the realizations of α and β, which would jointly satisfy (C_B). Teams form only from realizations in the overlap of these regions. The solid line, which runs through the interior of the team forming region, represents the locus of realizations of α and β for which discriminatory alignment equals unity—this locus of pairings will forms teams regardless of the synergy level. Comparing the two figures, one observes that as synergy increases, the team forming region expands to include more and more pairings besides just those with discriminatory alignment equal to 1. Note also that as beliefs about discrimination change, the boundaries of the team forming region created by (C_A) and (C_B) both move in the direction of the change in beliefs—the composition of teams (at least) loosely reflects society's discriminatory beliefs. What is not shown in the figures, but will be a focus of the analysis, is the probability density of abilities over the sample space—for it is the expectation of abilities over the team forming regions and regions of individualistic work that lead to the main results.

To illustrate the basic insights of the model, I analyze a simple two-type case. More general settings are analyzed in Section 5.1. For exposition assume the following team production function: $g(\alpha, \beta) = (\alpha + \beta)\,\kappa$ where κ measures synergy ($\kappa > 1$). Let $\alpha, \beta \in \{H, L\}$, where $H > L > 0$, be independently and identically distributed as follows:

$$\Pr\{\alpha = H\} = \Pr\{\beta = H\} = p \in [0, 1] \tag{2}$$

I first establish the intuitive result that if workers believe that no discrimination exists, then, in this completely symmetric world with fair management, none will. But such beliefs are

[11] Figure 1 was generated with $g_H(\alpha, \beta) = \alpha + \beta + \alpha\beta$ and $\gamma_H = \frac{5}{8}$. Figure 2 was generated with $g_L(\alpha, \beta) = \alpha + \beta + \frac{1}{4}\alpha\beta$ and $\gamma_L = \frac{7}{13}$.

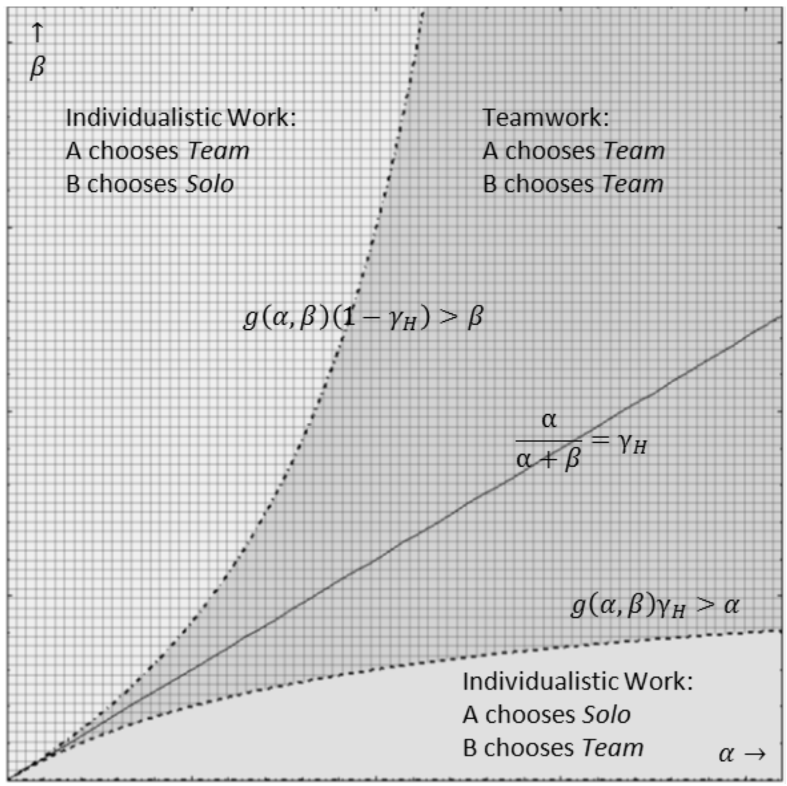

Figure 1: The joint ability sample space. Teams form in the (dark) overlapping region where both A chooses teamwork (meshed region with dashed border) and B chooses teamwork (shaded region with dot-dashed boarder).

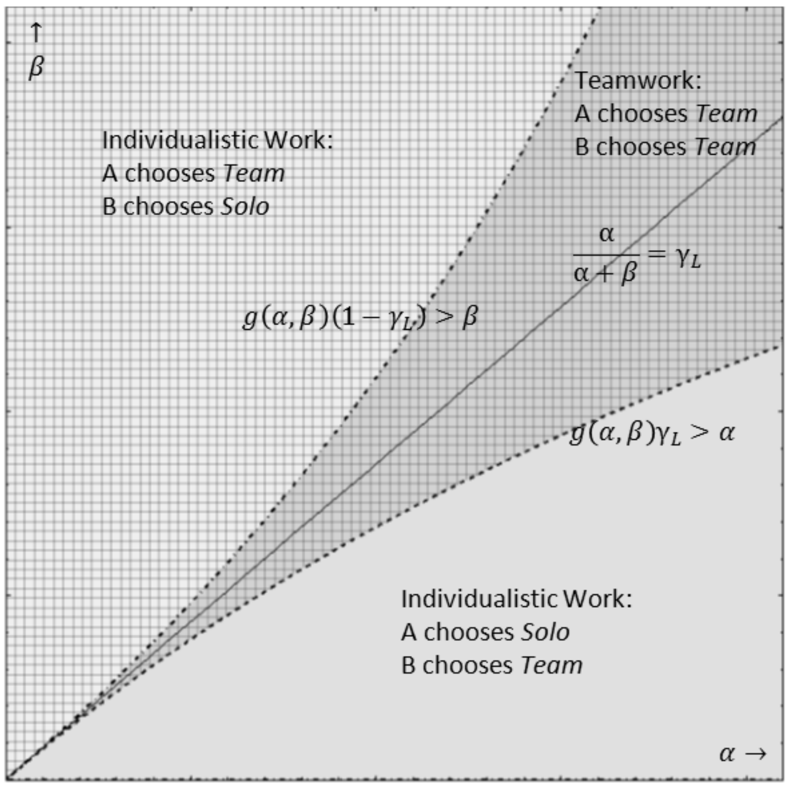

Figure 2: Belief that discrimination is lower rotates the team forming region toward the 45 degree line. Lower synergy shinks the team forming region around the central locus of pairings with discriminatory alignment = 1.

not enough to guarantee that talented individuals will cooperate with those less so—in fact, as we will see, egalitarianism may impede it.

Proposition 1 *(a) Egalitarianism (i.e. $\gamma = \frac{1}{2}$) is always an equilibrium. (b) Under egalitarianism, teams always form if synergy is strong enough (i.e. $\kappa > \kappa_4$, where $\kappa_4 = \frac{2H}{H+L}$), but low synergy (i.e. $\kappa \leq \kappa_4$) induces only homogeneous (i.e. $\alpha = \beta$) teams.*[12]

Proof. When $\kappa > \frac{2H}{H+L}$, (C_A) and (C_B) are satisfied for all α and β. When $\kappa \leq \frac{2H}{H+L}$, (C_A) and (C_B) are satisfied if and only if $\alpha = \beta$. In both cases $\frac{1}{2} = E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma = \frac{1}{2}\right]$ by symmetry. ■

Since workers have iid ability, if they believe management thinks observable characteristics are orthogonal to ability, workers will disregard observables in choosing teammates. Fair managers will then disregard observables too, and egalitarianism will be an equilibrium. Proposition 1 part (a) holds for all distributions and production functions (see the Appendix for a proof).

Even absent any discrimination, teams will not always form, though teamwork is efficient. Thus, egalitarianism does not alleviate adverse selection. Proposition 1 part (b) can be restated for general distributions and productions functions: *Under egalitarianism, a team will form iff the synergy of production is greater than the difference in worker abilities* (see the Appendix for a proof).

As synergy declines in an egalitarian world, only individuals of very similar ability will team up. If synergy results from labor specialization or leadership, this homogeneity may be undesirable. In fact, Hamilton, *et al.* (2003) find that, with average ability held constant, heterogeneous teams are more productive.[13] Figure 3 graphically depicts the team forming

[12] The notation for indexing key synergy levels can be thought of in the following way: κ_4 is the threshold above which all four possible ability realizations form a team under egalitarianism, κ_3 and $\kappa_{\underline{4}}$ represent the thresholds between which three but not four realizations will form teams if beliefs are those specified in Proposition 2. Finally $\kappa_{\underline{2}}$ represents the threshold below which one realization, but not two will form if beliefs are those specified in Proposition 3.

[13] The reader will observe that $g(\alpha, \beta) = (\alpha + \beta)\kappa$ does not increase with hetrogeneity of ability. Although a team production function with negative cross partials with respect to ability may reflect Hamilton et al's empirical findings better, the example function is conservative in that it overstates productivity under

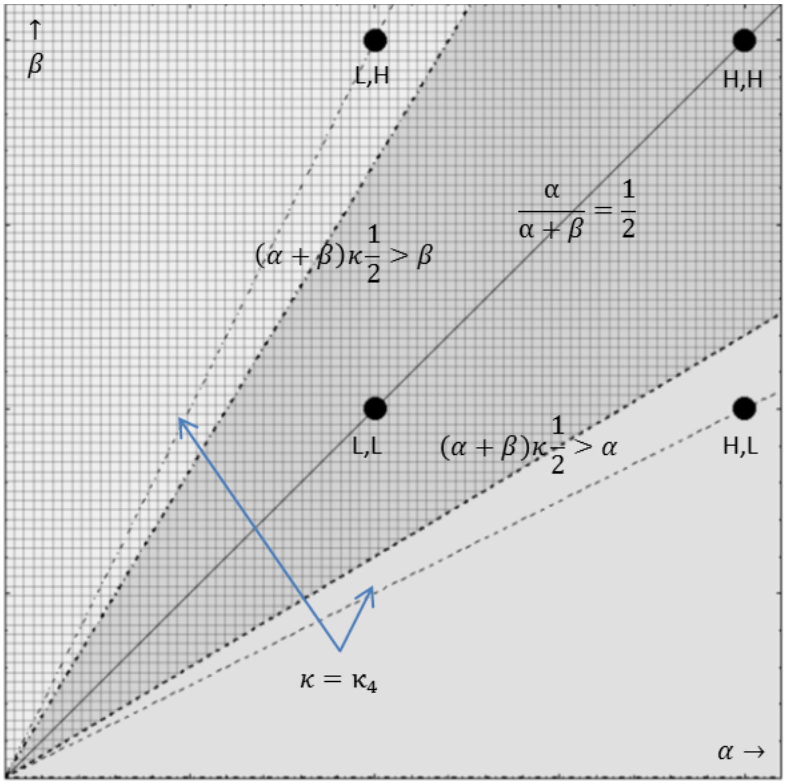

Figure 3: Egalitarianism is always an equilibrium, but when synergy is low ($\kappa \leq \kappa_4$) adverse selection prevents teamwork between heterogenous pairings.

region of the ability sample space when $\kappa < \kappa_4$; observe that heterogenous pairings $\langle H, L\rangle$ and $\langle L, H\rangle$ fall outside of the team forming region; they result in individualistic work.

Each of the following two propositions identifies a discriminatory equilibrium. That is, if workers believe that discrimination (of a particular level) exists, they will form teams such that a fair manager will reinforce those beliefs. The first equilibrium is mild, causing both homogeneous and heterogeneous ability teams to form. The second equilibrium is severely discriminatory, and only heterogeneous teams form. Figures 4 and 5 illustrate the shift in team forming regions of the ability sample space as beliefs change.

