

Measuring the Gains from Trade under Monopolistic Competition

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Abstract

Three sources of gains from trade under monopolistic competition are: (i) new import varieties available to consumers; (ii) enhanced efficiency as more productive firms begin exporting and less productive firms exit; (iii) reduced markups charged by firms due to import competition. We show how each of these gains can be measured. The first two sources of gains are analogous to “new goods” in a CES utility function for consumers or a constant-elasticity transformation curve for the economy, respectively. Alternatively, the first and third sources of gain can be measured using a translog expenditure function for consumers, which in contrast to the CES case, allows for finite reservation prices for new goods and endogenous markups.

1. Introduction

One of the great achievements of international trade theory in the last three decades is the incorporation of the monopolistic competition model. The need to include increasing returns to scale in trade theory was recognized as early as Graham (1923; see also Ethier 1982), and in the Canadian context, by Eastman and Stykolt (1967) and Melvin (1969). Still, it was not until the formalization of the monopolistic competition model by Dixit and Stiglitz (1977), in parallel with Spence (1976) and Lancaster (1979), that a set of global equilibrium conditions that avoided the problems of large firms and multiple equilibria could be developed.¹ That set of equilibrium conditions was first written down by Krugman (1979, 1980, 1981).²

There is no doubt that these developments have had important policy implications. For example, the simulation results of Harris (1984a,b) demonstrated large gains to Canada from free trade with the U.S., and were very influential in convincing policy makers to proceed with the Canada-U.S. free trade agreement in 1989; that agreement in turn paved the way for the North American free trade agreement in 1994. Subsequent empirical work for Canada by Trefler (2004), as well as Head and Ries (1999, 2001), confirmed the efficiency gains for Canada due to opening trade, though not in the manner predicted by Krugman's work. But a comprehensive empirical assessment of the gains from trade under monopolistic competition has not yet been made. The goal of this paper is to describe how these gains can be measured, using methods that draw heavily on duality theory from Diewert (1974, 1976).

The monopolistic competition model predicts three sources of gains from trade that are

¹ The multiple equilibria problem due to increasing returns was described by Chipman (1965). Ethier (1979) pointed out that this is avoided in a model with differentiated intermediate inputs, leading to a form of external (or "international") economies of scale. More recently, Grossman and Rossi-Hansberg (2008) argue that a careful specification of the market structure allows multiple equilibria to be avoided even with external economies.

² See also the early contributions of Dixit and Norman (1980, chapter 9), Lancaster (1980) and Helpman (1981); these various approaches were integrated by Helpman and Krugman (1985).

not present in traditional models. First, there are the consumer gains from having access to new import varieties of differentiated products. Those gains have recently been measured for the United States by Broda and Weinstein (2006), using the methods from Feenstra (1994), as described in section 2. Their approach assumes a constant elasticity of substitution (CES) utility function for consumers, in which case the import varieties are analogous to “new goods” in the utility function. We show that the gains from trade depend on the import share and the elasticity of substitution.

The extension of the monopolistic competition model to allow for heterogeneous firms, due to Melitz (2003), leads to a second source of gains from the self-selection of more efficient firms into export markets. This activity drives out less efficient firms and therefore raises overall productivity. We argue that this self-selection can still be interpreted as a gain from product variety, but now on the *export* side of the economy rather than for imports. Surprisingly, the consumer gains from new import varieties do not appear in this case, because they cancel out with disappearing domestic varieties. This finding, demonstrated in section 3, helps to explain the theoretical results of Arkolakis *et al* (2008a), where the gains from trade depend on the import share but are otherwise *independent* of the elasticity of substitution in consumption. Rather, the gains come from the production side of the economy, where the self-selection of firms leads to a constant-elasticity transformation curve between domestic and export varieties, with an elasticity depending on the Pareto parameter of productivity draws.

Third, the monopolistic competition model also allows for gains from a reduction in firm markups due to import competition. This third source of gains was stressed in Krugman (1979), but has been absent from much of the later literature due to the assumption of CES preferences, leading to constant markups. In section 4, I summarize my current research with David

Weinstein (Feenstra and Weinstein, 2009), that shows how a translog expenditure function leads to tractable formulas for the gains from product variety and the pro-competitive effect of imports on reducing markups. Conclusions are given in section 5.

2. Consumer Benefits from Import Variety

We start with the consumer gains from import variety. From a technical point of view, measuring the benefits of new import varieties is equivalent to the so-called “new goods” problem in index number theory. That has always been a favorite problem of Erwin Diewert’s, and arises because the price for a product before it is available is not observed, so we don’t know what price to enter in an index number formula. The answer given many years ago by Hicks (1940) was that the relevant price of a product before it is available is the “reservation price” for consumers, namely, a price so high that demand is zero. Once the product appears on the market then it has a lower price, determined by supply and demand. The fall in the price from its “reservation” level to the actual price can be used in an index number formula to obtain the consumer gains from the appearance of that new good.

For the constant elasticity of substitution (CES) utility function, we immediately run into a problem with implementing this suggestion because the reservation price for any good is infinite: the demand curve approaches the vertical axis as the price approaches infinity.³ But provided that the elasticity of substitution is greater than unity, then the area under the demand curve is bounded above, as shown in Figure 1, where the ratio of areas $A/B = 1/(\sigma - 1)$ is easily calculated for a demand curve with elasticity σ . The second problem we run into is how to express these consumer gains when there is not just one but many new goods available.

³ In Feenstra (2006), we show that an infinite reservation price leads to a well-behaved limit for the “quadratic mean of order r ” index number formula of Diewert (1976), providing an alternative proof of Theorem 1 below.

CES Utility Function

To address this problem, we will work with the non-symmetric CES function,

$$U_t = U(q_t, I_t) = \left[\sum_{i \in I_t} a_{it} q_{it}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1, \quad (1)$$

where $a_{it} > 0$ are tastes parameters that can change over time, and I_t denotes the set of goods available in period t at the prices p_{it} . The minimum expenditure to obtain one unit of utility is,

$$e(p_t, I_t) = \left[\sum_{i \in I_t} b_{it} p_{it}^{1-\sigma} \right]^{1/(1-\sigma)}, \quad \sigma > 1, \quad b_{it} \equiv a_{it}^\sigma. \quad (2)$$

For simplicity, first consider the case where $I_{t-1} = I_t = I$, so there is no change in the set of goods, and also $b_{it-1} = b_{it}$, so there is no change in tastes. We assume that the observed purchases q_{it} are optimal for the prices and utility, that is, $q_{it} = U_t(\partial e / \partial p_{it})$. Then the index number due to Sato (1976) and Vartia (1976) shows us how to measure the ratio of unit-expenditures:

Theorem 1 (Sato, 1976; Vartia, 1976)

If the set of goods available is fixed at $I_{t-1} = I_t = I$, taste parameters are constant, $b_{it-1} = b_{it}$, and observed quantities are optimal, then:

$$\frac{e(p_t, I)}{e(p_{t-1}, I)} = P_{SV}(p_{t-1}, p_t, q_{t-1}, q_t, I) \equiv \prod_{i \in I} \left(\frac{p_{it}}{p_{it-1}} \right)^{w_i(I)}, \quad (3)$$

where the weights $w_i(I)$ are constructed from the expenditure shares $s_{it}(I) \equiv p_{it} q_{it} / \sum_{i \in I} p_{it} q_{it}$ as,

$$w_i(I) \equiv \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right) / \sum_{i \in I} \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right). \quad (4)$$

The numerator in (4) is the “logarithmic mean” of the shares $s_{it}(\mathbf{I})$ and $s_{it-1}(\mathbf{I})$, and lies in-between these two shares, while the denominator ensures that the weights $w_i(\mathbf{I})$ sum to unity. The special formula for these weights in (4) is needed to precisely measure the ratio of unit-expenditures in (3), but in practice the Sato-Vartia formula will give very similar results to using other weights, such as $w_i(\mathbf{I}) = \frac{1}{2}[s_{it}(\mathbf{I}) + s_{it-1}(\mathbf{I})]$, as used for the Törnqvist price index. In both cases, the geometric mean formula in (3) applies. The important point from Theorem 1 is that goods with high taste parameters a_i will also tend to have high weights, so even without knowing the true values of a_i , the exact ratio of unit-expenditures is obtained.

Now consider the case where the set of goods is changing over time, but some of the goods are available in both periods, so that $I_{t-1} \cap I_t \neq \emptyset$. We again let $e(\mathbf{p}, \mathbf{I})$ denote the unit-expenditure function defined over the goods within the set \mathbf{I} , which is a non-empty subset of those goods available both periods, $\mathbf{I} \subseteq I_{t-1} \cap I_t \neq \emptyset$. We sometimes refer to the set \mathbf{I} as the “common set of goods.” Then the ratio $e(\mathbf{p}_t, \mathbf{I})/e(\mathbf{p}_{t-1}, \mathbf{I})$ is still measured by the Sato-Vartia index in the above theorem. Our interest is in the ratio $e(\mathbf{p}_t, I_t)/e(\mathbf{p}_{t-1}, I_{t-1})$, which can be measured as follows:

Theorem 2 (Feenstra, 1994)

Assume that $b_{it-1} = b_{it}$ for $i \in \mathbf{I} \subseteq I_{t-1} \cap I_t \neq \emptyset$, and that the observed quantities are optimal.

