Are Progressive Fiscal Rules Stabilizing?*

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Abstract

This paper studies how income-based, progressive taxes and transfers may reduce aggregate volatility by protecting the economy against expectation-driven business cycles. Eliminating "local" sunspots that are arbitrarily close to an indeterminate steady state requires, for sensible parameter values, strong levels of progressivity so as to make labor supply close to inelastic. However, progressive taxes and transfers are shown to be *ineffective* to rule out stable deterministic cycles (and the associated "global" sunspots) that are located close to a *determinate* steady state.

Our results are formalized within two benchmark models and show how the efficiency of progressive fiscal schemes as local automatic stabilizers depends on the fiscal base. In the first setting with heterogeneous agents and segmented asset markets in which wage income mostly finances consumption, we show that progressive taxes and transfers should be made dependent on *labor* income, so as to rule out local indeterminacy. On the contrary, progressive fiscal rules should be applied to *capital* income in an overlapping generations economy where consumption comes from savings income. Incidentally, the latter results suggest that capital income taxes may be desirable, when progressive, to make local expectation-driven fluctuations less likely. In both frameworks, key to the results is the property that progressive fiscal rules provide insurance in the presence of imperfect capital markets.

Keywords: progressive taxes and transfers, business cycles, sunspots, endogenous cycles.

Journal of Economic Literature Classification Numbers: D33, D58, E32, E62, H24, H30.

1 Introduction

Income-dependent taxes and transfers have been proposed as efficient automatic stabilizers since, at least, Musgrave and Miller [27] (see also Vickrey [37, 38], Slitor [34], Friedman [15]). In recent years, the development of dynamic general equilibrium models has proved useful to study in a precise manner how, in particular, *progressive* fiscal schedules may stabilize the economy's aggregate variables. This strand of literature specifically allows to evaluate the level of social insurance provided by given fiscal schemes in the presence of various shocks. In particular, Guo and Lansing [19], Guo [17], Guo and Harrison [18], Dromel and Pintus [12] have shown that progressive income taxes can rule out local indeterminacy and restore saddle-path convergence.

The present paper also studies how income-based, progressive taxes and transfers may reduce aggregate volatility by protecting the economy against expectation-driven fluctuations (e.g. sunspot or cyclical equilibria). Our main conclusion is twofold. First, fiscal progressivity reduces, in parameter space, the likelihood of sunspot equilibria that are arbitrarily close to an indeterminate steady state (as in Guo and Lansing [19]). Most importantly, however, progressive schedules are shown to be ineffective to rule out stable (Hopf or flip) deterministic cycles (and the associated "global" sunspots) that are located close to a *determinate* steady state. *Therefore, we establish that progressive fiscal schemes do not help immunizing the economy against "global" expectation-driven business cycles.* This is in contrast with the results reviewed above, which show that the usual "bifurcation" leading to local indeterminacy (whereby the steady state goes from a saddle to a sink, with eigenvalues going though infinity) can be prevented from occuring in the Ramsey model when progressivity is allowed.

Also in contrast to the literature is our result that fully eliminating local indeterminacy and sunspots requires, for sensible parameter values, *strong* progressivity. The main mechanism at the heart of this result is the following. When agents have optimistic expectations, they want to raise their consumption and, accordingly, they devote a higher fraction of their time endowment to work so as to increase their income, which originates an expansion. It follows that stabilizing labor supply movements requires low volatility of disposable income, which in turn imposes high fiscal progressivity. From this basic argument, one expects the efficiency of progressive taxes and transfers as automatic stabilizers to depend on the mechanisms leading to expectation-driven volatility and, consequently, on the fiscal base. In other words, if consumption is mostly financed by wage (resp. capital) income, then progressive taxes and transfers should apply to wage (resp. capital) income. We formalize this argument within two benchmark models and show how the power of progressive taxes and transfers to rule out local indeterminacy depends on the fiscal base. In both frameworks, key to the results is the property that progressive fiscal rules provide insurance in the presence of imperfect capital markets. We first show that progressive taxes and transfers should be made dependent on *labor* income in a heterogeneous agents economy with segmented asset markets in which wage income mostly finances consumption. On the contrary, we demonstrate that progressive fiscal rules should be applied to *capital* income in an overlapping generations (thereafter OLG) economy where consumption comes from savings income. In both models, progressive schemes are shown to reduce the range of parameter values that are compatible with local indeterminacy, which does not preclude the steady state being surrounded by stable cycles and global sunspots. In particular, the steady state turns out to be locally determinate if fiscal progressivity is larger than some threshold value. For sensible parameter values, it turns out that the threshold level of progressivity is, in the OLG model, half as high as the corresponding bound in the heterogeneous agents model: it takes higher progressivity to stabilize the economy with financial constraints. As a consequence, we argue that empirical evidence accords better with the (local) stabilizing effects of progressive fiscal rules in the OLG setting.

The two settings that we consider are admittedly limiting cases, as consumption is financed by labor income in the first model and capital income in the second one. Obviously, we have in mind the more realistic configuration that lies in between these two extremes but is closer to the former one. Our conclusions clearly predict that fiscal progressivity should operate mainly through labor income when consumption relies primarily on wage income, as seems to be the case in OECD countries. However, a slightly progressive tax and transfer rate on capital income may still be desirable, as long as non-wage income partially finances consumption expenditures, to make expectation-driven business cycles less likely. To our knowledge, this justification for taxing capital income has not been noticed by the literature. Of course, the historical record shows that stabilization concerns were not at the origin of income tax progressivity in advanced countries, about a century ago. However, our analysis indicates that this does not imply that progressive fiscal schedules do not have, *de facto*, some stabilizing properties. On the other hand, our study also suggests that progressive taxes and transfers are inefficient to rule out endogenous fluctuations when consumption is financed, even partially, by the returns from financial assets that would remain untaxed (it is easily shown to be the case, e.g. , in the monetary economy studied by Benhabib and Laroque [5]).

Few papers in the endogenous business-cycle literature study nonlinear tax schemes, all in the representative agent framework, following Guo and Lansing [19]. Only Guo [17] assumes, quite realistically, that capital and labor taxes have different statutory, progressivity features. We emphasize that several features differentiate our work from the latter papers. Most importantly, all models studied in this paper have *constant* returns to scale (that is, externalities and imperfect competition are absent), which makes stabilization policies more desirable *a priori* to improve welfare. Moreover, our results do not rely on either specifications for taxes, preferences or technology, numerical values or simulations, and they do consider both taxes and transfers. Also, our results stress the fact that identifying the relevant fiscal base is key when assessing the efficiency of progressive fiscal rules as built-in stabilizers. Labor supply movements reacting to waves of optimism or pessimism is here a key mechanism that may lead to sunspots and cycles, as in many papers in the literature. However, what the present paper does is to underline the power of progressive taxes and transfers on *factor* incomes to reduce *labor and consumption volatility*, which is hardly relevant in representative agent models where *total* income is supposed to finance consumption.