Proposition 2 *For intermediate synergy levels there exists a moderately discriminatory*

egalitarianism relative to discrimination. An in depth study of group production is beyond the scope of this paper.

equilibrium in which both homogeneous teams and heterogeneous teams form. Formally,

$$\kappa_3 < \kappa \leq \kappa_{\underline{4}} \implies \gamma_M = \frac{1}{2}p_e + \frac{H}{H+L}(1-p_e)$$

where

$$\kappa_3 = \frac{2H(1-p(1-p))}{H+L(1-2p(1-p))}, \; \kappa_{\underline{4}} = \frac{2H(1-p(1-p))}{H(1-2p(1-p))+L}$$

and

$$p_e = \Pr\{\alpha = \beta \mid team\} = \frac{1-2p(1-p)}{1-p(1-p)}$$

is a discriminatory equilibrium in which realizations $\langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle H, L \rangle, \langle L, L \rangle\}$ form teams.

Proof. Let $\gamma' = \frac{1}{2}p_e + \frac{H}{H+L}(1-p_e) = E\left[\frac{\alpha}{\alpha+\beta} \mid \langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle H, L \rangle, \langle L, L \rangle\}\right]$. Assume $\gamma = \gamma'$. (C_A) and (C_B) are satisfied for all $\langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle H, L \rangle, \langle L, L \rangle\}$ if and only if $\kappa > \kappa_3$. (C_B) is not satisfied for $\langle \alpha, \beta \rangle = \langle L, H \rangle$ if and only if $\kappa \leq \kappa_{\underline{4}}$. Thus, $E\left[\frac{\alpha}{\alpha+\beta} \mid team\right] = \gamma'$. ■

Proposition 3 *If synergy is not extremely strong, then there exists a severely discriminatory equilibrium in which only heterogeneous teams form. Formally,*

$$\kappa \leq \kappa_{\underline{2}} \implies \gamma_s = \frac{H}{H+L}$$

where

$$\kappa_{\underline{2}} = \frac{H+L}{2L}$$

is a discriminatory equilibrium in which realizations $\langle \alpha, \beta \rangle \in \{\langle H, L \rangle\}$form teams.

Proof. Let $\gamma' = \frac{H}{H+L} = E\left[\frac{\alpha}{\alpha+\beta} \mid \langle \alpha, \beta \rangle = \langle H, L \rangle\right]$. Assume $\gamma = \gamma'$. (C_A) and (C_B) are satisfied for $\langle \alpha, \beta \rangle = \langle H, L \rangle$ if and only if $\kappa > 1$. (C_B) is not satisfied for $\langle \alpha, \beta \rangle = \langle L, H \rangle$ if and only if $\kappa \leq \frac{H}{L}$. (C_B) is not satisfied for $\langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle L, L \rangle\}$ if and only if $\kappa \leq \kappa_{\underline{2}} = \frac{H+L}{2L} < \frac{H}{L}$. Thus, $E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma'\right] = \gamma'$. ■

Propositions 1-3 say that if synergy is so strong that everyone chooses teamwork all the time, egalitarianism is the only equilibrium, but as it declines (1) discriminatory and egalitarian equilibria coexist (note that $\kappa_3 < \kappa_4 < \kappa_{\underline{4}}$ and $\kappa_4 < \kappa_{\underline{2}}$) and (2) adverse selection prevents efficient team formation under egalitarianism and discrimination alike.

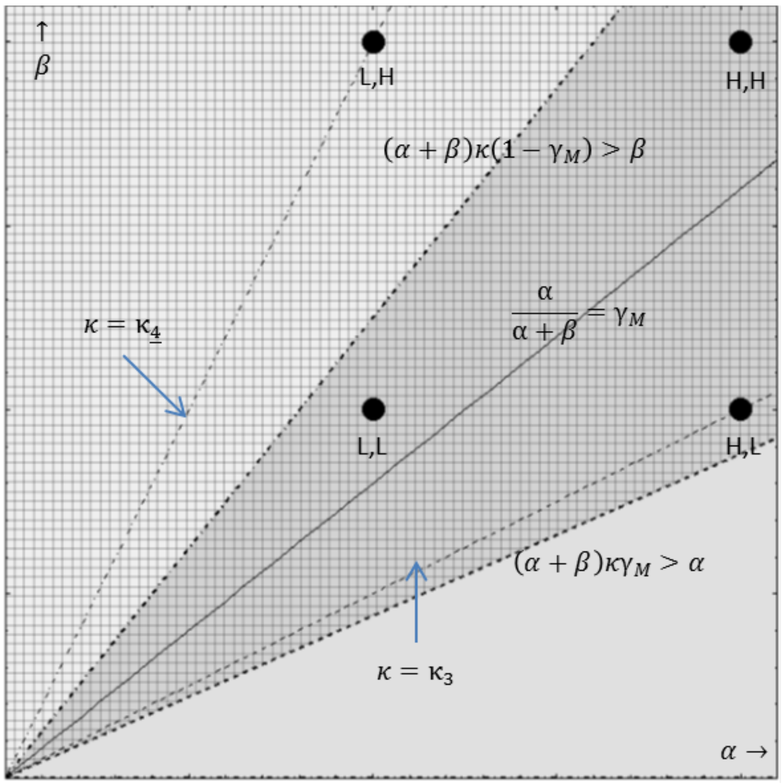

Figure 4: When synergy is moderate ($\kappa_3 < \kappa \leq \kappa_{\underline{4}}$) there exists a moderately discriminatory equilibrium, in which pairings $\langle \alpha, \beta \rangle \in \{\langle H, H \rangle, \langle H, L \rangle, \langle L, L \rangle\}$ collaborate. If synergy increases to $\kappa_{\underline{4}}$ all pairings team up. If synergy falls below κ_3 only homogenous teams form.

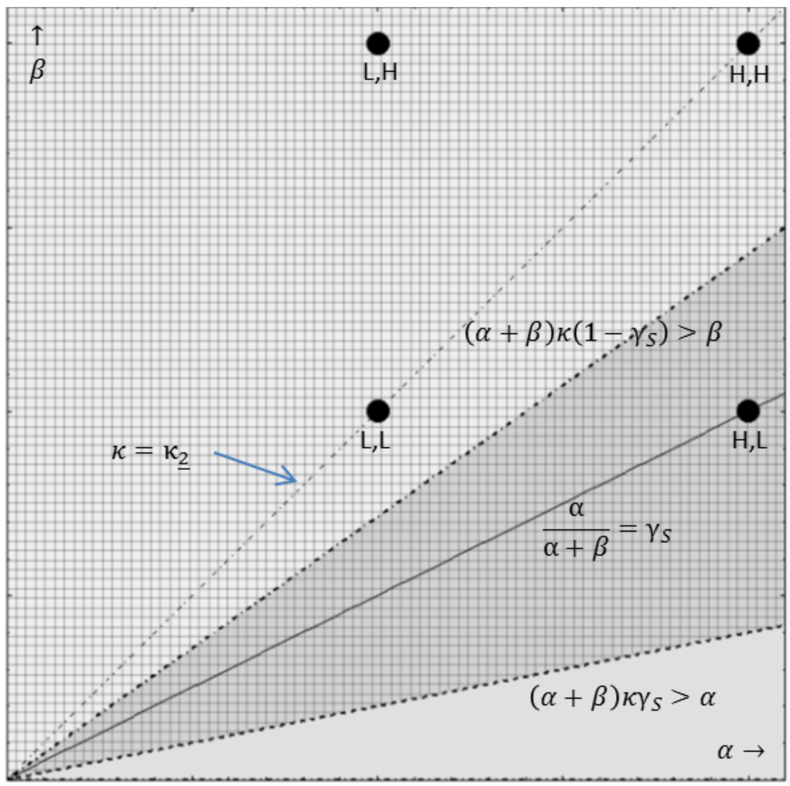

Figure 5: When synergy is very low ($\kappa \leq \kappa_{\underline{2}}$) and society believes discrimination is severe only pairings with discriminatory alignment = 1 will team up.

Conventional wisdom holds that discriminating against anyone on an attribute that has no direct causal link to productivity cannot improve output and may very well be counterproductive. Indeed, since any pair working together produces more than the two working alone, in a first-best world all possible parings would cooperate and egalitarianism would be the *only equilibrium*. But adverse selection creates a tension between social optimality and egalitarianism, because the severity of this adverse selection is not equal under both regimes—as κ falls below κ_4 and until it falls below κ_3 (always an open interval), more teamwork occurs under the discriminatory equilibrium of Proposition 2. In this case, conventional wisdom is specious:

Proposition 4 *(i) If synergy is moderate ($\kappa_3 < \kappa \leq \kappa_4$) there exists a discriminatory equilibrium ($\gamma = \frac{1}{2}p_e + \frac{H}{H+L}(1-p_e)$) that is socially preferable to egalitarianism. (ii) If minorities gain more from additional teams formed under discrimination than they loose from discrimination in teams that exist under egalitarianism*

$$(\kappa (H+L)(1-\gamma) - L)\, p\,(1-p) > \kappa \left(Hp^2 + L(1-p)^2\right)(2\gamma - 1)$$

then both minorities and non-minorities are better off under the discriminatory equilibrium than egalitarianism.

Proof. Part (i): Since every team that forms under egalitarianism also forms under discrimination, and teams always outproduce their members working individually, part (i) is immediate.

Part (ii): Minorities and non-minorities as groups alike, prefer discrimination if and only if their expected returns under that regime are higher than under egalitarianism. Clearly, non-minorities always prefer discrimination—their payoff is strictly higher under discrimination when teams form (i.e.$\langle \alpha, \beta \rangle \in \{\langle H, H\rangle, \langle H, L\rangle, \langle L, L\rangle\}$) and exactly the same when teams do not form under either regime (i.e..$\langle \alpha, \beta \rangle \in \{\langle L, H\rangle\}$). On the other hand, minorities, as a group, prefer discrimination to egalitarianism iff

$$\begin{aligned} & \kappa (H+H)(1-\gamma)\, p^2 + \kappa (H+L)(1-\gamma)\, p\,(1-p) \\ & + H(1-p)\, p + \kappa (L+L)(1-\gamma)(1-p)^2 \\ > \; & \kappa (H+H)\frac{1}{2}p^2 + Lp(1-p) + H(1-p)\, p + \kappa (L+L)\frac{1}{2}(1-p)^2 \end{aligned}$$

This simplifies to the form of the condition in part (ii) of the proposition. ■

The following simple numerical example verifies the existence of parameter sets which satisfy the conditions of Proposition 4 parts (i) and (ii):

Example 1 *If High ability is twice as potent as Low ability but equally common (i.e.* $H = 2, L = 1, p = \frac{1}{2}$*) and synergy is moderate (i.e.* $\kappa = \frac{5}{4}$*) society produces more under the discriminatory equilibrium (i.e.* $\gamma = \frac{5}{9}$*) than the egalitarian one. Under egalitarianism, non-minorities and minorities both earn* $\frac{27}{16}$*, on average. Under discrimination, non-minorities earn* $\frac{29}{16}$ *on average and minorities earn* $\frac{28}{16}$ *on average.*

Clearly discrimination harms many individuals—in the equilibrium of Proposition 4 minorities joining homogenous teams are harmed. By many moral frameworks, that alone justifies its eradication. If it also unequivocally harmed society's overall productivity, policy aimed at ending discrimination could be justified on purely economic (utilitarian) grounds. Lamentably, Proposition 4 dispels such an unambiguous justification: *with respect to productivity, egalitarianism is not always socially preferable to discrimination; in fact, discrimination may be preferred by both minorities and non-minorities as groups.*[14] Proposition 4 highlights the potential tension facing policy makers between efficiency and equity.

What intuition lies behind this unsettling result? An increase in discrimination induces some new teams to form and some existing ones to break up. The new teams' increased output is a societal gain, but society loses output from those individuals formerly working on teams. Whether or not the gain exceeds the loss depends on the specific distribution of ability and team production function.

If synergy is very low only those teams near the central locus will form; the productivity difference between egalitarianism and discrimination (and individual work) is minimal. As synergy increases, the productivity difference can grow. Although beyond the scope of this model, in which the manager is constrained to be fair, these observations suggest that

[14] Lundberg and Startz' (1983) conclude egalitarianism is always socially preferable. In their model, discrimination shifts training from individuals with lower costs for it to those with higher costs. In this one, discrimination may enable synergistic production when egalitarianism may not, and the increased productivity comes not from a change in the individual, but rather the choice of production technology.

managers in moderately synergistic industries may have incentive to be discriminatory and *unfair*, as it may be profitable.