Then for $\sigma > 1$:

$$\frac{e(\mathbf{p}_t, I_t)}{e(\mathbf{p}_{t-1}, I_{t-1})} = P_{SV}(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{q}_{t-1}, \mathbf{q}_t, \mathbf{I}) \left(\frac{\lambda_t(\mathbf{I})}{\lambda_{t-1}(\mathbf{I})} \right)^{1/(\sigma-1)}, \quad (5)$$

where the weights $w_i(\mathbf{I})$ are constructed from the expenditure shares $s_{it}(\mathbf{I}) \equiv p_{it}q_{it} / \sum_{i \in \mathbf{I}} p_{it}q_{it}$ as

in (4), and the values $\lambda_t(\mathbf{I})$ and $\lambda_{t-1}(\mathbf{I})$ are constructed as:

$$\lambda_{\tau}(I) = \left(\frac{\sum_{i \in I} p_{i\tau} q_{i\tau}}{\sum_{i \in I_{\tau}} p_{i\tau} q_{i\tau}} \right) = 1 - \left(\frac{\sum_{i \in I_{\tau}, i \notin I} p_{i\tau} q_{i\tau}}{\sum_{i \in I_{\tau}} p_{i\tau} q_{i\tau}} \right), \quad \tau = t-1, t. \quad (6)$$

Each of the terms $\lambda_{\tau}(I) \leq 1$ can be interpreted as the *period τ expenditure on the good in the common set I , relative to the period τ total expenditure*. Alternatively, this can be interpreted as *one minus the period τ expenditure on “new” goods (not in the set I), relative to the period τ total expenditure*. When there is a greater number of new goods in period t , this will tend to lower the value of $\lambda_t(I)$, which leads to a greater fall in the ratio of unit costs in (5), by an amount that depends on the elasticity of substitution.

The importance of the elasticity of substitution can be seen from Figure 2, where we suppose that the consumer minimizes the expenditure needed to obtain utility along the indifference curve AD. If initially only good 1 is available, then the consumer chooses point A with the budget line AB. When good 2 becomes available, the same level of utility can be obtained with consumption at point C. Then the drop in the cost of living is measured by the inward movement of the budget line from AB to the line through C, and this shift depends on the convexity of the indifference curve, or the elasticity of substitution.

Krugman (1980) Model

Turning to the international trade application, we will suppose that the utility function in (1) applies to the purchases of a good from various source countries $i \in I_t$. That is, the elasticity of substitution we are interested in is the Armington (1969) elasticity between the source countries for imports. We refer to the source countries as providing *varieties* of the differentiated

good, so the gains being measured in (5) are the gains from import variety. In this case, we can compare the formula in (5) with the gain from trade obtained in the model of Krugman (1980), as analyzed by Arkolakis *et al* (2008a).

In particular, suppose there are any number of countries, where the representative consumer in each has a CES utility function with elasticity $\sigma > 1$. Labor is the only factor of production and there is a single monopolistically competitive sector, with no other goods.⁴ Firms face a fixed cost of f to manufacture any good, and an iceberg transport cost to sell it abroad, but no other fixed cost for exports. Then it is well known that with profit-maximization and zero profits through free entry, the output of each firm is fixed at the amount:⁵

$$q = (\sigma - 1)f\varphi, \quad (7)$$

where φ is the productivity of the firm, i.e. the number of units of output per unit of labor. With the population of L , the full-employment condition is then:

$$L = N[(q/\varphi) + f] = N\sigma f, \quad (8)$$

which determines the number of product varieties produced in equilibrium as $N = L/\sigma f$. This condition holds under autarky or trade, so opening a country to trade has *no impact* on the number of varieties produced within a country.

The gains from opening trade can be measured by the ratio of real wages under free trade and autarky. With labor as the only factor of production we can normalize wages at unity, so the gains from trade are simply measured by the drop in the cost of living, which is the inverse of (5). The “common” set of goods are those domestic varieties that are available both in autarky and under trade. Then the Sato-Vartia index P_{SV} is just the change in the price of the domestic

⁴ In particular, we are ruling out the additively-separable numeraire good sometimes introduced into this model to obtain a “home market” effect; see Krugman (1980).

⁵ See Arkolakis *et al* (2008a, p. 3).

varieties, and with constant markups that is the change in home wages, which we have normalized to unity. So the gains from trade are measured by $(\lambda_t / \lambda_{t-1})^{-1/(\sigma-1)}$ in (5). The denominator of that ratio reflects the disappearance of domestic varieties, i.e. those varieties available in period $t-1$ but not in period t . As we have shown above, there are *no disappearing domestic varieties* in this model, so $\lambda_{t-1} = 1$. The numerator λ_t measures *the expenditure on the domestic varieties relative to total expenditure with trade, or one minus the import share*. The gains from trade are therefore $\lambda_t^{-1/(\sigma-1)}$, which is precisely the formula obtained by Arkolakis *et al* (2008b). While this formula is not too surprising, it will take on greater significance when we compare it to the results from the Melitz (2003) model, in the next section.

Broda and Weinstein (2006) measure these gains from trade for the U.S. They define a *good* as a 10-digit Harmonized System (HS) category, or before 1989, as a 7-digit Tariff Schedule of the United States (TSUSA) category. The imports from various source countries are the *varieties* available for each good. The ratio $(\lambda_t / \lambda_{t-1})$ is constructed for each good, using the expenditure on new and disappearing source countries. In addition, they estimate σ for each good, using the GMM method from Feenstra (1994), which exploits heteroskedasticity across countries to identify this elasticity. Putting these together, they measure $(\lambda_t / \lambda_{t-1})^{-1/(\sigma-1)}$ for 30,000 goods available in the HS and TSUSA data. For the TSUSA data they used 1972 as the base year and measured the gains from new supply countries up to 1988, and then for the HS data they used 1990 as the base year and measured the gains from new supplying countries up to 2001.⁶ Aggregating over goods, they obtain an estimate of the gains from trade for the US due to the expansion of import varieties, which amount to 2.6% of GDP in 2001.

⁶ 1989 is omitted because West and East Germany unified then, making comparisons with later years difficult.

Two features of Broda and Weinstein’s methods deserve special mention. First, by measuring the expenditure on new supplying countries relative to a base year, they are following the hypothesis of Theorem 2 that the “common” set of countries should be those with *constant* taste parameters. In contrast, when countries first start exporting goods, it is reasonable to expect that the demand curve in the importing country shifts out over some number of years, as consumers become informed about the product. Broda and Weinstein are allowing for such shifts for new and disappearing countries after the base year, and all such changes in demand for these countries are incorporated into the λ_τ terms in Theorem 2. That is the correct way to measure the gains from new import varieties.⁷

Second, Broda and Weinstein (2006) did not incorporate any changes in the number of U.S. varieties into their estimation, nor include the U.S. as a source country in the estimation of the elasticity of substitution for each good. That is the correct approach only under the limited case where *the number of U.S. varieties is constant*. While that is true under our assumptions in the model of Krugman (1980), it is certainly not the case in more general models: we could expect that increases in import variety would result in some reduction in domestic varieties. In that case, the gains from import varieties would be offset by the welfare loss from reduced domestic varieties. That potential loss was not addressed by Broda and Weinstein (2006), and we shall begin to address it in the remainder of the paper.

3. Producer Benefits from Output Variety

While we have so far restricted our attention to $\sigma > 1$ in the utility and expenditure functions (1) and (2), a wider range of values for this elasticity can be considered. In particular, if

⁷ In addition, countries that are suspected of selling a changing range of product varieties *within* each HS good should be excluded from the set I, and instead included in the λ_τ terms.

$\sigma < 0$ then instead of obtaining convex indifference curves from (1) for a fixed level of U_t , we obtain a concave transformation curve as shown in Figure 3.⁸ The parameter U_t in this case measures the resources devoted to production of the goods q_{it} , $i \in I_t$, and the elasticity of the transformation curve (measured as a positive number) equals $-\sigma$. This reinterpretation of (1) comes from Diewert (1976), who uses the general term “aggregator function” to refer to utility functions, production function, or transformation functions for an economy.