The literature discussed above consider decentralized equilibria that summarize the behavior of a representative agent, thereby ignoring the possible *conflicting* interests over the role of taxes. In contrast, Judd [20, 21], Kemp *et al.* [23], Alesina and Rodrik [1], Sarte [32], Lansing [25], Bénabou [3, 4], Saez [31] study the redistributive and growth effects of taxes. However, differently from these authors, we focus on the *stabilizing* power of progressive fiscal schemes. Our analysis also touches upon the much debated question of capital income taxation, as we show that capital taxes (however small) may be desirable to stabilize the economy. To our knowledge, this argument has not been noticed in the literature which strongly suggests that capital income taxes should be zero (see Chamley [10], Judd [20]). In contrast with most papers belonging to this strand of the literature, our two settings allow for elastic labor. Moreover, money is present in the first model (originally proposed by Woodford [39]) and it is held by a fraction of agents. These two features are certainly plausible. On one hand, there is a large evidence showing that labor employment moves at all frequencies in response to changes in real wages (moreover, theory also argues in favor of elastic labor; see, for instance, Boldrin and Horvath [6]). On the other hand, 59% of U.S. households did not hold, in 1989, any interest bearing assets (Mulligan and Sala-i-Martin [26, Table 1, p. 962]). Therefore, incorporating elastic labor and money, as a dominated asset, seems to provide relevant extensions of the current literature (see also Judd

[20, 21] on elastic labor). Our second benchmark case is a well-known version of the OLG model. The first setting studied in this paper is also indeed very close to a commonly used framework in the public finance literature studying redistributive taxation (see, for instance, Judd [20, 21], Kemp *et al.* [23], Alesina and Rodrik [1], Lansing [25]). Note that a recent paper by Seegmuller [33, s. 5.2.2] studies, in an example, the effects of nonlinear tax rates in the Woodford [39] model but restricts the analysis to regressive taxation.

The rest of the paper is organized as follows. Section 2 presents the heterogeneous agents economy with segmented asset markets and discusses how efficient progressive taxes and transfers are to make expectation-driven fluctuations less likely. Section 3 introduces progressive fiscal rules in an OLG economy with capital and elastic labor and shows how this feature may rule out local sunspots. Section 4 studies the quantitative implications of our analysis. Finally, some concluding remarks and directions of future research are gathered in Section 5, while three appendices present some proofs.

2 Stabilization Through Income-Based Taxes and Transfers in a Heterogeneous Agents Economy with Segmented Asset Markets

2.1 Fiscal Policy and Intertemporal Equilibria

A unique good is produced in the economy by combining labor $l_t \geq 0$ and the capital stock $k_{t-1} \geq 0$ resulting from the previous period. Production exhibits constant returns to scale, so that output is given by:

$$F(k,l) \equiv Alf(a), \tag{1}$$

where $A \geq 0$ is a scaling parameter and the latter equality defines the standard production function in intensive form defined upon the capital labor ratio $a = k/l$. On technology, we shall assume the following.

Assumption 2.1

The intensive production function $f(a)$ is continuous for $a \geq 0$, C^r for $a > 0$ and r large enough, with $f'(a) > 0$ and $f''(a) < 0$.

Competitive firms take real rental prices of capital and labor as given and determine their input demands by equating the private marginal productivity of each input to its real price. Accordingly, the real competitive equilibrium wage is:

$$\omega = \omega(a) \equiv A[f(a) - af'(a)], \tag{2}$$

while the real competitive gross return on capital is:

$$R = \rho(a) + 1 - \delta \equiv Af'(a) + 1 - \delta, \tag{3}$$

where $1 \geq \delta \geq 0$ is the constant depreciation rate for capital.

Fiscal policy is supposed to map market income x into disposable income $\phi(x)$. In this formulation, there are two benchmark cases. When $\phi(x)$ is proportional to x, then ϕ has unitary elasticity and taxes and transfers are linear. Decreasing the elasticity of $\phi(x)$ from one (when taxes and transfers are linear) to zero may be interpreted as *increasing fiscal progressivity.* More precisely, one can postulate the following (see Musgrave and Thin [28] for an early definition, and, for example, Lambert [24, chap. 7-8]).

Assumption 2.2

Disposable income $\phi(x)$ is a continuous, positive function of market income $x \geq 0$, with $\phi'(x) > 0$ and $0 \geq \phi''(x)$, for $x > 0$. The income tax-and-transfer scheme exhibits weak progressivity*, that is, $\phi(x)/x$ is non-increasing for $x > 0$ or, equivalently, $1 \geq \psi(x) \equiv x\phi'(x)/\phi(x)$.*

Then $\pi(x) \equiv 1 - \psi(x)$ is a measure of residual income progressivity. In particular, the fiscal schedule is linear when $\pi(x) = 0$, or $\psi(x) = 1$, for $x > 0$, and the higher $\pi(x)$, the more progressive the fiscal schedule.

One can reinterpret the condition $1 \geq \psi(x)$ as the property that the *marginal* tax-and-transfer rate $\tau_m \equiv \partial(x - \phi(x))/\partial x$ is larger than the average tax-and-transfer rate $\tau \equiv (x - \phi(x))/x$: it is easily shown that $\tau_m - \tau = \phi(x)/x - \phi'(x)$ so that $\tau_m \geq \tau$ when $1 \geq \psi(x)$ or $\pi(x) \geq 0$ for all positive x. Moreover, fiscal progressivity is naturally measured by $\pi \equiv 1 - \psi$ when one notes that $\pi = (\tau_m - \tau)/(1 - \tau)$.

To complete the description of the model, we now characterize the behavior of both classes of agents. A representative worker solves the following utility optimization problem, as derived in Appendix A:

$$\text{maximize } \{V_2(c^w_{t+1}/B) - V_1(l_t)\} \text{ such that } p_{t+1}c^w_{t+1} = p_t\phi(\omega_t l_t),\ c^w_{t+1} \geq 0,\ l_t \geq 0, \tag{4}$$

where $B > 0$ is a scaling factor, c^w_{t+1} is next period consumption, l_t is labor supply, $p_{t+1} > 0$ is next period price of output (assumed to be perfectly foreseen), $\omega_t > 0$ is real wage, and $\phi(\omega_t l_t)$ is disposable wage income, as described in Assumption 2.2. To keep things simple, we assume in this section that progressive taxes and transfers are depending on labor income only (capital taxes and subsidies are studied in the next section). We consider the case such that leisure and consumption are gross substitutes and assume therefore the following:

Assumption 2.3

The utility functions $V_1(l)$ and $V_2(c)$ are continuous for $l^* \geq l \geq 0$ and $c \geq 0$, where $l^* > 0$ is the (maybe infinite) workers' endowment of labor. They are C^r for, respectively, $0 < l < l^*$ and $c > 0$, and r large enough, with $V_1'(l) > 0$, $V_1''(l) > 0$, $\lim_{l \to l^*} V_1'(l) = +\infty$, and $V_2'(c) > 0$, $V_2''(c) < 0$, $-cV_2''(c) < V_2'(c)$ (that is, consumption and leisure are gross substitutes).