Despite the fact that the manager here cannot strategically *choose* to be discriminatory, both egalitarianism and discrimination are equilibria in the moderate synergy range ($\kappa_3 < \kappa \leq \kappa_{\underline{4}}$). The following evolutionary argument refines these equilibria. Suppose that the synergy of team production varies over a range of industries, each comprised of many competing firms. Some firms in each industry choose seniority based compensation (i.e. discriminatory) schemes while others compensate teammates equally. In industries with moderate ($\kappa_3 < \kappa \leq \kappa_{\underline{4}}$) synergy levels only seniority based firms survive due to the efficiency gains of discrimination, and, of course, seniority both reflects the relative contribution of teammates and is the right competitive "strategy" ex post. In industries with synergy outside this range, egalitarian firms dominate.

The productivity of society is the productivity of its members. The equilibrium definition describes the relative productivity of minorities and non-minorities on teams, but what about those working alone? And minorities versus non-minorities overall? Victims and beneficiaries remain statistically identical with respect to ability across the population at large, but not among those working on teams—this is precisely why a fair manager can discriminate after observing the teamwork decision.

Proposition 5 *Among those working individually, discrimination victims outperform beneficiaries on average.*

Proof. The proposition can be formally written

$$E\left[\alpha \mid solo\right] < E\left[\beta \mid solo\right]$$

which can be verified under each discriminatory equilibrium.

Case $\gamma = \frac{H}{H+L}$:

$$\frac{Hp^2 + Lp\left(1-p\right) + L\left(1-p\right)^2}{1 - p\left(1-p\right)} < \frac{Hp^2 + Hp\left(1-p\right) + L\left(1-p\right)^2}{1 - p\left(1-p\right)}$$

Case $\gamma = \frac{1}{2}p_e + \frac{H}{H+L}\left(1-p_e\right)$:

$$L < H$$

■

The model predicts greater success for self-selected minority entrepreneurs. Very able discrimination victims have the lowest discriminatory alignment, especially when potential partners are incompetent, and thus have the greatest incentive to work as individuals. Since a rejection of teamwork induces victim and beneficiary alike to work independently, victims have higher average ability in the solo working population. This intuition holds for general distributions and production functions (see the Appendix for a proof).

In the model presented here, a rejection by either teammate means both work alone. However, one of the teammates would have preferred teamwork. If either had another potential teamwork opportunity he might try again. Thus, in a market for teammates, the probability of rejection matters. The following two lemmas show that when teamwork is endogenous, adverse selection works causes two groups to eschew teamwork: (1) talented individuals and (2) discrimination victims.

Lemma 1 *Talented individuals reject teamwork more often.*

Proof. Let $\tilde{A}$ be the event that A rejects teamwork (i.e. $g(\alpha,\beta)\gamma \leq \alpha$) and $\tilde{B}$ be the event B rejects teamwork (i.e. $g(\alpha,\beta)(1-\gamma) \leq \beta$). Given that there is a rejection, the probability that the more able did it (recall that only one worker will reject due to synergy) can be written

$$\Pr\left\{\tilde{A} \mid \alpha > \beta\right\} + \Pr\left\{\tilde{B} \mid \alpha < \beta\right\} > \Pr\left\{\tilde{B} \mid \alpha > \beta\right\} + \Pr\left\{\tilde{A} \mid \alpha < \beta\right\}$$

where all probabilities are conditional on $\tilde{A} \cup \tilde{B}$. This holds iff

$$\begin{aligned}
& \Pr\left\{\tilde{A} \mid \alpha > \beta\right\} + 1 - \Pr\left\{\tilde{A} \mid \alpha < \beta\right\} > 1 - \Pr\left\{\tilde{A} \mid \alpha > \beta\right\} + \Pr\left\{\tilde{A} \mid \alpha < \beta\right\} \\
\iff \quad & \Pr\left\{\tilde{A} \mid \alpha > \beta\right\} > \Pr\left\{\tilde{A} \mid \alpha < \beta\right\} \\
\iff \quad & \Pr\left\{g(\alpha,\beta)\gamma \leq \alpha \mid \alpha > \beta\right\} > \Pr\left\{g(\alpha,\beta)\gamma \leq \alpha \mid \alpha < \beta\right\}
\end{aligned}$$

which is always true. ■

Lemma 2 *Discrimination victims reject teamwork (with beneficiaries) more often.*

Proof. By symmetry, $\gamma > \frac{1}{2} \Leftrightarrow \Pr\{g(\alpha,\beta)\gamma \leq \alpha\} < \Pr\{g(\alpha,\beta)(1-\gamma) \leq \beta\}$ ■

This suggests that talented individuals and discrimination victims reject teamwork more overall, choosing different work than discrimination beneficiaries and those of lesser ability. This may explain why Russia's entrepreneurs are smart and short (see introduction). Likewise, as suggested by the occupational segregation in sports (also highlighted in the introduction), even among those traditionally employed, discrimination victims and the gifted will choose individually measurable occupations over synergistic ones.

The natural context in which to empirically measure discrimination is when minorities and non-minorities are working side by side, such as within firms. Since, job duties can be compared and management is consistent, it is the setting in which legal disputes over workplace discrimination are most easily waged. And indeed, discrimination in these team settings is precisely what the model captures, but minority and non-minority workers in teams do not represent the populations as a whole, because occupational choice is endogenous. Thus, this measure of discrimination does not reflect the overall adversity faced by minorities in the population. Since, by Proposition 5, discrimination victims outperform beneficiaries when working individually, the following is immediate:

Proposition 6 *Discrimination victims receive more than* $1-\gamma$ *of total societal output on average.*

A high ability minority worker can opt out of discriminatory teams, say by choosing entrepreneurship or another occupation where individual contribution is more measurable. Thus, measuring discrimination as wage differences only within team settings overestimates the overall negative relative impact of discrimination on minorities—there is a selection bias due to the entrepreneurial choices of talented minorities. In fact, as the following example paradoxically shows, the total relative impact of discrimination on minorities need not be negative at all.

Example 2 *If High ability is rare (i.e.* $p = \frac{1}{8}$*), but very potent relative to Low ability (i.e.* $H = 20$ *and* $L = 1$*), and synergy is strong (i.e.* $\kappa = \frac{7}{4}$*), then* $\gamma = \frac{5}{9}$ *is a discriminatory equilibrium—non-minorities receive* 125% *(i.e.* $\frac{\gamma}{1-\gamma}$*) what minorities do in teams. Despite* *the high synergy of teamwork, the presence of discrimination and the relative rarity of individual work (i.e.* $\frac{7}{64} \approx 11\%$ *of the time), minorities receive over* 127% *of the credit that non-minorities receive overall.*

Proposition 5, Lemmas 1 and 2 as well as Proposition 6 hold under any discriminatory beliefs, even if those beliefs are not confirmed by a fair manager; that is, γ need not be a self-reinforcing BNE. For example, even if the manager has a preference for employees of one race over another (i.e. she is not fair) as is assumed in some other models of discrimination (e.g. Becker, 1957), talented minorities will *still strategically respond* by opting out of teams. We conclude the basic analysis by considering some comparative statics of the self reinforcing equilibrium of the model.

In one industry all workers may produce at similar levels, but in another the gap between highly productive workers and low productivity ones may be large. How does this production sensitivity to ability impact discrimination? Can different industries support different levels of statistical discrimination?

Proposition 7 *As the ability gap between High and Low types increases, so does (a) discrimination (in any discriminatory equilibrium) and (b) the maximum synergy, for which discriminatory equilibria exist.*

Proof. (a) $\frac{d\gamma}{dH} > 0$ and $\frac{d\gamma}{dL} < 0$ for both discriminatory equilibria. (b) $\frac{d\underline{\kappa_4}}{dH} > 0, \frac{d\underline{\kappa_4}}{dL} < 0, \frac{d\underline{\kappa_2}}{dH} > 0$ and $\frac{d\underline{\kappa_2}}{dL} < 0$. ■

Proposition 7 predicts that discrimination will be stronger in occupations, in which the (relevant) ability of workers exhibits higher variance. To see the intuition behind part (a) recall that discriminatory beliefs are an equilibrium, because on teams the expected type of beneficiaries is higher than victims'—either beneficiaries (on teams) are more likely to be High ability, less likely to be Low ability or both. Thus, if High increases or Low decreases, the expected ability gap increases and so does discrimination. The intuition behind part (b) is also simple. A High ability worker will work with a Low ability worker even if he believes he will be the victim of discrimination, *so long as synergy is high enough.* If this happens, a fair manager cannot discriminate (i.e. discrimination cannot be an equilibrium). But if the ability gap between these two increases even more, then these two may not work together, and now a fair manager must discriminate.

The model provides a way to analyze discriminatory settings in which existing theory says little. Although applications of the model are varied, one familiar to many readers is academic coauthoring. Einav and Yariv (2006) show that the probability of receiving tenure at a top economics department declines significantly with the alphabetic ordering of one's surname initial, even when accounting for country of origin, ethnicity and religion. This discrimination is difficult to analyze with existing economic models of discrimination—it is hard to imagine that a taste for individuals with last names beginning with A exists, that their discourse is somehow different from those with last names beginning with B, or that their childhood environment has been so different that they have endogenously acquired different human capital. It is not farfetched, though, that economics researchers with surname initials at the end of the alphabet may choose coauthors (teamwork) strategically, given the discriminatory convention of alphabetic surname ordering on economics publications. By casting the body of academic peers in the role of a merit-fair manager (credit allocator) the coauthoring (team-formation) decision may be analyzed with the predictions of the model. Proposition 5 predicts that the solo work of authors with surname initials near the end of the alphabet are of higher quality than the solo work of authors near the beginning of the alphabet. If the variance in ability at a department increases with its rank, as one would expect if top departments are drawing from the thin right tail of the talent distribution, then Proposition 7 predicts discrimination will be strongest within the highest tiered departments.

5 Extensions

The simple model can be generally extended in several ways: (1) ability distributions and production functions can be made general, (2) a market for teammates can be added, (3) workers may have incomplete information about potential teammates' ability, (4) the credit split can be made contractible on the output, and (5) the sharing rule can be changed. In this section, I consider these extensions individually—discriminatory equilibria survive all of them.

5.1 General Ability Distributions and Production Functions

One might worry that the existence of discriminatory equilibria is an artifact of the two type model or a very specific team production function. The next two propositions reassure that they are not. One additional definition is required:

Definition 1 *If an individual with no ability whatsoever joins a team and production is not improved (i.e.* $g(x,0) = x$*), and teamwork amplifies ability (i.e.* $\frac{d}{d\alpha} g(\alpha,\beta) > 1 \ \forall \alpha, \beta \geq 0$*) then team production is* regular.

Example 3 $g(\alpha,\beta) = \alpha + \beta + \kappa\alpha\beta$ *is a* regular *production function.*

Proposition 8 *If team production is regular, and ability is continuously distributed with support from* 0*, then at least one discriminatory equilibrium* ($\gamma \in \left(\frac{1}{2}, 1\right)$) *exists in which teams form.*[15]

Proof. The proof in the Appendix has the following steps: (1) $\lim_{\gamma \to 1} E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right] = 1$, (2) $\lim_{\gamma \to 1} \frac{d}{d\gamma} E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right] < 1$, (3) $\frac{d}{d\gamma} E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right] |_{\gamma=\frac{1}{2}} = 0 < 1$ and (4) this implies $E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]$ has a fixed point in $\left(\frac{1}{2}, 1\right)$. ■

Proposition 8 provides a sufficient but unnecessary condition for discriminatory equilibria to exist. There may be many such equilibria; Proposition 8 simply says that under reasonable conditions, *at least one* set of discriminatory beliefs exists such that workers will choose teamwork strategically such that a fair manager will confirm those beliefs. Regularity guarantees that the second step of the proof holds—many irregular production functions also satisfy the second step of the proof but must be handled on a case by case basis.