To make this reinterpretation explicit, when $\sigma < 0$ we will denote its positive value by $\omega \equiv -\sigma$, which is the elasticity of transformation. Then we will rewrite (1) using labor resources L_t to replace utility U_t , obtaining:

$$L_t = \left(\sum_{i \in I_t} a_{it} q_{it}^{(\omega+1)/\omega} \right)^{\omega/(\omega+1)}, \quad a_{it} > 0, \quad \omega > 0. \quad (9)$$

The maximum revenue obtained using one unit of labor resources, dual to (9), is then:

$$e(p_t, I_t) = \left[\sum_{i \in I_t} b_{it} p_{it}^{\omega+1} \right]^{1/(\omega+1)}, \quad b_{it} \equiv a_{it}^{-\omega}, \quad \omega > 0. \quad (10)$$

With this reinterpretation, Theorem 2 continues to hold as:

$$\frac{e(p_t, I_t)}{e(p_{t-1}, I_{t-1})} = P_{SV}(p_{t-1}, p_t, q_{t-1}, q_t, I) \left(\frac{\lambda_t(I)}{\lambda_{t-1}(I)} \right)^{-1/(\omega+1)}, \quad (11)$$

where the exponent appearing on $(\lambda_t/\lambda_{t-1})$ is now negative. In other words, the appearance of “new outputs,” so that $\lambda_t < 1$, will *raise revenue* on the producer side of the economy.

To understand where this increase in revenue is coming from, consider the transformation

⁸ Notice that the range $0 \leq \sigma \leq 1$ cannot be considered, since then all goods are *essential* in (1), with a zero quantity for any single good resulting in zero for the entire CES aggregate. In that case the welfare gain from a new good is infinite.

curve in Figure 3. If only good 1 is available, then the economy would be producing at the corner A, with revenue shown by the line AB. Then if good 2 becomes available to producers, the new equilibrium will be at point C, with an *increase* in revenue. This illustrates the benefits of output variety. In Figure 4 we illustrate the same idea in a partial equilibrium diagram, for a supply curve with constant elasticity ω . When the good becomes available for production, there is an effective price *increase* from the reservation price for producers (which is zero with a constant-elasticity supply curve) to the actual price. The gain in producer surplus is area C, and measured relative to total sales C+D, we can readily compute that $C/(C+D) = 1/(\omega+1)$.

While this reinterpretation of our earlier consumer model is mathematically valid, there is a problem in its application to international trade: the transformation curve between two outputs is often taken to be *linear* rather than strictly concave. That is the case in the Ricardian model, for example, or in the transformation curve (8) in Krugman's (1980) model. In that case, the gains from output variety vanish. So the question arises as to whether the strictly concave case we illustrate in Figure 3 has any practical application?

We will now argue that the case of a strictly concave transformation curve is indeed relevant, and in fact, arises in the generalization of the monopolistic competition model due to Melitz (2003). Melitz assumes that labor is the only factor of production, but he allows firms to differ in their productivities φ . In the equilibrium with zero expected profits, only firms above some cutoff productivity φ^* survive; and of these, only firms with productivities above $\varphi_x^* > \varphi^*$ actually export. We will argue that the endogenous determination of these cutoff productivities leads to a strictly concave constant-elasticity transformation curve between domestic and export varieties, adjusted for the quantity produced of each.

Melitz (2003) model

We outline here a two country version of the Melitz (2003) model that does not assume symmetry across the countries. We focus on the home country H, while denoting foreign variables with the superscript F. At home there is a mass of M firms operating in equilibrium. Each period, a fraction δ of these firms go bankrupt and are replaced by new entrants. Each new entrant pays a fixed cost of f_e to receive a draw φ of productivity from a cumulative distribution $G(\varphi)$, which gives rise to the marginal cost of w/φ , where w is the wage and labor is the only factor of production. Only those firms with productivity above a cutoff level φ^* find it profitable to actually produce (the cutoff level will be determined below). Letting M_e denote the mass of new entrants, then $[1 - G(\varphi^*)]M_e$ firms successfully produce. In a stationary equilibrium, these should replace the firms going bankrupt, so that:

$$[1 - G(\varphi^*)]M_e = \delta M . \quad (12)$$

Conditional on successful entry, the distribution of productivities for home firms is then:

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{[1 - G(\varphi^*)]} & \text{if } \varphi \geq \varphi^*, \\ 0 & \text{otherwise,} \end{cases} \quad (13)$$

where $g(\varphi) = \partial G(\varphi) / \partial \varphi$ is the density function.

Home and foreign consumers both have CES preferences that are symmetric over product varieties. Given home expenditure of wL , the revenue earned by a home firm from selling at the price $p(\varphi)$ is:

$$r(\varphi) = p(\varphi)q(\varphi) = \left[\frac{p(\varphi)}{P^H} \right]^{1-\sigma} wL, \quad \sigma > 1, \quad (14)$$

where $q(\varphi)$ is the quantity sold and P^H is the home CES price index. The profit-maximizing price from selling in the domestic market is the usual constant markup over marginal costs:

$$p(\varphi) = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w}{\varphi}. \quad (15)$$

Using this, we can calculate variable profits from domestic sales as $r(\varphi) - (w/\varphi)q(\varphi) = r(\varphi)/\sigma$.

The lowest productivity firm that just breaks even in the domestic market there satisfies the *zero-cutoff-profit* (ZCP) condition:

$$r(\varphi^*)/\sigma = wf \quad \Rightarrow \quad q(\varphi^*) = (\sigma - 1)f\varphi^*, \quad (16)$$

where f is the fixed labor cost. Note that this cutoff condition for the marginal firm is identical to what is obtained in Krugman's (1980) model, in (7), for all firms.

While firms with productivities $\varphi \geq \varphi^*$ find it profitable to produce for the domestic market, only those with higher productivities $\varphi \geq \varphi_x^* > \varphi^*$ find it profitable to export. A home exporting firm faces the iceberg transport costs of $\tau \geq 1$ meaning that τ units must be sent in order for one unit to arrive in the foreign country. Letting $p_x(\varphi)$ and $q_x(\varphi)$ denote the price received and quantity shipped at the factory-gate, the revenue earned by the exporter is:

$$r_x(\varphi) = p_x(\varphi)q_x(\varphi) = \left[\frac{p_x(\varphi)\tau}{P^F} \right]^{1-\sigma} w^*L^*, \quad (17)$$

where P^F is the aggregate CES price in the foreign country, and w^*L^* is foreign expenditure.

Again, the optimal export price is a constant markup over marginal costs:

$$p_x(\varphi) = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w}{\varphi}. \quad (18)$$

The variable profits from export sales are therefore $r_x(\varphi) - (w/\varphi)q_x(\varphi) = r_x(\varphi)/\sigma$, so the ZCP condition for the exporting firm is:

$$r_x(\varphi_x^*)/\sigma = wf_x \Rightarrow q_x(\varphi_x^*) = (\sigma - 1)f_x\varphi_x^*, \quad (19)$$

where f_x is the additional fixed labor cost for exporting. Provided that $r_x(\varphi)/f_x < r(\varphi)/f$, which we assume is the case, then the cutoff productivity for the exporting firm will exceed that for the domestic firm, $\varphi_x^* > \varphi^*$. Then the mass of exporting firms is computed as:

$$M_x \equiv \int_{\varphi_x^*}^{\infty} M\mu(\varphi)d\varphi < M. \quad (20)$$

To close the model, we use the full employment condition and also zero expected profits for any entrant. The labor needed for domestic sales for a firm with productivity φ is $[q(\varphi)/\varphi + f]$, and for export sales is $[q_x(\varphi)/\varphi + f_x]$, so the full employment condition is:

$$L = M_e f_e + M \int_{\varphi^*}^{\infty} [q(\varphi)/\varphi + f]\mu(\varphi)d\varphi + M_x \int_{\varphi_x^*}^{\infty} [q_x(\varphi)/\varphi + f_x]\mu_x(\varphi)d\varphi, \quad (21)$$

where the distribution of productivities conditional on exporting is $\mu_x(\varphi) \equiv g(\varphi)/[1 - G(\varphi_x^*)]$ if $\varphi \geq \varphi_x^*$, and zero otherwise. We can rewrite (21) by multiplying by w , and using the fact that $(w/\varphi)q(\varphi) = r(\varphi)(\sigma - 1)/\sigma$, and likewise for exporters, to obtain:

$$\begin{aligned} wL &= w(M_e f_e + Mf + M_x f_x) + \left(\frac{\sigma - 1}{\sigma}\right) \left[M \int_{\varphi^*}^{\infty} r(\varphi)\mu(\varphi)d\varphi + M_x \int_{\varphi_x^*}^{\infty} r_x(\varphi)\mu_x(\varphi)d\varphi \right] \\ &= w(M_e f_e + Mf + M_x f_x) + \left(\frac{\sigma - 1}{\sigma}\right) wL, \end{aligned}$$

where the second line is obtained using the definition of GDP, with zero expected profits. It follows immediately that there is a *linear* transformation curve between the mass of entering, domestic and exporting firms, that is:

$$L = \sigma(M_e f_e + M_f + M_x f_x). \quad (22)$$

To obtain further results, we assume a Pareto distribution for productivities:

$$G(\varphi) = 1 - \varphi^{-\theta}, \text{ with } \theta > \sigma - 1 > 0. \quad (23)$$

In that case, it can be shown (see the Appendix) that the number of entering firms is proportional to the labor force, $M_e = L(\sigma - 1) / \sigma \theta f_e$, which was assumed by Chaney (2008), for example. So the transformation curve between domestic and export varieties is further simplified as:

$$L = \frac{\sigma \theta}{(\theta - \sigma + 1)} (M_f + M_x f_x). \quad (24)$$

The fact that this transformation curve is linear between the mass of domestic and exported varieties is similar to that found in the Krugman (1980) model, in (7). But this fact does not tell us about the transformation curve between the economy's outputs, because we also need to take into account the *quantity* produced of each variety. In Krugman's model, the quantity produced by each firm is fixed, as in (6), so the transformation is *also* linear in the quantity produced by any groups of firms. But in the Melitz (2003) model, only the zero-profit-cutoff firm has output identical to that in Krugman's model, and the cutoff productivity φ^* itself is endogenously determined. So to determine the transformation curve for the economy, we first need to determine the correct measure of output used to adjust the varieties M and M_x .