The first-order condition of the above program (4) gives the optimal labor supply $l_t > 0$ and the next period consumption $c^w_{t+1} > 0$, which can be stated as follows.

$$v_1(l_t) = \psi(\omega_t l_t)v_2(c^w_{t+1}) \text{ and } p_{t+1}c^w_{t+1} = p_t\phi(\omega_t l_t), \tag{5}$$

where $v_1(l) \equiv lV_1'(l)$ and $v_2(c) \equiv cV_2'(c/B)/B$. Assumptions 2.2 and 2.3 implies that v_1 and v_2 are increasing while v_1 is onto $\mathbf{R}_+$. Therefore, Assumption 2.3 allows one to define, from Eqs. (5), $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$ (whose graph is the offer curve), which is a monotonous, increasing function only if the elasticity of ψ is either negative or not too large when positive.

Capitalists maximize the discounted sum of utilities derived from each period consumption. They consume $c_t^c \geq 0$ and save $k_t \geq 0$ from their income, which comes exclusively from real gross returns on capital and is not affected by fiscal rules (see next section for the case of capital income-based taxes and transfers). To fix ideas, we assume, following Woodford [39], that capitalists' instantaneous utility function is logarithmic. As easily shown (for instance by applying dynamic programming techniques), their optimal choices are then given by a constant savings rate:

$$c_t^c = (1 - \beta) R_t k_{t-1}, \; k_t = \beta R_t k_{t-1}, \tag{6}$$

where $0 < \beta < 1$ is the capitalists' discount factor and $R_t > 0$ is the real gross rate of return on capital.

As usual, equilibrium on capital and labor markets is ensured through Eqs. (2) and (3). Since workers save their wage income in the form of money, the equilibrium money market condition is:

$$\phi(\omega(a_t) l_t) = M_t / p_t, \tag{7}$$

where $M_t \geq 0$ is money supply and p_t is current nominal price of output. Finally, Walras' law accounts for the equilibrium in the good market. From the equilibrium conditions in Eqs. (2), (3), (5), (6), (7), one easily deduces that the variables c_{t+1}^w, c_t^w, l_t, p_{t+1}, p_t, c_t^c and k_t are known once (a_t, k_{t-1}) are given. This implies that intertemporal equilibria may be summarized by the dynamic behavior of both a and k.

Definition 2.1

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence (a_t, k_{t-1}) *of* $\mathbf{R}^2_{++}$, $t = 1, 2, \ldots$, *such that*

$$\begin{cases} v_2(\phi(\omega(a_{t+1}) k_t / a_{t+1})) &= v_1(k_{t-1}/a_t)/\psi(\omega(a_t) k_{t-1}/a_t), \\ k_t &= \beta R(a_t) k_{t-1}. \end{cases} \tag{8}$$

Note that the distribution of assets does not degenerate in this framework, as different agents hold money and capital (in contrast with Becker [2]).

In view of Eqs. (8) and recalling that $a = k/l$, the nonautarkic steady states are the solutions $(\overline{a}, \overline{l})$ in $\mathbf{R}^2_{++}$ of $v_2(\phi(\omega(\overline{a})\overline{l})) = v_1(\overline{l})/\psi(\omega(\overline{a})\overline{l})$ and $\beta R(\overline{a}) = 1$. Equivalently, in view of Eq. (3), the steady states are given by:

$$\begin{cases} v_2(\phi(\omega(\overline{a})\overline{l})) &= v_1(\overline{l})/\psi(\omega(\overline{a})\overline{l}), \\ \rho(\overline{a}) + 1 - \delta &= 1/\beta. \end{cases} \tag{9}$$

We shall solve the existence issue by choosing appropriately the scaling parameters A and B, so as to ensure that one stationary solution coincides with, for instance, $(\overline{a}, \overline{l}) = (1, 1)$. For sake of brevity, the proof is given in Appendix B.

2.2 Ruling Out Local Indeterminacy Through Labor Income-Based Fiscal Progressivity

We now study the dynamics of Eqs. (8) around one of its interior stationary points $(\overline{a}, \overline{k})$. These equations define locally a dynamical system of the form $(a_{t+1}, k_t) = G(a_t, k_{t-1})$ if the derivative of $\omega(a)/a$ with respect to a does not vanish at the steady state, or equivalently if $\varepsilon_\omega(\overline{a}) - 1 \neq 0$, where the notation ε_ω stands for the elasticity of $\omega(a)$ evaluated at the steady state under study. Then, the usual procedure to study the local stability of the steady states is to use the linear map associated to the Jacobian matrix of G, evaluated at the fixed point under study.

We assume that, in the neighborhood of the steady state that has been conveniently normalized by the procedure in Proposition B.1, ϕ has a *constant elasticity* $\psi = 1 - \pi$ with $1 > \pi \geq 0$. Our reason for restricting the analysis to this benchmark is twofold. Most importantly, economic theory does not place strong restrictions on how the *elasticity* $\psi(x)$ of after-tax income varies with pre-tax income x (see e.g. Lambert [24]). Therefore, we choose to be parcimonious and introduce fiscal progressivity through a *single* parameter, that is, $\pi = 1 - \psi$ that is locally constant. As it will soon appear, the following analysis could be easily adapted to account for a (locally) non-constant elasticity. A related but stronger assumption (that is, constant residual income progression) has been used in papers by Feldstein [14], Kanbur [22], Persson [29] in static models, and by Guo and Lansing [19], Bénabou [3, 4] in growth models.

Straightforward computations yield the following proposition.

Proposition 2.1 (Linearized Dynamics around a Steady State)

Under the assumptions of Proposition B.1, suppose that ϕ has constant elasticity in the neighborhood of the steady state $(\overline{a}, \overline{k})$ of the dynamical system in Eqs. (8), i.e. $\psi(x) = 1 - \pi$, with $0 < \pi < 1$ measuring fiscal progressivity based on labor income. Let ε_R, ε_ω, ε_γ be the elasticities of the functions $R(a)$, $\omega(a)$, $\gamma(l)$, respectively, evaluated at the steady state $(\overline{a}, \overline{k})$ and assume that $\varepsilon_\omega \neq 1$. The linearized dynamics for the deviations $da = a - \overline{a}$, $dk = k - \overline{k}$ are determined by the linear map:

$$\left\{ \begin{array}{rcccc} da_{t+1} & = & -\frac{\varepsilon_\gamma/(1-\pi)+\varepsilon_R}{\varepsilon_\omega - 1} da_t & + & \frac{\overline{a}}{\overline{k}} \frac{\varepsilon_\gamma/(1-\pi)-1}{\varepsilon_\omega - 1} dk_{t-1}, \\ dk_t & = & \frac{\overline{k}}{\overline{a}} \varepsilon_R da_t & + & dk_{t-1}. \end{array} \right. \tag{10}$$

The associated Jacobian matrix evaluated at the steady state under study has trace T and determinant D, where

$$T = T_1 - \frac{\varepsilon_\gamma - 1}{(1-\pi)(\varepsilon_\omega - 1)}, \quad \textit{with} \quad T_1 = 1 + \frac{|\varepsilon_R| - 1/(1-\pi)}{\varepsilon_\omega - 1},$$
$$D = \varepsilon_\gamma D_1, \quad \textit{with} \quad D_1 = \frac{|\varepsilon_R| - 1}{(1-\pi)(\varepsilon_\omega - 1)}.$$

Moreover, one has $T_1 = 1 + D_1 + \Lambda$, where $\Lambda \equiv -\pi|\varepsilon_R|/[(1-\pi)(\varepsilon_\omega - 1)]$.