The concept of BNE, considered so far, is precise—an equilibrium exists whenever $\gamma = E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]$. If private worker abilities can be uncovered, then empirical analysis can only tell us that γ is sufficiently close to $E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]$. This, as the following proposition highlights, is a much looser condition than BNE and may hold for a very wide set of beliefs that are not, strictly speaking, equilibria.

[15] Obviously, beliefs $\gamma \in \{0,1\}$ are always discriminatory equilibria; however, since no teams form under these beliefs, they are of little interest by themselves.

Proposition 9 *If ability is continuously distributed with support from* 0 *and synergy is low enough, the credit split of a fair manager will be arbitrarily close to confirming* any *beliefs. Formally, if synergy is measured by* $\kappa_{a\beta} = g(\alpha,\beta) - \alpha - \beta$*, then for all* $\varepsilon > 0, 0 < \kappa_{a\beta} \leq \varepsilon(\alpha+\beta) \implies \left|\gamma - E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]\right| < \varepsilon$.

Proof.

$$\begin{aligned}
& \left|\gamma - E\left[\tfrac{\alpha}{\alpha+\beta} \mid team, \gamma\right]\right| \\
= & \left|\gamma - E\left[\tfrac{\alpha}{\alpha+\beta} \mid g(\alpha,\beta)\gamma > \alpha, g(\alpha,\beta)(1-\gamma) > \beta\right]\right| \\
= & \left|\gamma - E\left[\tfrac{\alpha}{\alpha+\beta} \mid -\left(\frac{\kappa_{a\beta}}{\alpha+\beta}+1\right)\gamma < -\frac{\alpha}{\alpha+\beta} < \left(\frac{\kappa_{a\beta}}{\alpha+\beta}+1\right)(1-\gamma) - 1\right]\right| \\
= & \left|E\left[\gamma - \tfrac{\alpha}{\alpha+\beta} \mid -\gamma\tfrac{\kappa_{a\beta}}{\alpha+\beta} < \gamma - \tfrac{\alpha}{\alpha+\beta} < (1-\gamma)\tfrac{\kappa_{a\beta}}{\alpha+\beta}\right]\right|
\end{aligned}$$

Thus, for all $\kappa_{a\beta} \leq \varepsilon(\alpha+\beta)$ the above reduces to

$$= \left|E\left[\gamma - \tfrac{\alpha}{\alpha+\beta} \mid -\gamma\varepsilon \leq -\gamma\tfrac{\kappa_{a\beta}}{\alpha+\beta} < \gamma - \tfrac{\alpha}{\alpha+\beta} < (1-\gamma)\tfrac{\kappa_{a\beta}}{\alpha+\beta} \leq (1-\gamma)\varepsilon\right]\right| < \varepsilon$$

■

Low synergy produces not only few teams but ones in which discriminatory beliefs reflect workers' respective actual, not just expected, abilities quite precisely. Thus, when synergy is very low, any discrimination level is close to an equilibrium.

This observation presents an empirical challenge. For example, the 1963 Equal Pay Act says that US employers must pay employees equally for equal work; however, the burden of establishing a prima facie case that different wages are paid to employees of the opposite sex and that the employees perform substantially equal work belongs to the employee. While US law prohibits even statistical discrimination, this burden of proof amounts to showing that the proportion of wages paid to men γ is statistically different from their proportional (expected) work $E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]$—an employee must show $\left|\gamma - E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]\right| > \varepsilon$, where ε is the measurement error. Proposition 9 implies that establishing that meritocracy is not functioning may be more difficult in low synergy industries, because the measurement error must be smaller.

5.2 Frictionless Market for Teammates

The basic model implicitly assumes no market for teammates; a rejection by either teammate means both work alone. Of course, in the real world, the outside option of an individual deciding whether or not to join a team, is not usually limited to working alone, but rather includes working on one of several different teams. Here I examine the extreme opposite situation, namely that the market for teammates is frictionless—every worker in the economy is a potential teammate. I will show that, even in this extreme case, discriminatory equilibria can still exist, and thus we should expect them in a more realistic market with frictions.

Suppose all workers who reject teamwork (or are rejected) randomly draw new potential teammates from the pool of individual workers until every worker (a) finds a teammate, (b) rejects or is rejected by all remaining individual workers. Without loss of generality, assume minorities and majorities each have an even number of individuals with each supported ability.

Lemma 3 *In a frictionless market for teammates, no one works individually.*

Proof. Suppose someone chose to work individually. By symmetry another identical worker also did. These two could team up without facing discrimination and, because team production is synergistic, both be better off, a contradiction. ■

Can discrimination exist in a frictionless market for teammates? For simplicity, assume ability distributed as in (2) and that all workers assume discrimination exists. Clearly, High ability victims will always work together to avoid discrimination. Similarly Low ability beneficiaries will work together because no one else will work with them. Thus, if a heterogeneous team were to form, it could only be between a High ability beneficiary and a Low ability victim. A fair manager, seeing a team, would know this and divide credit accordingly: $\gamma = \frac{H}{H+L}$. These teams would form if and only if both parties prefer this heterogeneous team to working with their peers (i.e. those with identical discriminatory attribute and ability):

$$g(H,L)\frac{H}{H+L} > g(H,H)\frac{1}{2} \qquad (M_A)$$

$$g(H,L)\frac{L}{H+L} > g(L,L)\frac{1}{2} \qquad (M_B)$$

Thus, we have proved the following proposition:

Proposition 10 *In a frictionless market for teammates, discrimination in which A's and B's cooperate can exist if and only if heterogeneous teams are sufficiently more productive than homogeneous ones (i.e. (M_A) and (M_B) are satisfied); otherwise discriminatory beliefs completely segregate society.*

Example 4 $g(\alpha, \beta) = \alpha + \beta + |\alpha - \beta| \kappa$ *satisfies (M_A) and (M_B).*

As noted previously, Hamilton, *et al.* (2003) empirically find that heterogeneous teams produce more. So, one should not be surprised to find discriminatory compensation even if a perfect markets for teammates existed. Since, discriminatory equilibria exist both when the market for teammates does not exist and when it has no frictions, one can reasonably conclude that they exist in a more realistic imperfect market for teammates.

Lemma 2 suggested one cause of occupational segregation, namely that minorities will prefer occupations where individual contribution is more easily measured. Proposition 10 highlights a second possible cause of occupational segregation that exists even when there are no substantive differences between jobs—minorities may simply choose to team with other minorities to strategically eliminate the possibility of discrimination.

5.3 Incomplete Information

Although teammates almost certainly know each other's ability better than their manager, it could be difficult to know exactly how much a potential partner will contribute before production occurs. What if signals of a potential teammate's ability were noisy?

At the one extreme, workers may be able to communicate their types to each other almost perfectly. Suppose that there are finitely many ability types, and that with probability σ worker i observes j's true ability and with probability $1 - \sigma$ he observes a uniform draw from the type space instead. If the noise is small (i.e. σ is very near 1), then, since each worker type has a strict incentive to join a team (or not), the resulting team formation distribution will be arbitrarily close to the complete information specification. As a consequence, the manager's beliefs about the relative abilities of A and B workers will be arbitrarily close as well and all equilibria under complete information will persist in a model with slightly noisy signals.

At the other extreme, workers could observe nothing about each other's ability *ex ante.* In this extreme case, workers have the same information about the other worker as their manager. Here it is convenient to invoke the simple two type model again. The following lemma proves useful in analyzing this case:

Lemma 4 *Suppose workers have no specific information about potential teammates' types. In all equilibria, untalented A workers always choose teamwork.*

The above lemma follows directly from the fact that all teamwork is synergistic and A will capture at least half of the product.

From the lemma it is immediately clear that a number of the equilibria in identified in Section 4 no longer exist. The severely discriminatory equilibrium of Proposition 2 cannot survive—low ability A workers will sneak into teams too. Since low ability A workers will also sneak into teams with high ability Bs in the moderately discriminatory equilibrium of Proposition 3, this equilibrium also breaks. Given that both discriminatory equilibria from the complete information case no longer exist, one may doubt whether discrimination is possible when workers have no specific information about the ability of potential teammates. However, the following proposition shows that new discriminatory equilibria can arise when information is imperfect:

Proposition 11 *When workers have no specific information about potential teammates' ability and synergy is linear ($g(\alpha,\beta) = (\alpha+\beta)\kappa$) there always exists a non-empty synergy range*

$$\kappa_A = \frac{H}{(H+L)\gamma} < \kappa \leq \frac{H}{(2Hp + (H+L)(1-p))(1-\gamma)} = \kappa_{\overline{B}}$$

where

$$\gamma = \frac{H}{H+L}p + \frac{1}{2}(1-p)$$

is a discriminatory equilibrium, in which teams $\langle \alpha, \beta \rangle \in \{\langle H, L\rangle, \langle L, L\rangle\}$ form.

Proof. The proposition describes a pure strategy equilibrium—A always chooses teamwork and B chooses teamwork iff and only if his type is L. Therefore a team, if formed, will have composition

$$\langle \alpha, \beta \rangle = \left\{ \begin{array}{lll} \langle H, L \rangle & \text{with probability} & p \\ \langle L, L \rangle & \text{with probability} & 1-p \end{array} \right\}$$

and the equilibrium (if it exists) will consist of the beliefs specified in the proposition. Now check the participation constraints of all types.

From Lemma 4 untalented A workers (regardless of production function) choose teamwork.

Now I will show that for any team production function exhibiting increasing differences, including when synergy is linear, untalented Bs also choose teamwork. Formally,

$$\left(g\left(H, L\right) p + g\left(L, L\right)\left(1-p\right)\right)\left(1-\gamma\right) > L \tag{3}$$

After substituting the equilibrium beliefs γ, the second derivative of the left hand side (LHS) with respect to p is

$$-2\left(\frac{H}{H+L} - \frac{1}{2}\right)\left(g\left(H, H\right) - g\left(L, H\right)\right)$$

which is strictly negative, because g exhibits increasing differences. This concavity implies that the LHS of (3) has a minimum at $p \in \{0, 1\}$. When $p = 0$ the LHS of (3) reduces to $\frac{1}{2} g\left(L, L\right)$, which is strictly greater than the right hand side L. Similarly, when $p = 1$ the LHS of (3) reduces to $L \frac{g(H,L)}{H+L}$, which is also strictly greater than the right hand side L. Thus (3) always holds—untalented Bs never wish to deviate.

With linear synergy talented As choose teamwork iff

$$\left(H + L\right) \kappa \gamma > H$$

which reduces to the lower bound of the synergy range specified by the proposition.

Talented Bs choose individualistic work (i.e. they cannot deviate) iff

$$\left(\left(H + H\right) p + \left(L + H\right)\left(1-p\right)\right) \kappa \left(1-\gamma\right) \leq H$$

which reduces to the upper bound of the synergy range.

Finally, the synergy range of the lemma is non-empty if and only if the lower boundary is strictly less than the upper. Cross multiplying yields

$$\frac{2Hp + \left(H+L\right)\left(1-p\right)}{H+L} < \frac{\gamma}{1-\gamma} = \frac{2Hp + \left(H+L\right)\left(1-p\right)}{2Lp + \left(H+L\right)\left(1-p\right)}$$

which always holds. ■

Note that the parameters of Example 1 also lead to a discriminatory equilibrium when workers cannot communicate information about their ability, but discrimination is stronger ($\gamma = \frac{7}{12}$) now, because homogenous high ability teams no longer form—incomplete information increases the potential for adverse selection. This is intuitive, since talented minorities cannot afford the risk that their non-minority teammate may not be as gifted.