To determine the appropriate measure of quantity, it is convenient to invert the demand curve and treat revenue as a function of quantity, so from (14) we obtain:

$$r(\varphi) = A_d q(\varphi)^{\frac{\sigma-1}{\sigma}}, \text{ where } A_d \equiv P^H \left(\frac{wL}{P^H} \right)^{\frac{1}{\sigma}}. \quad (25)$$

We introduce the notation A_d as shift parameter in the demand curve facing home firms for their domestic sales. It depends on the CES price index P^H , and also on domestic expenditure wL .

Likewise, export revenue can be written as:

$$r_x(\varphi) = A_x q_x(\varphi)^{\frac{\sigma-1}{\sigma}}, \text{ where } A_x \equiv \left(\frac{P^F}{\tau} \right) \left(\frac{\tau_i w^* L^*}{P^F} \right)^{\frac{1}{\sigma}}. \quad (26)$$

Integrating domestic and export revenue over firms, we obtain GDP:

$$wL = A_d M \int_{\varphi^*}^{\infty} q(\varphi)^{\frac{\sigma-1}{\sigma}} \mu(\varphi) d\varphi + A_x M_x \int_{\varphi_x^*}^{\infty} q_x(\varphi)^{\frac{\sigma-1}{\sigma}} \mu_x(\varphi) d\varphi. \quad (27)$$

Thus, in order to measure GDP the mass of domestic and export varieties are multiplied by the quantities shown above. Feenstra and Kee (2008) demonstrate that the first-order conditions for maximizing GDP subject to the resource constraint for the economy, taking A and A_x as given, are precisely the monopolistic competition equilibrium conditions. So the quantities appearing in this expression are the “right” way to adjust the mass of domestic and export varieties.

We can simplify these quantities by noting that CES demand, combined with constant-markup prices in (15), imply that the quantity sold equals $q(\varphi) = (\varphi/\tilde{\varphi})^\sigma q(\tilde{\varphi})$ for any choice of reference productivity $\tilde{\varphi}$. We follow Melitz (2003) in specifying $\tilde{\varphi}$ as *average* productivity:

$$\tilde{\varphi} \equiv \left[\int_{\varphi^*}^{\infty} \varphi^{(\sigma-1)} \mu(\varphi) d\varphi \right]^{1/(\sigma-1)}, \quad (28)$$

and likewise for the average productivity $\tilde{\varphi}_x$ for exporters, computed using φ_x^* and μ_x . It

follows that GDP simply equals $(A_d \tilde{M} + A_x \tilde{M}_x)$, using the adjusted mass of varieties:

$$\tilde{M} \equiv M q(\tilde{\varphi})^{(\sigma-1)/\sigma} \quad \text{and} \quad \tilde{M}_x \equiv M_x q_x(\tilde{\varphi}_x)^{(\sigma-1)/\sigma}. \quad (29)$$

To simplify this expression for GDP further, we note that a property of the Pareto distribution is that an integral like (28) is always a constant multiple of the lower bound of integration. That is:

$$\tilde{\varphi} = \left[\frac{\theta}{(\theta - \sigma + 1)} \right]^{1/(\sigma-1)} \varphi^*, \quad (30)$$

as obtained by evaluating the integral in (28), which is finite provided that $\theta > \sigma - 1$. The cutoff productivity φ^* is in turn related to the mass of firms by $[1 - G(\varphi^*)]M_e = \delta M$, and using the mass of entering firms $M_e = L(\sigma - 1) / \sigma \theta f_e$ and the Pareto distribution, it follows that:

$$(\varphi^*)^{-\theta} = \frac{\delta \sigma \theta f_e}{L(\sigma - 1)} M. \quad (31)$$

Gathering together these results, we can use $q(\tilde{\varphi}) = (\tilde{\varphi} / \varphi^*)^\sigma q(\varphi^*)$ to compute that the adjusted mass of domestic varieties is:

$$\tilde{M} = M \left(\frac{\tilde{\varphi}}{\varphi^*} \right)^{\sigma-1} q(\varphi^*)^{\frac{\sigma-1}{\sigma}} = \frac{\theta M}{(\theta - \sigma + 1)} [(\sigma - 1) f \varphi^*]^{\frac{\sigma-1}{\sigma}} = k_1 f^{\frac{\sigma-1}{\sigma}} M^{1 - \frac{\sigma-1}{\theta\sigma}} \left(\frac{f_e}{L} \right)^{-\frac{(\sigma-1)}{\theta\sigma}},$$

where the second equality uses (30) and the ZCP condition $q(\varphi^*) = (\sigma - 1) f \varphi^*$, and the third follows from (31), where $k_1 > 0$ depends on the parameters θ , σ and δ . Thus, the adjusted mass of domestic varieties is an increasing but *nonlinear* function of the mass M . A similar expression holds for exports, but replacing f , M , and \tilde{M} with f_x , M_x , and \tilde{M}_x . Solving for M and M_x and substituting these into the linear transformation curve (24), we obtain a concave transformation curve between \tilde{M} and \tilde{M}_x , with elasticity $\omega \equiv \frac{\theta\sigma}{(\sigma-1)} - 1 > 0$:

$$L = k_2 f_e^{1/(\omega+1)} \left(\tilde{M}^{\frac{\omega+1}{\omega}} f^{\frac{1+(\omega+1)(\sigma-1)}{\omega\sigma}} + \tilde{M}_x^{\frac{\omega+1}{\omega}} f_x^{\frac{1+(\omega+1)(\sigma-1)}{\omega\sigma}} \right)^{\omega/(\omega+1)}, \quad (32)$$

where $k_2 > 0$ again depends on the parameters θ , σ and δ .

Summing up, from the Melitz (2003) model we have obtained a constant-elasticity transformation curve, with elasticity $\omega \equiv \frac{\theta\sigma}{(\sigma-1)} - 1 > 0$, just like in (9) as we initially asserted.

Our earlier results in Theorems 1 and 2 continue to apply to this transformation curve. In particular, consider the problem of maximizing $(A_d \tilde{M}_d + A_x \tilde{M}_x)$ subject to this transformation curve. This Lagrangian problem leads to the following solution, analogous to (10):

Theorem 3 (Feenstra and Kee, 2008)

Assume that the distribution of firm productivity is Pareto, as in (23). Then maximizing GDP subject to the transformation curve (32) results in $e(A_d, A_x)L$, where:

$$w = e(A_d, A_x) \equiv \frac{1}{k_2 f_e^{1/(\omega+1)}} \left[A_d^{\omega+1} f^{1-\frac{\theta}{(\sigma-1)}} + A_x^{\omega+1} f_x^{1-\frac{\theta}{(\sigma-1)}} \right]^{\frac{1}{(\omega+1)}}. \quad (33)$$

The function $e(A_d, A_x)$ is the revenue earned with $L = 1$ on the transformation curve, and equals wages. Note that the exponents appearing on the fixed costs f and f_x in (33) are obtained as $-[\omega + (1 + \omega)\frac{(\sigma-1)}{\sigma}] = 1 - \frac{\theta}{(\sigma-1)} < 0$. This expression also appears as the exponent on fixed costs in the gravity equation of Chaney (2008).