We shall assume throughout that a steady state exists in the whole range of parameter values that will be considered. To fix ideas, we may assume without loss of generality that the steady state has been normalized at $(\overline{a}, \overline{k}) = (1, 1)$ (see Proposition B.1).

Now fix the technology (i.e. ε_R and ε_ω), at the steady state, and vary the parameter representing workers' preferences $\varepsilon_\gamma > 1$. In other words, consider the parametrized curve $(T(\varepsilon_\gamma), D(\varepsilon_\gamma))$ when ε_γ describes $(1, +\infty)$. Direct inspection of the expressions of T and D in Proposition 2.1 shows that this locus is a half-line Δ that starts close to (T_1, D_1) when ε_γ is close to 1, and whose slope is $1 - |\varepsilon_R|$, as shown in Fig. 1. The value of $\Lambda = T_1 - 1 - D_1$, on the other hand, represents the deviation of the generic point (T_1, D_1) from the line (AC) of equation $D = T - 1$, in the (T, D) plane.

Insert Figures 1-2 here.

The main task we now face is locating the half-line Δ in the plane (T, D), i.e. its origin (T_1, D_1) and its slope $1 - |\varepsilon_R|$, as a function of the parameters of the system. The parameters we shall focus on are the depreciation rate for the capital stock $1 \geq \delta \geq 0$, the capitalist's discount factor $0 < \beta < 1$, the share of capital in total income $0 < s = \overline{a}\rho(\overline{a})/f(\overline{a}) < 1$, the elasticity of input substitution $\sigma = \sigma(\overline{a}) > 0$, and fiscal progressivity $1 > \pi \geq 0$, all evaluated at the steady state $(\overline{a}, \overline{k})$ under study. In fact, it is not difficult to get the following expressions.

$$\begin{aligned} &D_1 = (\theta(1-s) - \sigma)/[(1-\pi)(s-\sigma)], \quad && \Lambda = -\pi\theta(1-s)/[(1-\pi)(s-\sigma)], \\ &T_1 = 1 + D_1 + \Lambda, && \mathit{slope}_\Delta = 1 - \theta(1-s)/\sigma, \end{aligned} \tag{11}$$

where $\theta \equiv 1 - \beta(1-\delta) > 0$ and all these expressions are evaluated at the steady state under study.

Our aim now is to locate the half-line Δ, i.e. its origin (T_1, D_1) and its slope in the (T, D) plane when the capitalists' discount rate β, as well as the technological parameters δ, s, and the level of fiscal progressivity π at the steady state are fixed, whereas the elasticity of factor substitution σ is made to vary. One easily show that the benchmark economy *with constant fiscal rate* $\pi = 0$ (or $\psi = 1$) is equivalent to the *no-tax-and-transfer* case studied by Grandmont, Pintus, and de Vilder [16]: the origin (T_1, D_1) of Δ is located on the line (AC), i.e. $\Lambda \equiv 0$ (see Fig. 1). The immediate implication of the resulting geometrical representation is that indeterminacy and endogenous fluctuations emerge only for low values of σ ($\sigma < \sigma_I$, and indeed for σ less than s, the share of capital in output) while, on the contrary, local determinacy is bound to prevail for larger values of σ. One corollary of this is *that linear tax-and-transfer rates on wage income do not affect the range of parameter values that are associated with local indeterminacy and bifurcations.*

To anticipate our main result about fiscal progressivity (when $\pi > 0$), it is enough to notice from eqs. (10)

that $\varepsilon_\gamma/(1-\pi)$ enters the expressions of the trace and determinant. This implies that the half-line Δ can be alternatively generated by varying π, for fixed ε_γ. For example, fixing $\varepsilon_\gamma = 1$ and increasing π from zero to one would result in the exact configuration of Fig. 1. The most important implication of this fact is that *there exists a critical value for π such that Δ does not intersect the indeterminacy region* (that is, the ABC triangle). The next proposition shows that such a threshold exists for all σ's. The full charaterization of all possible cases that occur when one increases π are stated in Proposition C.1 (see Appendix C). The most important implication of Proposition C.1 is that there exists a threshold level of fiscal progressivity on labor income π_{min} above which the steady state is locally determinate, thereby excluding "local" sunspots.

Proposition 2.2 (Ruling Out Local Indeterminacy Through Progressive Fiscal Rules)

Under the assumptions stated in Proposition C.1, there exists a threshold level of fiscal progressivity on labor income $\pi_{min} \equiv 2[\theta(1-s) - s + \sqrt{s(s-\theta(1-s))}]/[\theta(1-s)]$ *such that the steady state is locally determinate (that is, a saddle or a source) when $\pi > \pi_{min}$ (see Fig. 2).*

Proof: See Appendix C.

Proposition 2.2 contains an important implication: it is easily shown (by L'Hôpital's rule) from the expression of π_{min} that it converges, from below, to one when θ tends to zero. In practice, $\theta = 1 - \beta(1-\delta)$ is bound to be close to zero when the period is commensurate with business-cycle length, as $\beta \approx 1$ and $\delta \approx 0$. *Therefore, for sensible parameter values, fully eliminating local sunspots may require close-to-maximal progressivity.* We examine quantitative aspects in Section 4.

We should emphasize that Proposition 2.2 does not rule out the case of a steady state being a source (locally) surrounded by a stable Hopf curve, in which case "global" sunspots and complex dynamics are possible (see Grandmont, Pintus, and de Vilder [16]). It is also possible that the steady state be a saddle surrounded by a stable period-two saddle created through a flip bifurcation. Here again, global sunspots could be constructed provided that the support contains the periodic orbit. Therefore, progressive schedules are ineffective to rule out stable deterministic cycles, and the associated global sunspot equilibria, that are located close to a *determinate* steady state. In other words, *although progressive fiscal schemes may rule out local indeterminacy, they do not help immunizing the economy against global expectation-driven business cycles.* In contrast, most results in the existing literature show that the usual "bifurcation" leading to local indeterminacy (whereby the steady state goes from a saddle to a sink, with eigenvalues going though infinity) can be prevented from occuring in the Ramsey model when progressivity is allowed.

2.3 Interpreting the Results

Our last step is to provide some intuitive explanation of the mechanisms at work that create some stabilizing power of progressive fiscal schedules. Roughly speaking, the main effect of progressivity tax rates can be interpreted as "taxing away the higher returns from belief-diven labor or investment spurts" (Guo and Lansing [19, p. 482]). However, note that in all models we are focusing on in this paper, returns to scale are *constant* both at the social level and at the firm level. In particular, the slope of labor demand (as a function of real wage) is negative so that fiscal progressivity does not reduce the likelihood of indeterminacy by changing the sign of labor demand's slope (in contrast with Guo and Lansing [19]). Moreover, we consider both taxes and transfers. Therefore, what remains to be elucidated is the sequence of events that make self-fulfilling expectations the driving force of the business cycle when taxes and transfers are progressive, even though externalities (or imperfect competition) are absent. More importantly, one would like to understand why *large* fiscal progressivity on factor income is required to stabilize the economy. As we now illustrate, key to the results is the fact that the more progressive taxes and transfers based on labor income, the more stable disposable wage income and, therefore, the less elastic workers' *labor supply*.