Thus, discriminatory equilibria may exist whether the signal noise is arbitrarily small or arbitrarily large,although the possible equilibria may, in fact, change with signal quality. More fundamentally, discrimination does not arise just because the workers are communicating something to each other that the manager cannot observe. Rather the opportunity for discrimination comes from the fact that the workers cannot credibly reveal information to the manager—not being able to reveal that information to teammates only make the potential for adverse selection and discrimination worse. Recognizing this fact, the implicit assumption that potential teammates cannot transfer utility to each other does not seem artificially limiting—when teammates cannot credibly communicate their type *ex ante,* offering a side payment for cooperation (to offset discrimination) is infeasible.

Incomplete information also increases adverse selection under egalitarian beliefs. Notice that Lemma 4 also implies that even under egalitarian beliefs high ability workers cannot guarantee that their potential team is similarly skilled. So, if synergy is not strong enough to induce teams of heterogenous ability ($\kappa \leq \frac{2H}{H+L} = \kappa_4$), high ability workers (both A and B) will eschew teamwork—their potential teammates may not be competent enough to justify splitting credit with them. Thus, without high synergy, only untalented workers will team up together under egalitarian beliefs and incomplete information.

Furthermore, it is always true that $\kappa_A < \kappa_4 < \kappa_B$. So, there always a non-empty synergy interval where the following two equilibria co-exist: (1) under egalitarian beliefs only untalented workers cooperate, and (2) under discriminatory beliefs both untalented workers cooperate and talented non-minorities cooperate with untalented minorities. The set of teams formed under egalitarianism is a proper subset of those formed under discrimination. So, the following version of Proposition 4 is immediate:

Proposition 4 (Incomplete Information) *When workers have no specific information* *about potential teammates' ability and synergy is linear* ($g(\alpha, \beta) = (\alpha + \beta)\kappa$)*, if* $\kappa_A < \kappa \leq \kappa_4$*, then society produces more and both minorities as well as non-minorities are better off under the discriminatory equilibrium than egalitarianism.*

Although discriminatory equilibria may still exist and Proposition 4 even sharpens when workers have no *ex ante* information about their teammates' abilities, notice that if synergy is low enough, discriminatory equilibria disappear altogether. Why? Intuitively, it is because the lowest ability workers always have the greatest incentives to join teams—although this is especially true of beneficiaries of discrimination, it is also true of untalented victims. Talented workers know this and, when informed about potential teammates' abilities, can cooperate selectively. Without this information, though, talented workers take a risk on the abilities of their teammates, a risk that can be offset by synergy; however, with very little synergy, only the lowest ability workers can afford the risk of blind cooperation. Thus, due to the symmetry of the model, only egalitarianism survives.

Of course, the manager may also get some signal of her employees' abilities. In this model, output itself is a strong signal of worker abilities. In the next section, I consider the impact of such a signal on equilibria.

5.4 Equilibria with Contractible Output

Previous analysis assumes that the fractional credit split did not depend on the realized output. The system of offering a salary to join a team and a proportional bonus based on company profitability fits this setting. The total bonus amount is tied to team output, but the fraction relative to one's peers is not. Similarly, corporate shares and options are typically divvied up before their exercisable worth is ever known; an output contractible split is impossible. Some compensation schemes, though, do recognize team output when splitting the reward. For example, Senior team member bonuses may more closely tied to team output than Junior members'.

Could the existence of discriminatory equilibria stem from the manager's inability to contract on the realized output? After all, the manager learns a great deal about her employees' abilities from the team's output. This subsection shows that discriminatory equilibria can exist even when the manager can contract on team output.

This requires that the credit split be a function of team output, $\gamma(Q)$, where $Q = g(\alpha, \beta)$. Theoretically, very little changes. The definition of equilibrium beliefs changes to

$$\gamma(Q) = E\left[\tfrac{\alpha}{\alpha+\beta} \mid g(\alpha,\beta)\gamma(Q) > \alpha, g(\alpha,\beta)(1-\gamma(Q)) > \beta\right]$$

This means the belief set is much more complex; workers must have beliefs for each possible output level. Without additional restrictions imposed on equilibria, workers may rationally believe that the manager severely discriminates if she observes low output, moderately discriminates if she observes typical output and does not discriminate at all for exceptional output. It is not difficult to see that, in general, describing even a single complete set of beliefs could be extremely cumbersome (not just as a researcher, but also as a manager or worker). Nevertheless, a few examples reveal that discriminatory equilibria may exist, even when the manager can contract on the extra information contained in the output itself.

Example 5 *Assume abilities are distributed $H > M = \frac{H+L}{2} > L > 0$ each arising with equal probability and linear synergy, $g(\alpha, \beta) = (\alpha + \beta)\kappa$. If the manager sees a team produce $Q = (H + L)\kappa = 2M\kappa$ then she does not know exactly what each worker contributed. It can be shown that if $\frac{2(H+L)}{H+3L} < \kappa \leq \frac{4H}{H+3L}$, then $\gamma(2M\kappa) = \frac{1}{2}\frac{H}{H+L} + \frac{1}{4}$ is an a equilibrium belief, which forms a team if and only if $\langle \alpha, \beta \rangle \in \{\langle H, L\rangle, \langle M, M\rangle\}$.*[16]

This example illustrates that although the manager may gain some information from output, she cannot, in general, completely resolve employee contributions, and thus the opportunity for discrimination persists. Furthermore, while it is clear that the above example is constructed to make this point, a manager would have even greater difficulty resolving contribution if there were many types or a continuum of types, from which many possible teams could produce identical output.

But perhaps surprisingly, even when output completely reveals the contribution of individual workers, discrimination can persist. Consider the severely discriminatory equilibrium

[16] Although the example illustrates the persistence of discriminatory equilibria with familiar linearly synergistic team production ($g(\alpha, \beta) = (\alpha + \beta)\kappa$), there is nothing special about that form. When the same ability distribution holds, but team production has constant synergy ($g(\alpha, \beta) = \alpha + \beta + \kappa$) then when synergy satisfies $\frac{H^2-L^2}{H+3L} \leq \kappa \leq 3\frac{H^2-L^2}{H+3L}$, $\gamma(2M + \kappa) = \frac{1}{2}\frac{H}{H+L} + \frac{1}{4}$ is an equilibrium belief, which forms the same teams.

of Proposition 2. In this case, the manager knows exactly what each teammate contributes before production occurs. Thus, observing the output adds no information above and beyond what discrimination tells her—the equilibrium is identical whether output is contractible or not. Of course, if beliefs were egalitarian and an (off-equilibrium) heterogenous team formed, then the manager would not know which worker was high ability and which was low before or after production.

Thus, despite the increased complexity of calculating complete belief sets contingent on output, versions of the previous propositions still hold with similar proofs. Also, although I have not explicitly modeled a separate signal of worker abilities available to the manager, the exercise here gives strong intuition that so long as the manager cannot perfectly observe worker types *without discriminating*, the potential for discrimination exists.

5.5 Shapley Values

In the basic model, a manager is fair if she divides team produce according to average ability. From the perspective that (in the model) ability is the only parameter an individual brings to the team this seems appropriate; however, if one is willing include parameters not associated with the individual alone, say by combining the firm's team production function with the mix of individual member abilities, then other sharing rules are possible. For example, by combining the possible ability levels of all team members together with information about the production function the manager may be able to calculate the average marginal contribution of each member and compensate according to that measure. In this subsection, I will show that discriminatory equilibria may still exist under such an allocation rule.

The most common way to fairly allocate cooperative gains is by using Shapley values (Shapley 1953)—they use the marginal contributions of members as inputs. Since a Shapley value represents a total allocation due a player in a coalition game, rather than a proportional measure like γ, they do not scale with output. Thus, using Shapley values, which completely distribute all surplus, requires that the level of output be fixed. Therefore, I restrict attention to settings where compensation is contractible on observed team output.

However, as was illustrated in Section 5.4, output alone is not enough to reveal the contributions of team members and is, thus, insufficient to generally determine team members' marginal contributions. Therefore, a manager is fair if she assigns each teammate his expected Shapley value for the observed level of output. It is a self-reinforcing equilibrium if, for a given level of team output, workers believe they will receive their expected Shapley value. Formally, if workers A and B believe that they will respectively receive $\nu_A(Q)$ and $\nu_B(Q)$ when jointly producing Q, a BNE exists whenever

$$\nu_A(Q) = E[\phi_A(\alpha,\beta) \mid team, Q] \text{ AND } \nu_B(Q) = E[\phi_B(\alpha,\beta) \mid team, Q]$$

where $\phi_A(\alpha,\beta)$ and $\phi_B(\alpha,\beta)$ are the Shapley values for A and B of abilities α and β respectively.

For simplicity, again assume the team production function and type distribution of Example 5. I will show that discrimination is an equilibrium when the manager sees output $Q = 2M\kappa$. For any given ability realization $\langle\alpha,\beta\rangle$ the A's and B's respective Shapley values may be calculated

$$\begin{aligned}\phi_A(\alpha,\beta) &= \frac{1}{2}((\alpha+\beta)\kappa-\beta)+\frac{1}{2}\alpha = \frac{1}{2}((\alpha+\beta)\kappa+\alpha-\beta)\\ \phi_B(\alpha,\beta) &= \frac{1}{2}((\alpha+\beta)\kappa-\alpha)+\frac{1}{2}\beta = \frac{1}{2}((\alpha+\beta)\kappa-\alpha+\beta)\end{aligned}$$

The Shapley values associated with team $\langle\alpha,\beta\rangle = \langle H,L\rangle$ are

$$\begin{aligned}\phi_A(H,L) &= \frac{1}{2}((H+L)\kappa+H-L)\\ \phi_B(H,L) &= \frac{1}{2}((H+L)\kappa-H+L)\end{aligned}$$

The Shapley values associated with team $\langle\alpha,\beta\rangle = \langle M,M\rangle$ can be simplified to

$$\phi_A(M,M) = \phi_B(M,M) = \frac{H+L}{2}\kappa$$

The manager, seeing output $Q = 2M\kappa$, know that the team is either $\langle H,L\rangle$ or $\langle M,M\rangle$ with equal probability, Therefore, the expected Shapley values (i.e. expectation over the possible teams that could have formed, given that a manager sees output $Q = 2M\kappa$) are

$$\begin{aligned}E[\phi_A \mid team, Q=2M\kappa] &= \frac{1}{2}\left(\frac{1}{2}((H+L)\kappa+H-L)\right)+\frac{1}{2}M\kappa = \frac{2(H+L)\kappa+H-L}{4}\\ E[\phi_B \mid team, Q=2M\kappa] &= \frac{1}{2}\left(\frac{1}{2}((H+L)\kappa-H+L)\right)+\frac{1}{2}M\kappa = \frac{2(H+L)\kappa-H+L}{4}\end{aligned}$$

These discriminatory payments are self-reinforcing if and only if beliefs that they will be paid induce teams to form in the assumed way. Formally, teams $\langle H, L\rangle$ form

$$(E[\phi_A \mid team, Q = 2M\kappa] > H) \text{ AND } (E[\phi_B \mid team, Q = 2M\kappa] > L)$$

teams $\langle M, M\rangle$ form

$$(E[\phi_A \mid team, Q = 2M\kappa] > M) \text{ AND } (E[\phi_B \mid team, Q = 2M\kappa] > M)$$

and teams $\langle L, H\rangle$ do not form

$$(E[\phi_A \mid team, Q = 2M\kappa] \leq L) \text{ OR } (E[\phi_B \mid team, Q = 2M\kappa] \leq H)$$

Since $E[\phi_A \mid team, Q = 2M\kappa] > E[\phi_B \mid team, Q = 2M\kappa]$ and $H > M > L$, showing the that the following subset of the above conditions hold suffices to guarantee the existence of a discriminatory equilibrium:

$$\begin{aligned}
E[\phi_A \mid team, Q = 2M\kappa] &= \frac{2(H+L)\kappa + H - L}{4} > H \\
E[\phi_B \mid team, Q = 2M\kappa] &= \frac{2(H+L)\kappa - H + L}{4} > \frac{H+L}{2} = M \\
E[\phi_B \mid team, Q = 2M\kappa] &= \frac{2(H+L)\kappa - H + L}{4} \leq H
\end{aligned}$$

These readily simplify to

$$\frac{3H+L}{2(H+L)} < \kappa \leq \frac{5H-L}{2(H+L)}$$

Thus, for moderate synergy (i.e. $\frac{3H+L}{2(H+L)} < \kappa \leq \frac{5H-L}{2(H+L)}$), there exists a discriminatory equilibrium for output level $Q = 2M\kappa$, in which teams $\langle \alpha, \beta\rangle \in \{\langle H, L\rangle, \langle M, M\rangle\}$ form—the existence of discriminatory equilibria is robust to expected Shapley value allocations.