We can now apply Theorem 2 to compute the gain from trade. Denoting autarky by $t-1$, the economy is at the corner of the transformation curve with $A_{xt-1} = \tilde{M}_{xt-1} = 0$, as illustrated by point A in Figure 5. Using t to denote the trade situation, under free trade we have $A_{xt} > 0$ and $\tilde{M}_{xt} > 0$, as at point C. We can therefore evaluate the gain from trade as the ratio of real wages in trade and under autarky:

$$\begin{aligned}
\frac{w_t / P_t^H}{w_{t-1} / P_{t-1}^H} &= \frac{e(A_{dt}, A_{xt})}{e(A_{dt-1}, 0)} \left(\frac{P_t^H}{P_{t-1}^H} \right)^{-1} \\
&= \left(\frac{A_{dt}}{A_{dt-1}} \right) \left(\frac{R_{dt}}{w_t L_t} \right)^{\omega+1} \left(\frac{P_t^H}{P_{t-1}^H} \right)^{-1} \\
&= \left(\frac{w_t / P_t^H}{w_{t-1} / P_{t-1}^H} \right)^{\frac{1}{\sigma}} \left(\frac{R_{dt}}{w_t L_t} \right)^{\omega+1}
\end{aligned} \tag{34}$$

where the first line follows from wages in Theorem 3; the second line follows from Theorem 2, using the domestic “price” A_d as the common good available both periods, with spending on domestic goods in period t of $R_{dt} \equiv A_{dt} \tilde{M}_t$; and the third line follows directly from the definition of A_d in (25).

We use this equation to solve for the ratio of real wages, obtaining the result:

Theorem 4 (Arkolakis, *et al*, 2008a)

The gains from trade in the Melitz (2003) model are:

$$\frac{w_t / P_t^H}{w_{t-1} / P_{t-1}^H} = \left(\frac{R_{dt}}{w_t L_t} \right)^{\omega+1} \left(\frac{\sigma}{\sigma-1} \right)^{-1} = \left(\frac{R_{dt}}{w_t L_t} \right)^{-\frac{1}{\theta}}, \tag{35}$$

where the final equality is obtained because $\omega \equiv \frac{\theta\sigma}{(\sigma-1)} - 1$, so $\frac{1}{(\omega+1)} \left(\frac{\sigma}{\sigma-1} \right) = \frac{1}{\theta}$.

Note that the ratio of domestic expenditure R_{dt} to total income $w_t L_t$ is equal to one minus the import share, so this formula is identical to the gains from trade in the Krugman (1980) model, except that we replace the exponent $\frac{-1}{(\sigma-1)}$ in that case with $-\frac{1}{\theta}$ in (35). This result is precisely the result derived by Arkolakis *et al* (2008a), and remarkably, the elasticity of substitution σ does not enter the formula at all (except insofar as it affects the import share). Our

derivation gives some intuition as to where this simple formula comes from. Namely, the movement from a corner of the transformation curve A in Figure 5, with exports equal to zero, to an interior position like C, gives rise to gains equal to one minus the import (or export) share with the exponent $\frac{-1}{(\omega+1)}$, which is a straightforward application of Theorem 2 on the production side of the economy. We might interpret these gains as due to *export* variety. These gains are shown in the second line of (34), and reflect the increase in wages due to the productivity improvement as the exporting firms drive out less productive domestic firms. But in addition, this productivity improvement drives down prices, and therefore further increase *real* wages: that is shown as we substitute for the endogenous value of A_d , and thereby solve for real wages in (35). Through these two channels, the gains equal one minus the import (or export) share with the exponent $-\frac{1}{\theta}$, which exceeds $\frac{1}{(\omega+1)} = \frac{(\sigma-1)}{\theta\sigma}$ in absolute value.

But what about any *further* gain due to *import* variety? Now we must be careful, because the Melitz model leads to the exit of domestic firms and therefore a reduction in domestic varieties, which must be weighted against the increase in import variety. Baldwin and Forslid (2004) argue that the total number of product varieties falls with trade liberalization, whereas Arkolakis *et al* (2008) show that it can rise or fall. But simply counting the total number of varieties is not the right way to evaluate the welfare gains: instead, we need to take the ratio $(\lambda_t / \lambda_{t-1})^{-1/(\sigma-1)}$ on the consumption side of the economy, as in Theorem 2. As we now show, this ratio turns out to be *unity: the gains due to new import varieties are exactly offset for reduced domestic varieties*. Therefore, the production-side gains we have already identified in Theorem 4 are all that is available.

To obtain this result, we use the CES price index for the Melitz model:

$$P^H = \left[\int_{\varphi^*}^{\infty} p(\varphi)^{1-\sigma} M \mu(\varphi) d\varphi + \int_{\varphi_x^{F*}}^{\infty} p^F(\varphi)^{1-\sigma} M^F \mu^F(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}}, \quad (36)$$

where φ_x^{F*} denotes the zero-profit-cutoff for the foreign exporters, with prices $p^F(\varphi)$. This CES price index is conceptually identical to what we referred to as the unit-expenditure function in (2). The average prices of domestic goods appearing in (36) are:

$$\left[\int_{\varphi^*}^{\infty} p(\varphi)^{1-\sigma} M \mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}} = \left(\frac{\sigma}{\sigma-1} \right) \left(\frac{w}{\tilde{\varphi}} \right) M^{\frac{-1}{\sigma-1}}, \quad (37)$$

which uses the prices (15) together with the definition of average productivity in (28).

When comparing autarky (denoted by $t-1$) with free trade (denoted by t), we need to take into account the changing price of domestic goods and their changing variety, as in (37), along with the fact the all imported goods are new. Applying Theorem 2 gives rise to the following ratio of unit-expenditures:

$$\frac{P_t^H}{P_{t-1}^H} = \left(\frac{w_t / \tilde{\varphi}_t}{w_{t-1} / \tilde{\varphi}_{t-1}} \right) \left(\frac{R_{dt} / w_t L_t}{M_t / M_{t-1}} \right)^{\frac{1}{\sigma-1}}. \quad (38)$$

The first term appearing on the right of (38) is just the change in the average price of domestic goods, reflecting the change in wages and in average productivity. The aggregate domestic good is available in both periods, so the first term reflects the Sato-Vartia index P_{SV} over the “common” good in Theorem 2. The numerator of the second term on the right is the spending on domestic goods relative to total spending in period t ; this equals λ_t in Theorem 2, or one minus the share of spending on new imported varieties. The denominator of the second term is λ_{t-1} in Theorem 2, and reflects the reduction in the number of domestic varieties, $M_t < M_{t-1}$.

We now show that $M_t / M_{t-1} = R_{dt} / w_t L_t$ in (38), so the reduction in the number of domestic varieties just cancels with share of spending on new imported varieties, and there are no further consumption gains. This result is obtained from the ZCP condition for domestic firms, in (16). The second expression appearing in (16) is $q(\varphi^*) = (\sigma - 1)f\varphi^*$, which is familiar from the Krugman model – see (7). We will combine this with the first expression appearing in (16), $r(\varphi^*) / \sigma = wf$, which can be rewritten using the inverse demand curve in (25), to obtain:

$$\left[\frac{A_{dt} q(\varphi_t^*)^{\frac{\sigma-1}{\sigma}}}{A_{dt-1} q(\varphi_{t-1}^*)^{\frac{\sigma-1}{\sigma}}} \right] = \left(\frac{w_t}{w_{t-1}} \right).$$

Using the definition $A_d \equiv P^H (wL / P^H)^{1/\sigma}$, we readily simplify this expression as:

$$\left[\frac{q(\varphi_t^*)}{q(\varphi_{t-1}^*)} \right] = \left(\frac{w_t / P_t^H}{w_{t-1} / P_{t-1}^H} \right).$$

Now using the ZCP condition that $q(\varphi^*) = (\sigma - 1)f\varphi^*$, we immediately obtain:

$$\left(\frac{\varphi_t^*}{\varphi_{t-1}^*} \right) = \left(\frac{w_t / P_t^H}{w_{t-1} / P_{t-1}^H} \right), \quad (39)$$

so that the increase in real wages reflects the increase in the ZCP productivities. From (30) the ratio of ZCP productivities equals the ratio of average productivities, $(\tilde{\varphi}_t / \tilde{\varphi}_{t-1})$, then comparing (38) with (39) we immediately see that $M_t / M_{t-1} = R_{dt} / w_t L_t$, as we intended to show.

This finding that there are no additional consumption gains from variety in the Melitz (2003) model, which is implicit in Arkolakis *et al* (2008a), is discussed explicitly by di Giovanni and Levchenko (2009), who argue that if the distribution of firm size follows Zipf's Law then the extensive margin of imports accounts for a vanishing small portion of the total gains from trade. Their model differs somewhat from our discussion above because firms also use differentiated

intermediate inputs, but they still assume a Pareto distribution for productivities. This assumption implies that the distribution of firms by size follows a power distribution, which correspond to Zipf's Law as $\theta \rightarrow (\sigma - 1)$. That is the case where they find that the extensive margin of imports has a vanishing contribution to the gains from trade. In comparison, our results above are more general because we show that the extensive margin of imports has a welfare contribution that just cancels with the reduced extensive margin of domestic goods, and this result holds for all $\theta > (\sigma - 1)$.