It is helpful to start with the benchmark case of a linear fiscal rate (which also covers the case with zero taxes and transfers) on labor income. In that case, workers' decisions are summarized by Eqs. (5) that may be written as follows, as ϕ reduces to the identity function and $\psi = 1$:

$$v_1(l_t) = v_2(p_t\omega_t l_t/p_{t+1}). \tag{12}$$

The latter first-order condition shows that when workers expect, in period t, that the price of goods p_{t+1} will go, say, down tomorrow, they wish to increase their consumption at $t+1$ and, therefore, to work more today (remember that gross substitutability is assumed) so as to save more in the form of money balances to be consumed tomorrow. Moreover, the dynamical system in Eqs. (8) may be written as follows:

$$\begin{cases} v_2(\omega_{t+1}l_{t+1}) & = & v_1(l_t), \\ k_t & = & \beta R(k_{t-1}/l_t)k_{t-1}. \end{cases} \tag{13}$$

A higher labor supply l_t will lead to greater output, larger consumption and a smaller capital-labor ratio k_{t-1}/l_t and, therefore, to a higher return on capital R_t, so that, from Eqs. (13), capital demand k_t and investment will increase. Moreover, a larger capital stock k_t tomorrow will tend to increase tomorrow's real wage ω_{t+1} which will trigger an increase in tomorrow's labor supply. However, a higher capital stock will also tend to increase the ratio of capital/labor and, eventually, the effect on capitalists' savings will turn negative: a higher capital-labor ratio leads to a lower rate of return on capital and, therefore, to lower capital demand and investment. This will lead to lower wage, lower labor supply, etc: the economy will experience a downturn. Note that this intuitive description relies on the presumption that both wage and interest rate are elastic enough to the capital-labor ratio: the elasticity of input substitution σ must be small enough.

Now, we would like to shed some light on why progressive fiscal rates makes the occurrence of self-fulfilling fluctuations less likely. Assume again, for simplicity, that ϕ has constant elasticity around the steady state. In that case, Eqs. (5) reduce to:

$$\gamma(l_t) = p_t\phi(\omega_t l_t)/p_{t+1}, \tag{14}$$

where $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$. When π increases from zero to one, the volatility of wage income decreases to zero: eventually, a highly progressive fiscal rate on labor income (that is, π close to one) leads to an almost constant wage bill, which in turn leads to *a more stable consumption and, thereby, a smaller reaction of labor* *supply to optimistic expectations*, in comparison to the case of linear taxes and transfers. More specifically, Eq. (14) shows that *a large progressivity π decreases the elasticity of labour supply to expected inflation and real wage.* To see this, differentiate Eqs. (14) to get:

$$(\varepsilon_\gamma - 1 + \pi)\frac{dl}{l} = -\frac{d\pi^e}{\pi^e} + (1-\pi)\frac{d\omega}{\omega}, \tag{15}$$

where π^e denotes expected inflation, that is, $\pi^e_{t+1} \equiv p_{t+1}/p_t$. Eq. (15) clearly shows how the higher fiscal progressivity π, the less elastic labor supply to both expected inflation and real wage, thereby limiting both the initial impact of expectations movements and their subsequent effect on labor supply. *Therefore, optimistic expectations (say, a reduction in p_{t+1}) lead to a smaller increase of consumption and labor when fiscal policy is highly progressive so that local indeterminacy is ruled out.* We now show how the results differ when fiscal rules are based on capital income.

2.4 Capital Income-Based Fiscal Progressivity

Our description of the mechanisms that account for the stabilizing power of progressive fiscal rules based on labor income also suggest that taxing/subsidizing capital income in a progressive manner is *not* expected to rule out local indeterminacy and bifurcations: in a nutshell, taxing (resp. subsidizing) capital income merely amounts to reducing (resp. increasing) capitalists' disposable revenues, thereby affecting their consumption (which is negligible when the discount factor is close enough to one) and investment demands, as seen from Eqs. (6). This does not alter workers' consumption and labor supply movements. In other words, only the second equation of the dynamical system (8) is affected. However, our analysis and interpretation in Sections 2.2 and 2.3 have shown that variations in workers' consumption and labor supply are at the origin of expectation-driven fluctuations so that taxing or subsidizing capitalists' income (even progressively) is not expected to rule out such business cycles. This is what we now formalize. If we apply our non-linear fiscal schedule (under Assumption 2.2) to capital income, it is not difficult to derive the dynamical system that now summarizes intertemporal equilibria (the proof is available from the authors upon request):

Definition 2.2

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence (a_t, k_{t-1}) *of* $\mathbf{R}^2_{++}$, $t = 1, 2, \ldots$, *such that*

$$\begin{cases} v_2(\omega(a_{t+1})k_t/a_{t+1}) & = & v_1(k_{t-1}/a_t), \\ k_t & = & \beta\psi(R(a_t)k_{t-1})\phi(R(a_t)k_{t-1}). \end{cases} \tag{16}$$

For sake of brevity, we assume that a normalized steady state exists and, moreover, that ϕ has constant elasticity $\psi = 1 - \pi$. One then easily derives:

$$T = T_1 - \frac{\varepsilon_\gamma - 1}{\varepsilon_\omega - 1}, \quad \text{with} \quad T_1 = 1 - \pi + \frac{(1-\pi)|\varepsilon_R| - 1}{\varepsilon_\omega - 1}, \tag{17}$$

$$D = \varepsilon_\gamma D_1, \quad \text{with} \quad D_1 = (1-\pi)\frac{|\varepsilon_R| - 1}{\varepsilon_\omega - 1}. \tag{18}$$

Insert Figure 3 here.

The impact of fiscal progressivity is summarized in Fig. 3. Starting from Fig. 1 (when $\pi = 0$), one can see from Fig. 3 that increasing π reduces, here again, the slope of Δ_1 (the locus such that $\varepsilon_\gamma = 1$). However, the intersection of Δ_1 with the line (AC) (when $\sigma = +\infty$) now moves *south-west* along (AC) when π increases. The main implication is that the qualitative picture is not much affected by the presence of capital income taxes: more precisely, local indeterminacy still prevails as long as labor supply is not too inelastic (that is, if ε_γ is not too large) and inputs are not too substitutable (that is, $\sigma < \sigma_I$), see Fig. 3. The expression of σ_I (such that $1 + T_1(\sigma) + D_1(\sigma) = 0$) turns out to be as follows.

Proposition 2.3 (Local Indeterminacy in Spite of Progressive Fiscal Rules)

Under Assumptions 2.1, 2.2 and 2.3, progressive taxes and transfers on capital income do not rule out local indeterminacy and bifurcations of the steady state of Eqs. (16).