Although changing the allocation rule to expected Shapley values would alter the details of the proofs related to the other results, the basic intuition that *talented minorities have the greatest incentive to eschew teamwork* remains, and so one may be confident that analogous proofs exist. Likewise, the basic consequence of this intuition is that discrimination puts pressure on teams to form in ways that mimic consensus beliefs. So, while there are many alternative allocation rules, of which Shapley values are but one, that a person could analyze, most of them can be expected to exhibit the feature that an average measure of individual contributions in teams resembles beliefs—when the resemblance is exact, it is a Bayesian Nash Equilibrium, and the beliefs self-reinforce.

6 Conclusion

Career choice is a fundamental life decision. The model highlights how discriminatory expectations can shape these choices. While minorities are free to choose either teamwork or entrepreneurship in the model, i.e., they face no "closed doors," they may worry about the allocation of credit in team environments where individual contributions cannot be readily measured. When talented minorities expect to receive too little credit, they opt out of team-based careers in favor of entrepreneurship. In contrast, less talented minorities opt in since they enjoy productive gains from teamwork and receive reasonably fair credit for their contributions. As a result, expectations about minority contributions in teams are self-fulfilling.

Importantly, the model implies that measures of discrimination based on wage differentials in firms overstate the economic effects of discrimination. The bias in the estimates stems from selection—talented minorities fight discrimination by opting out of the system and pursuing entrepreneurship. The "dual" to this implication is that minorities should outperform beneficiaries when working individually—a fact which is broadly consistent with a number of studies.

The model also reveals that meritocracy does not eliminate discrimination *per se*. Managers in the model fairly allocate credit based on their information. Indeed, in a severely discriminatory equilibrium, credit is being correctly allocated, even *ex post*. Thus, meritorious individuals enjoy the credit they deserve, yet, based on selection expectations, the result is still discriminatory.

In other models of statistical discrimination, leveling the playing field between victims and beneficiaries may remedy the situation. For instance, if the problem is that the discourse system of minorities leads to noisier signals of ability, then the appropriate policy solutions involve either educating majorities to better understand minority discourse or, more realistically, training minorities to communicate in ways that majorities can understand. If the problem is underinvestment in human capital, then subsidy schemes, such as minority-targeted scholarships, can remedy the problem. In my setting, minorities and majorities are equally skilled and equally able to convey this information at the time of employment. The apparent economic policy solution implied by the model—constrain the occupational choices of individuals so that talented minorities can no longer eschew teamwork—harms minorities even further in the near term. This may be seen as negative, but given that populations of minorities and non-minorities in the model are statistically identical at all times, a change in beliefs suffices to break the discriminatory equilibrium, without enhancing the current generation of minorities' abilities. Thus, the model may suggest cultural rather than economic policy remedies.

The model presented here explains some instances of discrimination that are outside the domain of other models. For instance, the alphabetical discrimination in the market for academic economists highlighted by Einav and Yariv is hard to explain using standard models of discrimination. It seems unlikely that individuals whose last names begin later in the alphabet systematically underinvest in human capital or use a different dialect than those whose last names come earlier. My model, however, offers an expectational rationale for this phenomenon. Moreover, it is not simply a "just so story"—the model offers sharp out of sample predictions as well.

Alchian and Demsetz (1972) observed that the fundamental incentive structure within teams changes, because individual actions are unobservable. That work spawned a substantial theoretical investigation of moral hazard in exogenously formed teams. By endogenizing the teamwork decision, the model here uniquely exposes an adverse selection problem created by the same unobservability of individual contribution in teams.

This model captures one force among many potentially operating in teams (and outside). A great deal of complexity could be added to the model, but Section 5 illustrates that the fundamental force survives an array of extensions. Its simplicity facilitates incorporation into models with richer institutional details, and indeed, studying this force in a specific setting may dictate adding such intricacies, even potentially at a loss of clarity or tractability. This paper's aims are more general.

Like other discrimination models it is static. Future work will examine teamwork decisions in a dynamic setting—when future potential teammates may share the attribute of discrimination or not, and ability may be revealed over time.

7 Appendix

Throughout the appendix, the following notational definition is useful:

Definition 2 $\psi(\gamma) = E\left[\frac{\alpha}{\alpha+\beta} \mid team, \gamma\right]$

Proposition 1 (General) *(a) Egalitarianism (i.e. $\gamma = \frac{1}{2}$) is always a team forming equilibrium. (b) Under egalitarianism, a team will form iff the synergy of production is greater than the difference in worker abilities.*

Proof. (a) Since $g(\alpha, \beta) > \alpha + \beta$, a team forms whenever A and B have identical ability. Thus, equation (1) must hold:

$$E\left[\tfrac{\alpha}{\alpha+\beta} \mid g(\alpha,\beta)\frac{1}{2} > \alpha, g(\alpha,\beta)(1-\frac{1}{2}) > \beta\right] = \frac{1}{2}$$

where the last equality results because α and β are iid and g is symmetric.

(b) Let synergy be measured by $\kappa_{a\beta} = g(\alpha, \beta) - \alpha - \beta > 0$. Then under egalitarianism, a team forms iff $\alpha + \beta + \kappa_{a\beta} > 2\alpha$ and $\alpha + \beta + \kappa_{a\beta} > 2\beta$. This can be rewritten $\kappa_{a\beta} > \alpha - \beta$ and $\kappa_{a\beta} > \beta - \alpha$, which reduces to $\kappa_{a\beta} > |\alpha - \beta|$. ■

Proposition 5 (General) *Discrimination victims produce better average solo work than beneficiaries.*

Proof. Abbreviate $g(\alpha, \beta)$ as g and $1-\gamma$ as $\overline{\gamma}$ for notational simplicity. When no team forms, exactly one worker objects to teamwork because $g > \alpha+\beta$. Therefore, the expectation of a random variable x over the solo sample space can be partitioned as follows:

$$E[x \mid Solo] = E[x \mid g\gamma \leq \alpha]\Pr\{g\gamma \leq \alpha\} + E[x \mid g\overline{\gamma} \leq \beta]\Pr\{g\overline{\gamma} \leq \beta\}$$

The second term can be further partitioned

$$\begin{aligned} E[x|Solo] \;\; = \;\; & E[x \mid g\gamma \leq \alpha]\Pr\{g\gamma \leq \alpha\} + E[x \mid g\gamma \leq \beta, g\overline{\gamma} \leq \beta]\Pr\{g\gamma \leq \beta, g\overline{\gamma} \leq \beta\} \\ & + E[x \mid g\gamma > \beta, g\overline{\gamma} > \alpha, g\overline{\gamma} \leq \beta]\Pr\{g\gamma > \beta, g\overline{\gamma} > \alpha, g\overline{\gamma} \leq \beta\} \\ & + E[x \mid g\gamma > \beta, g\overline{\gamma} \leq \alpha, g\overline{\gamma} \leq \beta]\Pr\{g\gamma > \beta, g\overline{\gamma} \leq \alpha, g\overline{\gamma} \leq \beta\} \end{aligned}$$

Observe (1) $\gamma > \frac{1}{2}$ and $g\gamma \leq \beta$ imply $g\overline{\gamma} \leq \beta$ and (2) $g > \alpha + \beta$ and $g\overline{\gamma} \leq \alpha$ imply $g\gamma > \beta$. Thus, the conditional expected abilities can be simplified as follows:

$$\begin{aligned} E\left[x|Solo\right] &= E\left[x \mid g\gamma \leq \alpha\right] \Pr\left\{g\gamma \leq \alpha\right\} + E\left[x \mid g\gamma \leq \beta\right] \Pr\left\{g\gamma \leq \beta\right\} \\ &+ E\left[x \mid \alpha < g\overline{\gamma} \leq \beta < g\gamma\right] \Pr\left\{\alpha < g\overline{\gamma} \leq \beta < g\gamma\right\} \\ &+ E\left[x \mid g\overline{\gamma} \leq \alpha, g\overline{\gamma} \leq \beta\right] \Pr\left\{g\overline{\gamma} \leq \alpha, g\overline{\gamma} \leq \beta\right\} \end{aligned}$$

Because α and β are iid and g is symmetric with respect to α and β the difference in the expected ability of Bs who work alone from the expected ability of As who work alone is

$$E\left[\alpha \mid Solo\right] - E\left[\beta \mid Solo\right] = E\left[\alpha - \beta \mid \alpha < g\overline{\gamma} \leq \beta < g\gamma\right] \Pr\left\{\alpha < g\overline{\gamma} \leq \beta < g\gamma\right\} < 0$$

■

Definition 3 *Given α and γ, the lowest type B who will (be permitted by A to) join a team is defined by $L(\alpha,\gamma)$ satisfying $g(\alpha, L(\alpha,\gamma))\gamma = \alpha$. Similarly, the highest type B who will (willingly) join a team is defined by $H(\alpha,\gamma)$ satisfying $g(\alpha, H(\alpha,\gamma))(1-\gamma) = H(\alpha,\gamma)$.*

Remark 1 *Observe that as γ approaches 1, any A type will permit any B type to join the team, although none but the lowest B types will be willing to do so. Formally,*

$$\lim_{\gamma \to 1} H(\alpha,\gamma) = \lim_{\gamma \to 1} L(\alpha,\gamma) = 0 \tag{4}$$

Remark 2 *Applying the Implicit Function Theorem to the definitions of $H(\alpha,\gamma)$ and $L(\alpha,\gamma)$ yields*

$$\begin{aligned} H_\gamma(\alpha,\gamma) &= -\frac{\frac{\partial}{\partial\gamma}\left(g(\alpha,\beta)(1-\gamma)-\beta\right)}{\frac{\partial}{\partial\beta}\left(g(\alpha,\beta)(1-\gamma)-\beta\right)}\Big|_{\beta=H(\alpha,\gamma)} = \frac{g(\alpha,H(\alpha,\gamma))}{g_\beta(\alpha,H(\alpha,\gamma))(1-\gamma)-1} \\ L_\gamma(\alpha,\gamma) &= -\frac{\frac{\partial}{\partial\gamma}\left(g(\alpha,\beta)\gamma-\alpha\right)}{\frac{\partial}{\partial\beta}\left(g(\alpha,\beta)\gamma-\alpha\right)}\Big|_{\beta=L(\alpha,\gamma)} = -\frac{g(\alpha,L(\alpha,\gamma))}{g_\beta(\alpha,L(\alpha,\gamma))\gamma} \end{aligned}$$

Definition 4 *Define the expectation of a random variable $\zeta(\alpha,\beta)$ conditional on a team forming given A's ability α and beliefs about discrimination γ*

$$Z(\alpha,\gamma) = E_\beta\left[\zeta(\alpha,\beta) \mid L(\alpha,\gamma) < \beta < H(\alpha,\gamma)\right] = \frac{\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} \zeta(\alpha,\beta)\, dF(\beta)}{\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)} dF(\beta)} \tag{5}$$

where F is distribution of (α and) β. Then by the Quotient Rule

$$Z_{\gamma}(\alpha,\gamma)=\frac{\left(\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)}dF(\beta)\right)\left(\frac{d}{d\gamma}\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)}\zeta(\alpha,\beta)dF(\beta)\right)-\left(\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)}\zeta(\alpha,\beta)dF(\beta)\right)\left(\frac{d}{d\gamma}\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)}dF(\beta)\right)}{\left(\int_{L(\alpha,\gamma)}^{H(\alpha,\gamma)}dF(\beta)\right)^{2}}$$