These results from the Melitz (2003) model obviously challenge the empirical finding of Broda and Weinstein (2008), who treated domestic varieties as unchanged. In the next section, we consider an alternative framework to CES that allows for changes in domestic varieties as well as changes in the markups charged by firms. Changing markups have already been introduced in theory by Melitz and Ottaviano (2008), using a quadratic utility function with an additively separable numeraire good, leading to linear demand functions. As useful as that framework is, its zero income elasticities suggest that in empirical application it is best suited for partial equilibrium analysis. We will consider instead a translog expenditure function, which has income elasticities of unity and price elasticities that are not constant.

Before turning to the translog case, we conclude by noting that the gains from trade in the Melitz (2003) model have been estimated on the production side of the economy. Intuitively, movements along the transformation curve in Figure 5 due to greater export variety will be associated with higher GDP and productivity. That hypothesis is strongly confirmed empirically by Feenstra and Kee (2008). They analyze 48 countries exporting to the U.S. over 1980–2000, and find that average export variety to the United States increases by 3.3% per year, so it nearly doubles over these two decades. That total increase in export variety is associated with a

cumulative 3.3% productivity improvement for exporting countries, i.e. after two decades, GDP is 3.3% higher than otherwise due to growth in export variety, on average. That estimate is *greater* than the welfare gains for the U.S. found by Broda and Weinstein (2006), which was that after 30 years, real GDP was 2.6% higher than otherwise due to growth in import variety. Of course, because the U.S. has a low import share we might expect to find greater gains to exporters, but these results still demonstrate that the gains on the production side of the economy can be substantial.

4. Translog Expenditure Function

We turn now to consider a translog unit-expenditure function. In a monopolistic competition model we need to be explicit about which goods are available and which are not, so let \tilde{N} denote the maximum number of goods conceivably available, which we treat as *fixed*. The translog unit-expenditure function (Diewert, 1976) is defined as:⁹

$$\ln e = \alpha_0 + \sum_{i=1}^{\tilde{N}} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln p_i \ln p_j, \text{ with } \gamma_{ij} = \gamma_{ji} \text{ and } \alpha_i > 0. \quad (40)$$

Note that the restriction that $\gamma_{ij} = \gamma_{ji}$ is made without loss of generality. To ensure that the expenditure function is homogenous of degree one, we add the conditions that:

$$\sum_{i=1}^{\tilde{N}} \alpha_i = 1, \quad \text{and} \quad \sum_{i=1}^{\tilde{N}} \gamma_{ij} = 0. \quad (41)$$

The share of each good in expenditure is obtained by differentiating (40) with respect to $\ln p_i$, obtaining:

$$s_i = \alpha_i + \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln p_j. \quad (42)$$

⁹ The translog direct and indirect utility functions were introduced by Christensen, Jorgenson and Lau (1975), and the expenditure function in (40) was proposed by Diewert (1976, p. 122).

These shares must be non-negative, of course, but we will allow for a subset of goods to have zero shares because they are not available for purchase. To be precise, suppose that $s_i > 0$ for $i=1, \dots, N$, while $s_j = 0$ for $j=N+1, \dots, \tilde{N}$. Then for the latter goods, we set $s_j = 0$ within the share equations (42), and use these $(\tilde{N} - N)$ equations to solve for the reservation prices \tilde{p}_j , $j=N+1, \dots, \tilde{N}$, in terms of the observed prices p_i , $i=1, \dots, N$.

Solving for the reservation prices introduces a level of complexity that did not arise in the CES case, where reservation prices are infinite: in the expenditure function (2), an infinite reservation price raised to the negative power $(1 - \sigma)$ simply vanishes. To solve for finite reservation prices in the translog case, it is essential to simplify the translog by imposing the additional “symmetry” requirements:

$$\gamma_{ii} = -\gamma \left(\frac{\tilde{N} - 1}{\tilde{N}} \right) < 0, \text{ and } \gamma_{ij} = \frac{\gamma}{\tilde{N}} > 0 \text{ for } i \neq j, \text{ with } i, j = 1, \dots, \tilde{N}. \quad (43)$$

It is readily confirmed that the restrictions in (43) satisfy the homogeneity conditions (41), and also guarantee that the reservation prices are finite. Because \tilde{N} is a fixed number, (43) simply says that the Γ matrix has a negative constant on the diagonal, and a positive constant on the off-diagonal, chosen so that the rows and columns sum to zero.

The restrictions in (43) are not familiar from the translog literature, but are essential to solve for reservation prices for goods not available. Note that we have *not* restricted the $\alpha_i > 0$ parameters, though they must sum to unity as in (41), so there are $\tilde{N} - 1$ free α_i parameters.¹⁰ In addition, we have the free parameter α_0 in (40) as well as $\gamma > 0$ in (43), so there are a total of $\tilde{N} + 1$ free parameters in this “symmetric” translog function. That is the same number of free

¹⁰ Feenstra (2003) adds an additional symmetry restriction on the α_i parameters, but Bergin and Feenstra (2009) show that Theorem 5 below can be obtained without that restriction.

parameters in our “non-symmetric” CES function (1), where we allowed for \tilde{N} parameters $a_i > 0$ (possibly changing over time) along with the elasticity $\sigma > 1$. So in describing the translog case as “symmetric” we are comparing it to the empirical version that does not use (43); while in describing the CES function as “non-symmetric” we are comparing it to the theoretical version in monopolistic competition models that assumes $a_i \equiv 1, i = 1, \dots, \tilde{N}$. In fact, both the CES function in (1) and the translog in (40) have the same number of free parameters, or degree of symmetry, which we have chosen to be tractable in a monopolistic competition framework.

The usefulness of the symmetric restrictions in (43) is shown by the following result:

Theorem 5 (Feenstra, 2003; Bergin and Feenstra, 2009)

Using the symmetry restrictions (43), suppose that only the goods $i=1, \dots, N$ are available, so the reservation prices \tilde{p}_j for $j=N+1, \dots, \tilde{N}$ are used. Then the unit-expenditure function equals:

$$\ln e = a_0 + \sum_{i=1}^N a_i \ln p_i + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N b_{ij} \ln p_i \ln p_j, \quad (44)$$

where:
$$b_{ii} = -\gamma \frac{(N-1)}{N} < 0, \text{ and } b_{ij} = \frac{\gamma}{N} > 0 \text{ for } i \neq j \text{ with } i, j = 1, \dots, N, \quad (45a)$$

$$a_i = \alpha_i + \frac{1}{N} \left(1 - \sum_{i=1}^N \alpha_i \right), \text{ for } i = 1, \dots, N, \quad (45b)$$

$$a_0 = \alpha_0 + \left(\frac{1}{2\gamma} \right) \left\{ \sum_{i=N+1}^{\tilde{N}} \alpha_i^2 + \left(\frac{1}{N} \right) \left(\sum_{i=N+1}^{\tilde{N}} \alpha_i \right)^2 \right\}. \quad (45c)$$

Notice that the expenditure function in (44) looks like a conventional translog function defined over the goods $i=1, \dots, N$, while the symmetry restrictions continue to hold in (45a), but are defined now using the number of available goods N , which can change over time. As N grows, for example, we will find that the price elasticity of demand also grows (as shown

below), because goods are closer substitutes. To interpret (45b), it implies that each of the coefficient α_i is increased by the same amount to ensure that the coefficients a_i sum to unity over $i=1,\dots,N$. The final term a_0 , appearing in (45c), incorporates the coefficients α_i of the unavailable products. If the number of available products N rise, then a_0 falls, indicating a welfare gain from increasing the number of available products.

Theorem 5 is a promising start towards using the translog function in monopolistic competition models, and shows how the functional form changes as N grows. For theoretical work, this result is all that is needed. But for empirical work, the gains from new varieties suggested by (45c) does not allow for the direct measurement of welfare gain, because it depends on the unknown parameters α_i . We now report results from Feenstra and Weinstein (2009), who develop an alternative formula for the welfare gain that depends on the observable expenditure shares on goods, and can therefore be implemented.

Let us distinguish two periods $t-1$ and t , and continue to assume that the goods $i=N+1,\dots,\tilde{N}$ are not available in either period. The goods $\{1,\dots,N\}$ are divided into two (overlapping) sets: the set $i \in I_\tau$ is available in period $\tau = t-1,t$, and has N_τ goods; with union $I_{t-1} \cup I_t = \{1,\dots,N\}$ and intersection $I_{t-1} \cap I_t \neq \emptyset$. We shall let $\bar{I} \subseteq I_{t-1} \cap I_t \neq \emptyset$ denote any non-empty subset of their intersection, with $\bar{N} > 0$ goods within the set \bar{I} . We need to solve for the reservation prices \tilde{p}_{it-1} for new goods available only in period t , and \tilde{p}_{it} for disappearing goods available only in period $t-1$. These reservation prices are again defined by the respective shares equaling zero, where the share equations are obtained by differentiating (44) with respect to $\ln p_{it}$, while making use of (45):

$$s_{it} = \alpha_i + a_t - \gamma \left(\ln p_{it} - \overline{\ln p_t} \right), \quad i \in I_t, \quad (46)$$

where $a_t = \frac{1}{N_t}(1 - \sum_{i \in I_t} \alpha_i)$ is a time-effect which ensures that $\sum_{i \in I_t} (\alpha_i + a_t) = 1$, and

$\overline{\ln p_t} = \frac{1}{N_t} \sum_{i \in I_t} \ln p_{it}$ is the *average* log-price of all available goods in period t .