More precisely, the steady state is a sink (locally indeterminate) if labor supply is sufficiently elastic (ε_γ

not too large) and if the elasticity of capital-labor substitution is small enough, that is, $\sigma < \sigma_I = s/2 + \theta(1-s)(1-\pi)/(2-\pi)$*, where* s *is the share of capital income (see Fig. 3). In particular,* $\sigma_I = s/2$ *when progressivity is maximal (that is, when* $\pi = 1$*).*

Again, as argued at the end of Subsection 2.2, θ is close to zero when the period is short (say, a year or less) so that *a progressive fiscal rules based on capital income reduces the scope of local indeterminacy, but only to a negligible extent.* We now show that different conclusions are obtained in an OLG economy in which progressive capital income taxes and transfers may stabilize labor supply and, thereby, immunize the economy against local indeterminacy.

3 Stabilization Through Income-Based Taxes and Transfers in an OLG Economy

3.1 Progressive Capital Income-Based Fiscal Rules

In the competitive, non-monetary economy studied in this section (see Reichlin [30]), a unique good is produced, which can be either consumed or saved as investment by a constant population of households living two periods. Agents are identical within each generation, supply labor and save their wage income in the form of capital when young, to be consumed when old. Using the same notation as in Section 2, agents born at time t solve the following program:

$$\text{maximize } \{V_2(c_{t+1}/B) - V_1(l_t)\} \text{ such that } k_t = \omega_t l_t, c_{t+1} = \phi(R_{t+1}k_t), c_{t+1} \geq 0, l_t \geq 0. \quad (19)$$

We start by imposing that taxes and transfers depend on savings income only (see next section for the case of fiscal rules based on labor income). Under Assumptions 2.1, 2.2 and 2.3, it is easily shown that the first-order conditions of the above problem are:

$$v_1(l_t) = \psi(R_{t+1}\omega_t l_t)v_2(c_{t+1}) \text{ and } c_{t+1} = \phi(R_{t+1}\omega_t l_t). \tag{20}$$

Therefore, intertemporal equilibria may here again be summarized by the dynamic behavior of both a and k, as follows.

Definition 3.1

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence (a_t, k_{t-1}) *of* $\mathbf{R}^2_{++}$, $t = 1, 2, \ldots$, *such that*

$$\left\{ \begin{array}{rcl} v_2(\phi(R(a_{t+1})k_t)) & = & v_1(k_{t-1}/a_t)/\psi(R(a_t)k_{t-1}), \\ k_t & = & \omega(a_t)k_{t-1}/a_t. \end{array} \right. \tag{21}$$

By comparing Eqs. (8) and (21), one uncovers the correspondence existing between the model with segmented asset markets and the present OLG model: the dynamical system (21) summarizing equilibria in the OLG economy can be obtained from (8) by changing the distribution of factor incomes. While young agents *save* their wage income and old agents *consume* their capital income, in the OLG setting, the reverse is true in the model of Section 2: workers consume their (real) wage income whereas capitalists save their capital income (entirely if they are extremely patient, that is, $\beta = 1$). This correspondence will obviously help to interpret the results and, in particular, to explain why capital income taxes and transfers may be desirable to stabilize the economy. But it does also lead one to derive easily the jacobian matrix of (21) and, therefore, the analog of Proposition 2.1: what is needed is simply to replace $\varepsilon_\omega - 1$ by ε_R and *vice-versa*. By adapting the procedure in Proposition B.1, one establishes the existence of a normalized steady state[1] and the following statements hold.

Proposition 3.1 (Linearized Dynamics around a Steady State)

[1]For sake of brevity, the proof is omitted.

Under Assumptions 2.1, 2.2 and 2.3, suppose that ϕ has constant elasticity in the neighborhood of a steady state $(\overline{a}, \overline{k})$ of the dynamical system in Eqs. (21) that is assumed to exist, i.e. $\psi(x) = 1 - \pi$, with $0 < \pi < 1$ measuring fiscal progressivity based on capital income. Let ε_R, ε_ω, ε_γ be the elasticities of the functions $R(a)$, $\omega(a)$, $\gamma(l)$, respectively, evaluated at the steady state $(\overline{a}, \overline{k})$. The linearized dynamics for the deviations $da = a - \overline{a}$, $dk = k - \overline{k}$ are determined by the linear map:

$$\begin{cases} da_{t+1} = -\frac{\varepsilon_\gamma/(1-\pi)+\varepsilon_\omega-1}{\varepsilon_R} da_t + \frac{\overline{a}}{\overline{k}} \frac{\varepsilon_\gamma/(1-\pi)-1}{\varepsilon_R} dk_{t-1}, \\ dk_t = \frac{\overline{k}}{\overline{a}}(\varepsilon_\omega - 1) da_t + dk_{t-1}. \end{cases} \tag{22}$$

The associated Jacobian matrix evaluated at the steady state under study has trace T and determinant D, where

$$T = T_1 + \frac{\varepsilon_\gamma - 1}{(1-\pi)|\varepsilon_R|}, \quad \textit{with} \quad T_1 = 1 + \frac{\varepsilon_\omega - 1 + 1/(1-\pi)}{|\varepsilon_R|},$$
$$D = \varepsilon_\gamma D_1, \quad \textit{with} \quad D_1 = \frac{\varepsilon_\omega}{(1-\pi)|\varepsilon_R|}.$$

Moreover, one has $T_1 = 1 + D_1 + \Lambda$, where $\Lambda \equiv \pi(1 - \varepsilon_\omega)/[(1-\pi)|\varepsilon_R|]$.

Proposition 3.1 reveals that the geometrical configuration is here qualitatively similar to the one that we have derived in Section 2. In particular, fixing technology and varying the elasticity of the offer curve ε_γ generates, in the (T, D) plane, a half-line Δ that starts close to (T_1, D_1) when ε_γ is close to one and whose slope is $\varepsilon_\omega = s/\sigma$.

<u>Insert Figures 4-5 here.</u>

It is then straightforward to derive the following expressions which critically depend on the elasticity of input substitution $\sigma \geq 0$ and fiscal progressivity $0 < \pi < 1$.

$$\begin{aligned} &D_1 = [1 - \delta(1-s)]/[(1-\pi)(1-s)], \quad \Lambda = \pi(\sigma - s)[1 - \delta(1-s)]/[(1-s)(1-\pi)], \\ &T_1 = 1 + D_1 + \Lambda, \qquad\qquad \textit{slope}_\Delta = s/\sigma. \end{aligned} \tag{23}$$

In the benchmark case with *linear (or no) taxes and transfers* (that is, $\pi = 0$ or $\psi = 1$), the situation is as in Fig. 4: the steady state is a sink and looses stability through Hopf bifurcations only when $\sigma < s$. The immediate implication is that endogenous cycles and sunspot equilibria occur only for low values of σ, that is, only if capital and labor are complementary enough. On the contrary, local determinacy prevails when $\sigma > s$. Just as in our first model of Section 2, it turns out that linear tax-and-transfer rates on capital income do not modify the range of parameter values compatible with local indeterminacy and bifurcations. However, inspection of Eqs. (23) shows that increasing π from zero to one increases D_1 from a positive value $[1 - \delta(1 - s)]/(1 - s)$ (which is plausibly assumed to be less than one when δ is close to one; see Fig. 4) to infinity. In other words, there exists a value π_{min}, illustrated by Fig. 5, above which the steady state is locally determinate (either a saddle or a source). This value is simply defined by the condition that $D_1 = 1$, or equivalently, $\pi_{min} \equiv (\delta(1 - s) - s)/(1 - s)$.