For fixed α and $0<\gamma<1$ Leibniz Rule may be applied

$$Z_{\gamma}(\alpha,\gamma)=\frac{(\zeta(\alpha,H(\alpha,\gamma))-Z(\alpha,\gamma))H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))-(\zeta(\alpha,L(\alpha,\gamma))-Z(\alpha,\gamma))L_{\gamma}(\alpha,\gamma)F'(L(\alpha,\gamma))}{F(H(\alpha,\gamma))-F(L(\alpha,\gamma))}\tag{6}$$

Lemma 5 *If $Z(\alpha,\gamma)$ is defined as in (5) then as γ approaches unity $Z(\alpha,\gamma)$ converges pointwise*

$$\lim_{\gamma\to 1}Z(\alpha,\gamma)=\zeta(\alpha,0)$$

Proof. Since from (5) both numerator and denominator of $Z(\alpha,\gamma)$ approach 0, apply L'Hôpital's Rule once

$$\lim_{\gamma\to 1}Z(\alpha,\gamma)=\lim_{\gamma\to 1}\frac{\zeta(\alpha,H(\alpha,\gamma))H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))-\zeta(\alpha,L(\alpha,\gamma))L_{\gamma}(\alpha,\gamma)F'(L(\alpha,\gamma))}{H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))-L_{\gamma}(\alpha,\gamma)F'(L(\alpha,\gamma))}$$

From (4) $\lim_{\gamma\to 1}\zeta(\alpha,H(\alpha,\gamma))=\lim_{\gamma\to 1}\zeta(\alpha,L(\alpha,\gamma))=\zeta(\alpha,0)$:

$$\lim_{\gamma\to 1}Z(\alpha,\gamma)=\zeta(\alpha,0)\lim_{\gamma\to 1}\frac{H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))-L_{\gamma}(\alpha,\gamma)F'(L(\alpha,\gamma))}{H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))-L_{\gamma}(\alpha,\gamma)F'(L(\alpha,\gamma))}=\zeta(\alpha,0)$$

■

Lemma 6 *If $Z(\alpha,\gamma)$ is defined as in (5) then as γ approaches unity $Z_{\gamma}(\alpha,\gamma)$ converges pointwise*

$$\lim_{\gamma\to 1}Z_{\gamma}(\alpha,\gamma)=\frac{H_{\gamma}(\alpha,1)+L_{\gamma}(\alpha,1)}{2}\zeta_{\gamma}(\alpha,0)$$

Proof. Observe from (4) and Lemma 5 that the numerator and denominator of (6) both go to 0 as γ approaches 1. Apply L'Hôpital's Rule once. The derivative of the first term of the numerator with respect to γ

$$\begin{aligned}
&\frac{d}{d\gamma}\left(\zeta(\alpha,H(\alpha,\gamma))-Z(\alpha,\gamma)H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))\right)\\
=\;&H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))\frac{d}{d\gamma}\left(\zeta(\alpha,H(\alpha,\gamma))-Z(\alpha,\gamma)\right)\\
&+\left(\zeta(\alpha,H(\alpha,\gamma))-Z(\alpha,\gamma)\right)\frac{d}{d\gamma}H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))\\
=\;&H_{\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))\left(H_{\gamma}(\alpha,\gamma)\zeta_{\gamma}(\alpha,H(\alpha,\gamma))-Z_{\gamma}(\alpha,\gamma)\right)\\
&+\left(\zeta(\alpha,H(\alpha,\gamma))-Z(\alpha,\gamma)\right)\left(H_{\gamma\gamma}(\alpha,\gamma)F'(H(\alpha,\gamma))+H_{\gamma}(\alpha,\gamma)^{2}F''(H(\alpha,\gamma))\right)
\end{aligned}$$

Take the limit as γ approaches 1 and simplify using (4) and Lemma 5

$$\begin{aligned}
&\lim_{\gamma\to 1}\frac{d}{d\gamma}\left(\zeta\left(\alpha,H\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)H_{\gamma}\left(\alpha,\gamma\right)F'\left(H\left(\alpha,\gamma\right)\right)\right)\\
=\;& H_{\gamma}\left(\alpha,1\right)F'\left(0\right)\left(H_{\gamma}\left(\alpha,1\right)\zeta_{\gamma}\left(\alpha,0\right)-\lim_{\gamma\to 1}Z_{\gamma}\left(\alpha,\gamma\right)\right)
\end{aligned}$$

Similarly, the derivative of the numerator's second term with respect to γ as γ approaches 1

$$\begin{aligned}
&\lim_{\gamma\to 1}\frac{d}{d\gamma}\left(\zeta\left(\alpha,L\left(\alpha,\gamma\right)\right)-Z\left(\alpha,\gamma\right)\right)L_{\gamma}\left(\alpha,\gamma\right)F'\left(L\left(\alpha,\gamma\right)\right)\\
=\;& L_{\gamma}\left(\alpha,1\right)F'\left(0\right)\left(L_{\gamma}\left(\alpha,1\right)\zeta_{\gamma}\left(\alpha,0\right)-\lim_{\gamma\to 1}Z_{\gamma}\left(\alpha,\gamma\right)\right)
\end{aligned}$$

The derivative of the denominator with respect to γ as γ approaches 1

$$\lim_{\gamma\to 1}\frac{d}{d\gamma}\left(F\left(H\left(\alpha,\gamma\right)\right)-F\left(L\left(\alpha,\gamma\right)\right)\right)=H_{\gamma}\left(\alpha,1\right)F'\left(0\right)-L_{\gamma}\left(\alpha,1\right)F'\left(0\right)$$

Thus,

$$\begin{aligned}
\lim_{\gamma\to 1}Z_{\gamma}\left(\alpha,\gamma\right) &= \frac{H_{\gamma}(\alpha,1)\left(H_{\gamma}(\alpha,1)\zeta_{\gamma}(\alpha,0)-\lim_{\gamma\to 1}Z_{\gamma}(\alpha,\gamma)\right)-L_{\gamma}(\alpha,1)\left(L_{\gamma}(\alpha,1)\zeta_{\gamma}(\alpha,0)-\lim_{\gamma\to 1}Z_{\gamma}(\alpha,\gamma)\right)}{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}\\
&= \frac{H_{\gamma}(\alpha,1)H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)L_{\gamma}(\alpha,1)}{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}\zeta_{\gamma}\left(\alpha,0\right)-\frac{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}{H_{\gamma}(\alpha,1)-L_{\gamma}(\alpha,1)}\lim_{\gamma\to 1}Z_{\gamma}\left(\alpha,\gamma\right)\\
&= \frac{H_{\gamma}\left(\alpha,1\right)+L_{\gamma}\left(\alpha,1\right)}{2}\zeta_{\gamma}\left(\alpha,0\right)
\end{aligned}$$

■

Remark 3 *Since $g_{\beta}\left(\alpha,\beta\right)>1$ and $H_{\gamma}\left(\alpha,\gamma\right)<0$ for all γ satisfying $g_{\beta}\left(\alpha,H\left(\alpha,\gamma\right)\right)\left(1-\gamma\right)<1$, there exists some γ^{ε} for all $\varepsilon>0$ such that for all $\gamma^{\varepsilon}\leq\gamma\leq 1$ the following holds: $g_{\beta}\left(\alpha,H\left(\alpha,\gamma\right)\right)\left(1-\gamma\right)<\varepsilon$.*

Remark 4 *Since F has finite variance, $H\left(\alpha,\gamma\right)>L\left(\alpha,\gamma\right)>0$ and $\lim_{\gamma\to 1}H\left(\alpha,\gamma\right)=\lim_{\gamma\to 1}L\left(\alpha,\gamma\right)=0$, there exists some γ^{ε} for all $\varepsilon>0$ such that for all $\gamma^{\varepsilon}\leq\gamma\leq 1$ the following holds: $\overline{F}'\left(\gamma\right)-\underline{F}'\left(\gamma\right)<\varepsilon$, where*

$$\begin{aligned}
\overline{F}\left(\gamma\right)' &= \max\left\{F'(x):x\in\left(0,H\left(\alpha,\gamma\right)\right)\right\}\\
\underline{F}'\left(\gamma\right) &= \min\left\{F'(x):x\in\left(0,H\left(\alpha,\gamma\right)\right)\right\}
\end{aligned}$$

Definition 5 *Define $\hat{\gamma}$ such that for all $\gamma>\hat{\gamma}$ (1) $g_{\beta}\left(\alpha,H\left(\alpha,\gamma\right)\right)\left(1-\gamma\right)<\frac{1}{2}$ and (2) $\frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})}<2$. From Remarks 3 and 4 such a $\hat{\gamma}$ always exists.*

Lemma 7 *If $Z(\alpha,\gamma)$ is defined as in (5), $\zeta_\alpha(\alpha,\beta) > 0$ and $\zeta_\alpha(\alpha,\beta) > 0$ then for all $\gamma > \hat{\gamma}$, $Z_\gamma(\alpha,\gamma)$ is bounded as follows:*

$$0 < Z_\gamma(\alpha,\gamma) \leq \frac{\zeta(\alpha, L(\alpha,\gamma)) - \zeta(\alpha, H(\alpha,\gamma))}{H(\alpha,\gamma) - L(\alpha,\gamma)} \left(-H_\gamma(\alpha,\gamma) - L_\gamma(\alpha,\gamma)\right) \frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})} \tag{7}$$

Proof. From (6)

$$Z_\gamma(\alpha,\gamma) = \frac{(Z(\alpha,\gamma)-\zeta(\alpha,H(\alpha,\gamma)))(-H_\gamma(\alpha,\gamma))F'(H(\alpha,\gamma))+(\zeta(\alpha,L(\alpha,\gamma))-Z(\alpha,\gamma))(-L_\gamma(\alpha,\gamma))F'(L(\alpha,\gamma))}{F(H(\alpha,\gamma))-F(L(\alpha,\gamma))} > 0$$

Observe that $\zeta(\alpha, H(\alpha,\gamma)) \leq Z(\alpha,\gamma) \leq \zeta(\alpha, L(\alpha,\gamma))$ and every factor in the numerator and the denominator are always positive for all $\gamma > \hat{\gamma}$. Thus,

$$Z_\gamma(\alpha,\gamma) \leq \frac{\zeta(\alpha, L(\alpha,\gamma)) - \zeta(\alpha, H(\alpha,\gamma))}{F(H(\alpha,\gamma)) - F(L(\alpha,\gamma))} \left(-H_\gamma(\alpha,\gamma) F'(H(\alpha,\gamma)) - L_\gamma(\alpha,\gamma) F'(L(\alpha,\gamma))\right)$$

The form of the lemma results from applying the Mean Value Theorem to the denominator and bounding the F' terms in both numerator and denominator. ■

Corollary 1 *If $Z(\alpha,\gamma)$ is defined as in (5) and $\zeta(\alpha,\beta) = \frac{\alpha}{\alpha+\beta}$, then for all $\gamma > \hat{\gamma}$, $Z_\gamma(\alpha,\gamma)$ is bounded by the following Lebesgue integrable function:*

$$|Z_\gamma(\alpha,\gamma)| \leq \frac{1}{\alpha}\left(2g(\alpha, H(\alpha,\hat{\gamma})) + \frac{g(\alpha, L(\alpha,\hat{\gamma}))}{\hat{\gamma}}\right) \frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})} = \Theta(\alpha) \tag{8}$$