In general, the Törnqvist price index is exact for the translog expenditure function (Diewert, 1974), which means that the ratio of the unit-expenditure functions is measured by:

$$\ln\left(\frac{e_t}{e_{t-1}}\right) = \sum_{i=1}^N \frac{1}{2}(s_{it} + s_{it-1})(\ln p_{it} - \ln p_{it-1}). \quad (47)$$

Using this formula, and substituting for the reservation prices for new and disappearing goods, we obtain the following result:

Theorem 6 (Feenstra and Weinstein, 2009)

Let $\bar{I} \subseteq I_{t-1} \cap I_t \neq \emptyset$ denote any non-empty subset of goods available in both periods, and using the symmetry restriction (43) and share equations (46) to solve for the reservation prices for new and disappearing goods. Then the ratio of unit-expenditure functions equals:

$$\ln\left(\frac{e_t}{e_{t-1}}\right) = \sum_{i \in \bar{I}} \frac{1}{2}(\bar{s}_{it} + \bar{s}_{it-1})(\ln p_{it} - \ln p_{it-1}) + V, \quad (48a)$$

where,

$$V \equiv -\left(\frac{1}{2\gamma}\right) \left\{ \sum_{i \notin \bar{I}} (s_{it}^2 - s_{it-1}^2) + \left(\frac{1}{N}\right) \left[\left(\sum_{i \notin \bar{I}} s_{it} \right)^2 - \left(\sum_{i \notin \bar{I}} s_{it-1} \right)^2 \right] \right\}, \quad (48b)$$

and the shares \bar{s}_{it-1} and \bar{s}_{it} are defined as:

$$\bar{s}_{i\tau} \equiv s_{i\tau} + \frac{1}{N} \left(1 - \sum_{i \in \bar{I}} s_{i\tau} \right), \text{ for } i \in \bar{I}, \text{ and } \tau = t-1, t. \quad (49)$$

To interpret this result, notice that the constructed shares $\bar{s}_{i\tau}$ apply to the \bar{N} goods within the set \bar{I} , i.e. a subset of those available in both periods. The constructed shares simply take the

observed shares $s_{i\tau}$ (which sum to unity over the set I_τ) and additively increase each of them by the amount needed so that they sum to unity across the products $i \in \bar{I}$. This transformation of shares means that the term $\sum_{i \in \bar{I}} \frac{1}{2}(\bar{s}_{it} + \bar{s}_{it-1})(\ln p_{it} - \ln p_{it-1})$, appearing in (48a), is the Törnqvist price index defined over products available in both periods. The term V defined in (48b) is therefore the *extra* impact on the exact price index from having the new and disappearing goods.

In the CES case reported in Theorem 2, the welfare impact from a changing set of goods depends on the *share* of new products as compared to disappearing products. Now from (48b) we see that the welfare impact depends on the *sum of squared shares* for new and disappearing goods (appearing first on the right), and also the *sum of shares squared* for new and disappearing goods (appearing second on the right, divided by \bar{N}). As new goods become available, with shares exceeding that for disappearing goods, then there will be a welfare gain. Since we expect the *squared* shares to change less than shares themselves in response to the addition of new goods, it follows that we might expect the welfare gains to be smaller in the translog than in the CES case. This is consistent with the idea that the welfare gain under the demand curve, area A in Figure 1, should be smaller when the reservation price is finite.

However this intuition is not entirely correct because we cannot identify V as the “total” welfare effect of new goods. Rather, new goods can also contribute to lower prices for existing goods by reducing their markups. In other words, the translog expenditure function allows for a “pro-competitive” effect of imports that is entirely absent in the CES case. So imports from new supplying countries can raise welfare from both the “partial” welfare effect V and the pro-competitive effect: the “total” welfare impact would have to sum these two effects.

Pro-competitive Effect

To illustrate the pro-competitive effect, we follow Feenstra and Weinstein (2009) in writing the optimal prices for firms as the familiar markup over marginal costs:

$$p_{it} = c_{it} \left(\frac{\eta_{it}}{\eta_{it} - 1} \right),$$

where c_{it} denotes the time-dependant marginal costs, and η_{it} is the elasticity of demand.

Since demand equals $q_{it} = E_t s_{it} / p_{it}$, for given expenditure E_t , the elasticity of demand is computed from the share equation (46) as:

$$\eta_{it} = 1 - \left(\frac{\partial \ln s_{it}}{\partial \ln p_{it}} \right) = 1 + \frac{\gamma(N_t - 1)}{s_{it} N_t}. \quad (51)$$

Notice that the elasticity of demand is inversely related to the share of the firm s_{it} . If all firms have the same share then $s_{it} = 1/N_t$, and the elasticity becomes $\eta_{it} = 1 + \gamma(N_t - 1)$, which increases as the number of available products rises. This confirms that goods are stronger substitutes as more of them become available, as we asserted above. Using the elasticity (51) in the pricing equation, and taking logs, we obtain:

$$\ln p_{it} = \ln c_{it} + \ln \left[1 + \frac{s_{it} N_t}{\gamma(N_t - 1)} \right]. \quad (52)$$

The pricing equation in (52) will be the key to identifying the pro-competitive effect.

Notice that a reduction in the share s_{it} of good i lowers the markup and price of that good.

Feenstra and Weinstein (2009) show that the U.S. market share falls for many goods, leading to a fall in markups. In order to cumulate all these various pro-competitive effects we substitute (52) into the Törnqvist price index in (48) to obtain:

$$\sum_{i \in \bar{I}} \frac{1}{2} (\bar{s}_{it} + \bar{s}_{it-1}) (\ln p_{it} - \ln p_{it-1}) = \sum_{i \in \bar{I}} \frac{1}{2} (\bar{s}_{it} + \bar{s}_{it-1}) (\ln c_{it} - \ln c_{it-1}) + P, \quad (53)$$

$$\text{where,} \quad P \equiv \sum_{i \in \bar{I}} \frac{1}{2} (\bar{s}_{it} + \bar{s}_{it-1}) \ln \left[\frac{1 + s_{it} N_t / \gamma (N_t - 1)}{1 + s_{it-1} N_{t-1} / \gamma (N_{t-1} - 1)} \right]. \quad (54)$$

The first term on the right of (53) is a Törnqvist index defined over the marginal costs of goods available in both periods, while the second term P indicates the change in the average markups on these goods. We expect these markups to fall as new products becomes available, so $P < 0$, which is the pro-competitive effect.

To summarize, the change in the exact price index for the translog case can be decomposed into three terms:

$$\ln \left(\frac{e_t}{e_{t-1}} \right) = \sum_{i \in \bar{I}} \frac{1}{2} (\bar{s}_{it} + \bar{s}_{it-1}) (\ln c_{it} - \ln c_{it-1}) + V + P. \quad (55)$$

The first term reflects the drop in marginal costs for products supplied in both periods; the second term V in (48b) reflects the “partial” welfare effect of new and disappearing goods; and the third term P in (54) reflects the change in markups for goods available both periods. Feenstra and Weinstein (2009) implement the decomposition in (55) for the United States, using both domestic sales and import sales for each good. That is, they allow consumers to substitute between domestic and import varieties of each good, unlike Broda and Weinstein (2006), with finite reservation prices for each variety. Increased shares of imports and reduced U.S. shares can lead to reduced U.S. markups, unlike the CES case. For both reasons, the translog case offers a promising theoretical and empirical framework that goes beyond the limitations of the CES case.