Proposition 3.2 (Local Stability and Bifurcations of the Steady State)

Under the assumptions of Proposition 3.1, the following generically holds when $\delta(1-s) > s$ *and* $\pi < \pi_{min} \equiv (\delta(1 - s) - s)/(1 - s)$ *(that is, fiscal progressivity is not too large).*

1. $0 < \sigma < s$*: the steady state is a sink for* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*, where* $\varepsilon_{\gamma H} \equiv (1 - \pi)(1 - s)/[1 - \delta(1 - s)]$ *is the value of* ε_γ *for which* Δ *crosses* $[BC]$*. Then the steady state undergoes a Hopf bifurcation (the complex characteristic roots cross the unit circle) at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$*, and is a source when* $\varepsilon_\gamma > \varepsilon_{\gamma H}$*.*

2. $s < \sigma$*: the steady state is a saddle when* $\varepsilon_\gamma > 1$*.*

3.2 Progressive Capital Income Taxes and Transfers as Automatic Stabilizers

The most important implication of Proposition 3.2 is that the half-line Δ intersects the indeterminacy triangle ABC only if π is small enough: in other words, *there exists a threshold level of fiscal progressivity*

π_{min} above which the steady state is locally determinate, thereby here again excluding local sunspots but, however, not stable cycles and global sunspots.

Proposition 3.3 (Ruling Out Local Indeterminacy Through Progressive Fiscal Rules)

Under the assumptions of Proposition 3.2, there exists a threshold level of fiscal progressivity on capital income $\pi_{min} \equiv (\delta(1-s)-s)/(1-s)$ such that the steady state is locally determinate (that is, a saddle or a source) when $\pi > \pi_{min}$ (see Fig. 5).

Our last step aims at providing an intuitive description of the mechanisms that account for the stabilizing power of progressive taxes and transfers on capital income. It is relevant to recall the correspondence noticed above between the two models considered in this paper. Roughly speaking, one goes from one model to the other by inversing the distribution of factor incomes. This implies that if fiscal schemes that are progressive and based on *labor* income rules out local indeterminacy and bifurcations in the heterogeneous agents economy, as we have shown, one expects that fiscal schemes related to *capital* income should have the same effects in the OLG setting. We now go a little further to be more specific about the basic mechanisms at work. As in the previous model (in Section 2), tomorrow's consumption and today's labor supply movements following a wave of optimism are necessary to sustain expectation-driven equilibria. Specifically, the first-order conditions in Eqs. (20) can be rewritten as:

$$\gamma(l_t) = \phi(R_{t+1}\omega_t l_t), \tag{24}$$

where $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$ and ϕ has, for simplicity, constant elasticity around the steady state. Therefore, direct inspection of Eq. (24) reveals that *the more progressive the fiscal rate on capital income, the more stable consumption and labor supply.* More precisely, if young agents expect, in period t, the return on capital R_{t+1} to go, say, up, they wish to increase tomorrow's consumption and, therefore, to work more today. However, a larger capital stock following the investment boom will eventually depress the return on capital and lower capital demand, turning the economy into a recession because of pessimistic expectations.

When π increases from zero to one, the volatility of capital income decreases to zero: eventually, a highly progressive fiscal rate (that is, π close to one) leads to almost constant capital income and tomorrow's consumption, which in turn leads to a much smaller reaction of labor supply to optimistic expectations, in comparison to the case of linear taxes. More specifically, with π close to one, ϕ is almost constant and, in that case, Eq. (24) show that *a large progressivity π decreases the elasticity of labour supply to expected return on capital and real wage.* To see this, differentiate Eqs. (24) to get:

$$(\varepsilon_\gamma - 1 + \pi)\frac{dl}{l} = (1-\pi)[\frac{dR}{R} + \frac{d\omega}{\omega}]. \tag{25}$$

Eq. (25) clearly shows how the higher capital income fiscal progressivity π, the less elastic labor supply to both expected capital return and real wage, thereby limiting both the initial impact of expectations movements and their subsequent effect on labor supply. *Therefore, optimistic expectations (say, an increase in R_{t+1}) lead to a smaller increase of consumption and labor when the capital fiscal rate is highly progressive so that local indeterminacy are ruled out.*

In view of the above discussion, similar results are expected to hold in an extended version of the OLG model in which young agents may consume a fraction of their wage income. More precisely, it is known that the higher the propensity to consume out of wage income, the less likely local indeterminacy and bifurcations (see Cazzavillan and Pintus [9]). Therefore, one expects that taxing (in a progressive way) capital income in such setting would still stabilize *old agents'* consumption and make local indeterminacy less likely. However, allowing for consumption in the first period of life would make the analysis heavier.

Moreover, our intuitive discussion also gives a hint on why progressive taxes and transfers on *labor* income do not rule out local indeterminacy: this is simply not helpful to stabilize tomorrow's consumption and today's labor supply. In fact, it turns out that this *enlarges* the range of parameters values compatible with sunspots and cycles, as shown in Fig. 6.

Insert Figure 6 here.

In that case, one can show (computations are available from the authors upon request) that the steady state is a sink (locally indeterminate) when ε_γ is not too large and $\sigma < \sigma_I \equiv s(1-s)/[1-\delta(1-s)]$ (see Fig. 12, which covers the case $\pi > \pi_{min}$). For instance, $\sigma_I = 1-s > s$ when, plausibly, $\delta = 1$ and $s < 1/2$. Therefore, in comparison with the case with linear or progressive taxes and transfers on capital income, opposite results are obtained, as the range of σ's for which local indeterminacy and bifurcations occur is enlarged by the introduction of progressive fiscal rules based on labor income. In view of our intuitive discussion above, this is straightforward to explain. In that case, the first-order conditions of young agents' problem are:

$$\gamma(l_t) = R_{t+1}\phi(\omega_t l_t), \tag{26}$$

as only wage income is affected by fiscal policy. Eq. (26) clearly shows that wage income is almost stable when the fiscal rate on labor income is highly progressive. However, *this is not enough to stabilize labor supply and tomorrow's consumption that depend also on movements of the interest rate.* Therefore, consumption and labor supply will still be subject to endogenous volatility, however progressive taxes and transfers on labor income are. Now, to explain that progressive fiscal schemes based on labor income make local indeterminacy *more* likely requires recalling that, in this model, cyclical paths (be they deterministic or stochastic) arise because of two conflicting effects on savings that operate through wage and interest rate (see Cazzavillan and Pintus [9]). More precisely, when the capital stock increases (say, from its steady state value), this triggers an increase in wage and savings that will be, eventually, reversed by a lower interest rate that, on the contrary, depresses savings. For a given deviation of the capital stock, the (disposable) wage effect will be smaller when progressive taxes and transfers on labor income are introduced so that the effect of the interest rate will be more likely to reverse it.