Proof. Substitute $\zeta(\alpha,\beta) = \frac{\alpha}{\alpha+\beta}$, $H_\gamma(\alpha,\gamma)$ and $L_\gamma(\alpha,\gamma)$ from Lemma 2

$$Z_\gamma(\alpha,\gamma) \leq \frac{\frac{\alpha}{\alpha+L(\alpha,\gamma)} - \frac{\alpha}{\alpha+H(\alpha,\gamma)}}{H(\alpha,\gamma) - L(\alpha,\gamma)} \left(\frac{g(\alpha, H(\alpha,\gamma))}{1 - g_\beta(\alpha, H(\alpha,\gamma))(1-\gamma)} + \frac{g(\alpha, L(\alpha,\gamma))}{g_\beta(\alpha, L(\alpha,\gamma))\,\gamma}\right) \frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})}$$

Simplifying the first factor and then using the facts that $H(\alpha,\gamma) > L(\alpha,\gamma) > 0$, $1 - g_\beta(\alpha, H(\alpha,\gamma))(1-\gamma) \leq \frac{1}{2}$, $g_\beta(\alpha,\beta) > 1$ to bound each factor yields the form of the corollary. ■

Lemma 8 *If ability is continuously distributed with support from 0, then γ*

$$\lim_{\gamma\to 1} \psi(\gamma) = 1$$

Proof. Define $Z(\alpha,\gamma)$ as in (5) where $\zeta(\alpha,\beta) = \frac{\alpha}{\alpha+\beta}$. Then

$$\lim_{\gamma\to 1} \psi(\gamma) = \lim_{\gamma\to 1} \int_{-\infty}^{\infty} Z(\alpha,\gamma)\, dF(\alpha)$$

From Lemma 5 $Z(\alpha,\gamma)$ converges pointwise to $\frac{\alpha}{\alpha+0}=1$ for all α, and $Z(\alpha,\gamma)$ is dominated by 1 (i.e. $|Z(\alpha,\gamma)|\leq 1$). Thus, by Lebesgue's Dominated Convergence Theorem

$$\lim_{\gamma\to 1}\psi(\gamma)=\int_{-\infty}^{\infty}1dF(\alpha)=1$$

■

Lemma 9 *If team production is regular and ability is continuously distributed with support from* 0*, then*

$$\lim_{\gamma\to 1}\psi'(\gamma)<1$$

Proof. Define $Z(\alpha,\gamma)$ as in (5) where $\zeta(\alpha,\beta)=\frac{\alpha}{\alpha+\beta}$. Then (1) $Z(\alpha,\gamma)$ is a Lebesgue integrable function for all $\gamma\in(\hat{\gamma},1)$, (2) for almost all α, $Z_\gamma(\alpha,\gamma)$ exists for all $\gamma\in(\hat{\gamma},1)$ and (3) by Lemma 1 $Z_\gamma(\alpha,\gamma)$ is dominated by $\Theta(\alpha)$ as defined in (8) for all $\gamma\in(\hat{\gamma},1)$. Thus, by Leibniz' Rule (see Folland 1999, Theorem 2.27.b for the measure theory version)

$$\lim_{\gamma\to 1}\psi'(\gamma)=\lim_{\gamma\to 1}\int_{-\infty}^{\infty}Z_\gamma(\alpha,\gamma)\,dF(\alpha)$$

From Lemma 6 $Z_\gamma(\alpha,\gamma)$ converges pointwise to

$$-\frac{H_\gamma(\alpha,1)+L_\gamma(\alpha,1)}{2\alpha}$$

for all α, and from Corollary 1 $Z_\gamma(\alpha,\gamma)$ is dominated by $\frac{1}{\alpha}\left(2g(\alpha,H(\alpha,\hat{\gamma}))+\frac{g(\alpha,L(\alpha,\hat{\gamma}))}{\hat{\gamma}}\right)\frac{\overline{F}'(\hat{\gamma})}{\underline{F}'(\hat{\gamma})}$ for all $\gamma>\hat{\gamma}$. Thus, by Lebesgue's Dominated Convergence Theorem

$$\lim_{\gamma\to 1}\psi'(\gamma)=-\int_{-\infty}^{\infty}\frac{H_\gamma(\alpha,1)+L_\gamma(\alpha,1)}{2\alpha}dF(\alpha)\tag{9}$$

From Remark 2 $H_\gamma(\alpha,1)=-g(\alpha,0)$ whenever $H(\alpha,\gamma)$ is interior. Observe $H(\alpha,\gamma)$ is always interior when γ is near unity, since $g\left(\alpha,\alpha\frac{1-\gamma}{\gamma}\right)(1-\gamma)\geq\left(\alpha+\alpha\frac{1-\gamma}{\gamma}\right)(1-\gamma)=\alpha\frac{1-\gamma}{\gamma}$. Similarly $L_\gamma(\alpha,1)=-\frac{g(\alpha,0)}{g_\beta(\alpha,0)}$ whenever $L(\alpha,\gamma)$ is interior.
Case $L(\alpha,1)$ is interior: Simplify (9)

$$\lim_{\gamma\to 1}\psi'(\gamma)=-\int_{-\infty}^{\infty}\frac{-g(\alpha,0)+-\frac{g(\alpha,0)}{g_\beta(\alpha,0)}}{2\alpha}dF(\alpha)=\frac{1}{2}\int_{-\infty}^{\infty}\left(1+\frac{1}{g_\beta(\alpha,0)}\right)dF(\alpha)<1$$

where the second equality follows because $g(\alpha,0)=\alpha$ by the regularity assumption and the inequality follows because $g_\beta(\alpha,\beta)>1$ for all α and β.

Case $L(\alpha, 1)$ is not interior: Simplify (9) using $H_\gamma(\alpha, 1) = -g(\alpha, 0)$ and $L_\gamma(\alpha, 1) = 0$ (because $L(\alpha, 1)$ is not interior).

$$\lim_{\gamma \to 1} \psi'(\gamma) = -\int_{-\infty}^{\infty} \frac{-g(\alpha, 0)}{2\alpha} dF(\alpha) = \frac{1}{2} \int_{-\infty}^{\infty} dF(\alpha) = \frac{1}{2} < 1$$

where the last equality follows because $g(\alpha, 0) = \alpha$ by the regularity assumption. ■

Lemma 10

$$\psi'\left(\frac{1}{2}\right) = 0$$

Proof. Rotate the ability sample space Ω by angle $-\frac{\pi}{4}$:

$$\begin{aligned} \widetilde{\alpha} &= \alpha \cos\left(-\tfrac{\pi}{4}\right) - \beta \sin\left(-\tfrac{\pi}{4}\right) = \tfrac{\alpha+\beta}{\sqrt{2}} \\ \widetilde{\beta} &= \alpha \sin\left(-\tfrac{\pi}{4}\right) + \beta \cos\left(-\tfrac{\pi}{4}\right) = \tfrac{-\alpha+\beta}{\sqrt{2}} \end{aligned}$$

This yields $\alpha = \frac{\widetilde{\alpha}-\widetilde{\beta}}{\sqrt{2}}$, $\beta = \frac{\widetilde{\alpha}+\widetilde{\beta}}{\sqrt{2}}$ and $\widetilde{\zeta}\left(\widetilde{\alpha}, \widetilde{\beta}\right) = \frac{\widetilde{\alpha}-\widetilde{\beta}}{2\widetilde{\alpha}} = \frac{\alpha}{\alpha+\beta}$. Thus $\widetilde{Z}\left(\alpha, \frac{1}{2}\right) = \frac{1}{2}$, $\widetilde{\zeta}\left(\widetilde{\alpha}, \widetilde{\beta}\right) - \widetilde{Z}\left(\alpha, \frac{1}{2}\right) = \frac{-\widetilde{\beta}}{2\widetilde{\alpha}}$ and from (6)

$$\widetilde{Z}_\gamma\left(\alpha, \frac{1}{2}\right) = \frac{\frac{-\widetilde{H}\left(\widetilde{\alpha}, \frac{1}{2}\right)}{2\widetilde{\alpha}} \widetilde{H}_\gamma\left(\widetilde{\alpha}, \frac{1}{2}\right) \widetilde{F}'\left(\widetilde{H}\left(\widetilde{\alpha}, \frac{1}{2}\right)\right) - \frac{-\widetilde{L}\left(\widetilde{\alpha}, \frac{1}{2}\right)}{2\widetilde{\alpha}} \widetilde{L}_\gamma\left(\widetilde{\alpha}, \frac{1}{2}\right) \widetilde{F}'\left(\widetilde{L}\left(\widetilde{\alpha}, \frac{1}{2}\right)\right)}{\widetilde{F}\left(\widetilde{H}\left(\widetilde{\alpha}, \frac{1}{2}\right)\right) - \widetilde{F}\left(\widetilde{L}\left(\widetilde{\alpha}, \frac{1}{2}\right)\right)}$$

Observe that when $\gamma = \frac{1}{2}$ the team forming region (i.e. $\Omega \ni g\left(\widetilde{\alpha}, \widetilde{\beta}\right) \gamma \geq \widetilde{\alpha}$ and $g\left(\widetilde{\alpha}, \widetilde{\beta}\right)(1-\gamma) \geq \widetilde{\beta}$) is symmetric about the $\widetilde{\alpha}$-axis. Thus $-\widetilde{L}\left(\widetilde{\alpha}, \frac{1}{2}\right) = \widetilde{H}\left(\widetilde{\alpha}, \frac{1}{2}\right)$, $\widetilde{f}\left(\widetilde{\alpha}, \widetilde{L}\left(\widetilde{\alpha}, \frac{1}{2}\right)\right) = \widetilde{f}\left(\widetilde{\alpha}, \widetilde{H}\left(\widetilde{\alpha}, \frac{1}{2}\right)\right)$ and $-\widetilde{L}_\gamma\left(\widetilde{\alpha}, \frac{1}{2}\right) = \widetilde{H}_\gamma\left(\widetilde{\alpha}, \frac{1}{2}\right)$. Thus, $\widetilde{Z}_\gamma\left(\widetilde{\alpha}, \frac{1}{2}\right) = 0$. By Leibniz' Rule, then

$$\psi'\left(\frac{1}{2}\right) = \int_{-\infty}^{\infty} \widetilde{Z}_\gamma\left(\widetilde{\alpha}, \frac{1}{2}\right) dF(\widetilde{\alpha}) = 0$$

■

Theorem 1 (Fixed Point) *If ψ is continuous and differentiable at distinct fixed points a and c, and $sign(1-\psi'(a)) = sign(1-\psi'(c))$ then there exists another fixed point b strictly between a and c.*

Proof. Define $\chi(\gamma) = \gamma - \psi(\gamma)$. Then $\chi(\gamma) = 0 \iff \gamma = \psi(\gamma)$. $\exists \delta > 0 \ni \forall 0 < \varepsilon < \delta, sign(\chi(a+\varepsilon)) = sign(1-\psi'(c)), sign(\chi(c-\varepsilon)) = -sign(1-\psi'(c))$. Thus if

$sign\,(1-\psi'(a)) = sign\,(1-\psi'(c))$, then 0 lies between $\chi(a+\varepsilon)$ and $\chi(c-\varepsilon)$. Then by the Intermediate Value Theorem there exists $b \in (a+\varepsilon, c-\varepsilon)$ such that $\chi(b)=0$. ■

Proposition 8 *If team production is regular, continuous and ability is continuously distributed with support from* 0*, then at least one discriminatory equilibrium* ($\gamma \in \left(\frac{1}{2},1\right)$) *exists in which teams form.*

Proof. $\gamma = \frac{1}{2}$ is always an equilibrium by Proposition 1 (General). $\lim_{\gamma\to 1}\psi(\gamma) = 1$ by Lemma 8. $\lim_{\gamma\to 1}\psi'(\gamma) < 1$ by Lemma 9. $\psi'\left(\frac{1}{2}\right) = 0 < 1$ by Lemma 10. Thus, $sign\left(1-\psi'\left(\frac{1}{2}\right)\right) = " - " = sign\,(1-\psi'(1))$. Theorem 1 implies $\psi(\gamma)$ has a fixed point in $\left(\frac{1}{2},1\right)$. ■

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