5. Conclusions [TO BE COMPLETED]

Appendix

Using $L = \sigma(M_e f_e + M f + M_x f_x)$ and the full employment condition, we have that:

$$\left(\frac{\sigma-1}{\sigma}\right)L = M \int_{\varphi^*}^{\infty} [q(\varphi)/\varphi] \mu(\varphi) d\varphi + M_x \int_{\varphi_x^*}^{\infty} [q_x(\varphi)/\varphi] \mu_x(\varphi) d\varphi,$$

Evaluating these integrals:

$$\begin{aligned} \int_{\varphi^*}^{\infty} [q(\varphi)/\varphi] \mu(\varphi) d\varphi &= \int_{\varphi^*}^{\infty} \frac{q(\varphi^*)}{\varphi} \left(\frac{\varphi}{\varphi^*}\right)^{\sigma} \mu(\varphi) d\varphi \\ &= \frac{q(\varphi^*)}{\varphi^*} \int_{\varphi^*}^{\infty} \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \frac{\theta \varphi^{-\theta-1}}{(\varphi^*)^{-\theta}} d\varphi \\ &= \frac{q(\varphi^*)}{\varphi^*} \frac{\theta}{(\sigma-\theta-1)} \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-\theta-1} \Bigg|_{\varphi^*}^{\infty} \\ &= f \frac{(\sigma-1)\theta}{(\theta-\sigma+1)}, \end{aligned}$$

where the first line uses $q(\varphi) = (\varphi/\varphi^*)^{\sigma} q(\varphi^*)$ and the last line uses $q(\varphi^*)/\varphi^* = (\sigma-1)f$.

Likewise,
$$\int_{\varphi_x^*}^{\infty} [q_x(\varphi)/\varphi] \mu_x(\varphi) d\varphi = f_x \frac{(\sigma-1)\theta}{(\theta-\sigma+1)}.$$

Substituting these in to the full employment condition above we obtain:

$$L = \frac{\sigma\theta}{(\theta-\sigma+1)} (M f + M_x f_x),$$

from which it follows that $M_e = L(\sigma-1)/\sigma\theta f_e$.

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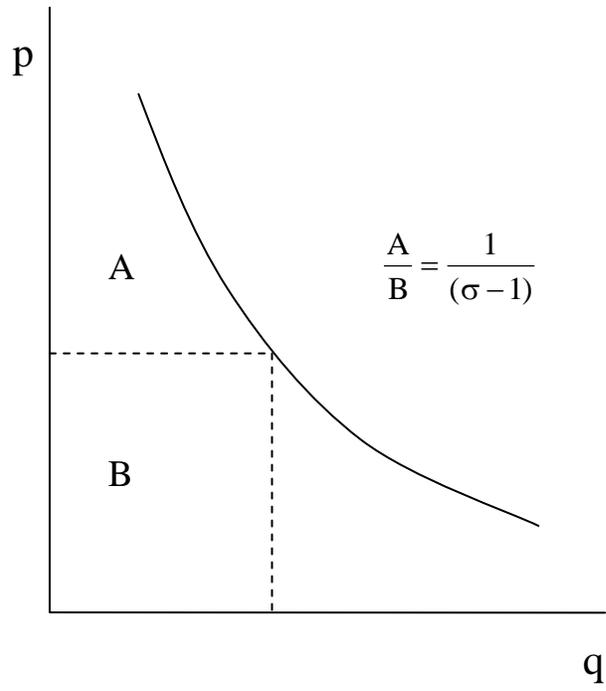


Figure 1 : CES Demand

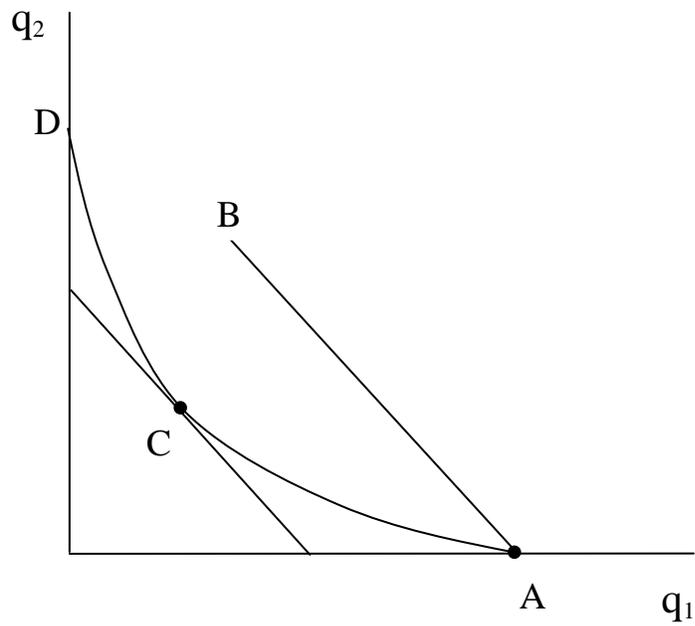


Figure 2: CES Indifference Curve

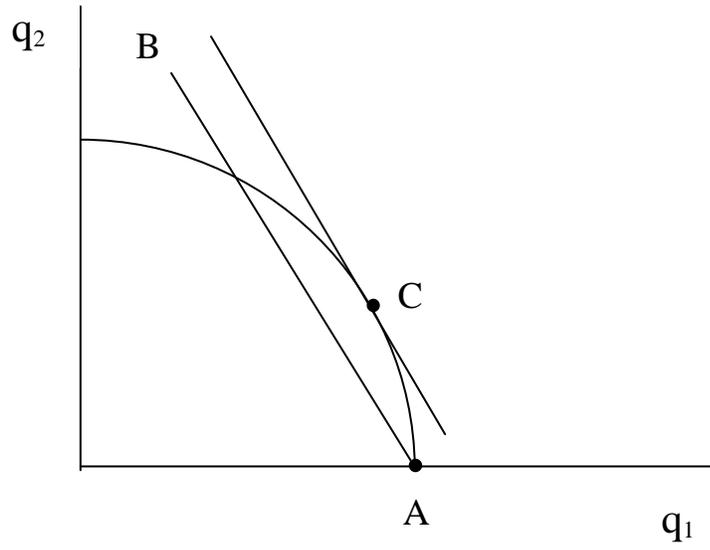


Figure 3: Constant-Elasticity Transformation Curve

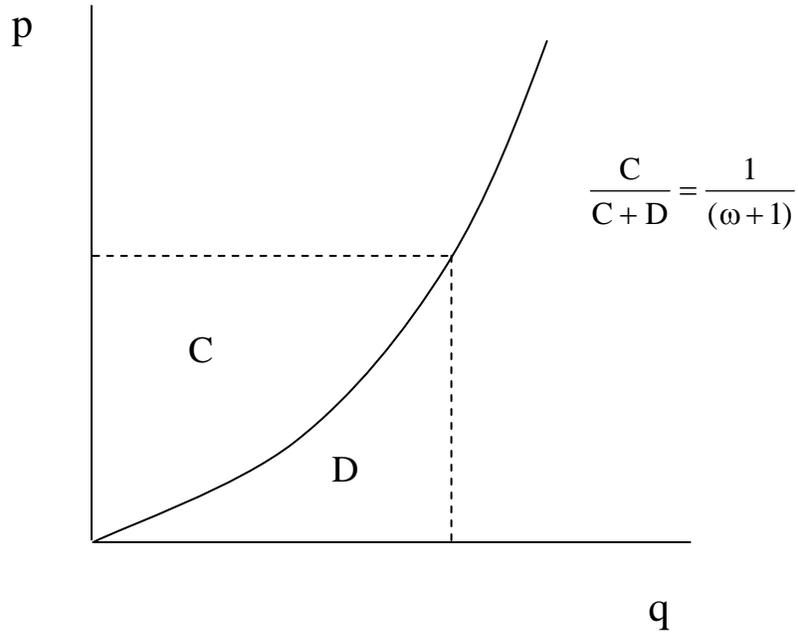


Figure 4: Constant-Elasticity Supply Curve

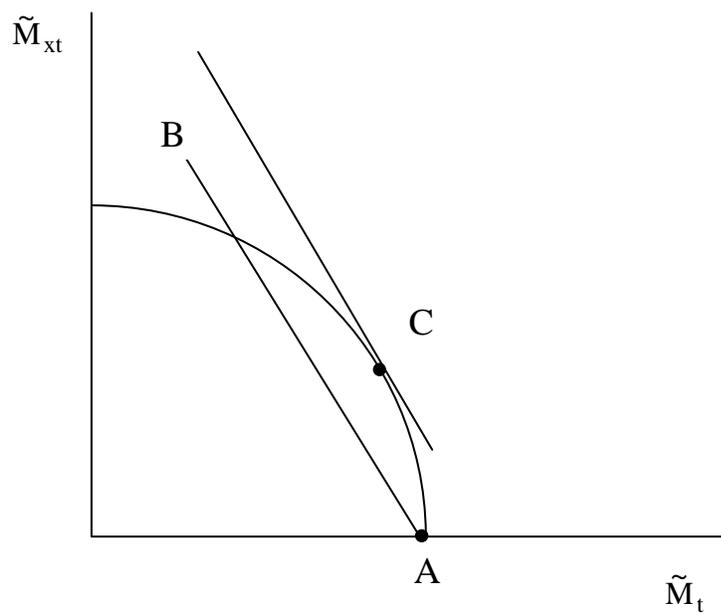


Figure 5: Constant-Elasticity Transformation Curve from Melitz (2003)