4 Assessing the Stabilization Effect of Progressive Fiscal Rules

In this section, we derive some quantitative implications about the (local) stabilization effect of progressive taxes and transfers, when some parameter values are considered. Let us start with the model of Section 2. Consider the standard values $\beta = 0.96$, $\delta = 0.1$ (based on annual data) and $s = 1/3$. Then, from Proposition 2.2, one has that $\pi_{min} \approx 0.92$ (moreover, $1 - \theta(1-s)/s \approx 0.73$ and $[s - \theta(1-s)]/[s - \theta(1-s)/2] \approx 0.84$). If one focuses on taxes, this seems a high value, e.g. with respect to actual US data on average marginal tax rates. For example, our computations from Stephenson [35, Table 1, p. 391] deliver that income tax progressivity has ranged in $4\% - 11\%$ over the last sixty years (in accord with some recent evaluations by Cassou and Lansing [7], for example). With alternative measures in mind, Caucutt, Imrohoroglu and Kumar [8, p. 550] argue in favor of values approaching 50%, while Bénabou [4, pp. 501-2] concludes that 20% is a reasonable estimate for tax progressivity.

On the other hand, transfers are typically found to be more progressive than taxes. For instance, Englund and Persson [13] report a value of $\pi = 63\%$ for Sweden (see also Davidson and Duclos [11] who present related results for Canada). In our analysis, it is total progressivity (of *both* taxes and transfers) that is relevant. In view of the reported evidence, one may conclude that the level of progressivity that is required to rule out local indeterminacy in the heterogenous-agent model of Section 2 seems too high. More precisely, case 1 in Proposition C.1 (see Fig. C1) is most likely to occur. However, one is led to a different assessment of the plausibility of progressive fiscal rules as automatic stabilizers in the OLG model of Section 3. By adopting the standard values $s = 1/3$ and $\delta = 0.96$ (which corresponds to an annual depreciation rate of 0.1 if the period length is 30 years), Proposition 3.3 yields $\pi_{min} \approx 0.46$. *The critical level of fiscal progressivity is, in the OLG setting, half as high as the threshold value obtained in the heterogenous-agent model. Moreover, it falls within the range of estimates that include taxes and transfers we just discussed.*[2]

[2] Our numerical examples are reminiscent of early discussions about the practical importance of "built-in flexibility", e.g. in Musgrave and Miller [27], Vickrey [37, 38], Slitor [34].

5 Conclusion

In agreement with the recent literature, we have shown that income-based taxes and transfers may immunize the economy against local sunspots, provided though that the level of progressivity is high enough and if progressive fiscal rules are applied to the relevant fiscal base so as to stabilize labor supply movements. More surprisingly, however, we have established that progressive fiscal schemes do not help immunizing the economy against "global" cycles (originated though local bifurcations) and sunspots. Incidentally, our results suggest that capital income taxes and transfers may be desirable, when progressive, to reduce the likelihood of expectation-driven volatility. We have argued that progressivity is, in the real world, probably lower than the threshold values predicted by the heterogeneous-agent model, above which local determinacy of the steady state prevails. However, fiscal schemes may rule out local indeterminacy and bifurcations in the OLG setting for admissible levels of progressivity. Moreover, our analysis suggests that less-than-maximal progressivity may still be helpful to reduce the range of parameter values that are compatible with local sunspots.

Some directions for future research naturally follow. It would be useful to generalize the analysis to the realistic case whereby progressivity is increasing with income. In the OLG setting, it seems relevant to introduce consumption/savings choices in the first period of life. In that context, one expects that a smaller level of progressivity on *both* incomes could rule out local indeterminacy and bifurcations by stabilizing both young and old agents' consumptions. It remains to be seen if this is more in line with actual levels of fiscal progressivity. It is also expected that progressive taxes and transfers are inefficient to rule out endogenous fluctuations when consumption is financed, even partially, by the returns from financial assets that would remain untaxed (e.g. in the monetary economy studied by Benhabib and Laroque [5]). Moreover, although we have emphasized the stabilizing power of capital taxation in the OLG setting, similar results are expected in other frameworks, e.g. in models with credit-constrained firms and collateral requirements related to cash-flows. Finally, it would also be helpful to derive the main features of optimal tax-and-transfer schedules in our two benchmark models. We plan to further pursue this line of research in the near future.

A Progressive Labor Income-Based Taxes and Transfers and Workers' Choices

In this section, we show how workers' decisions can be reduced to a two-period problem. Workers solve the following problem:

$$\max \sum_{t=1}^{+\infty} \beta_w^{t-1} V_2(c_t^w/B) - \beta_w^t V_1(l_t), \tag{27}$$

subject to

$$M_{t-1}^w + (r_t + (1-\delta)p_t)k_{t-1}^w + p_t\phi(\omega_t l_t) \geq p_t c_t^w + p_t k_t^w + M_t^w, \tag{28}$$

$$M_{t-1}^w + (r_t + (1-\delta)p_t)k_{t-1}^w \geq p_t c_t^w + p_t k_t^w, \tag{29}$$

where $B > 0$ is a scaling parameter, $0 < \beta_w < 1$ is the discount factor, $c_t^w \geq 0$ is consumption, $l_t \geq 0$ is labor supply. On the other hand, $M_{t-1}^w \geq 0$ and $k_{t-1}^w \geq 0$ are respectively money demand and capital holdings at the beginning of period t, $p_t > 0$ is the price of consumption goods, $w_t > 0$ is nominal wage, $r_t > 0$ is nominal return on capital and $1 \geq \delta \geq 0$ is capital depreciation, while $p_t\phi(\omega_t l_t)$ is disposable wage income (see Section 2) and $\omega = w/p$ defines real wage.

Define $\lambda_t \geq 0$ and $\epsilon_t \geq 0$ as the Lagrange multipliers associated, respectively, to (28) and (29) at date t. Necessary conditions are then the following.

$$\begin{aligned} 0 &\geq \beta_w^{t-1} V_2'(c_t^w/B)/B - (\lambda_t + \epsilon_t)p_t, && = 0 \text{ if } c_t^w > 0, \\ 0 &\geq -(\lambda_t + \epsilon_t)p_t + (\lambda_{t+1} + \epsilon_{t+1})(r_{t+1} + (1-\delta)p_{t+1}), && = 0 \text{ if } k_t^w > 0, \\ 0 &\geq -\lambda_t + \lambda_{t+1} + \epsilon_{t+1}, && = 0 \text{ if } M_t^w > 0, \\ 0 &\geq -\beta_w^t V_1'(l_t) + \lambda_t p_t \omega_t \phi'(\omega_t l_t), && = 0 \text{ if } l_t > 0. \end{aligned} \tag{30}$$

Therefore, capital holdings are zero at all dates ($k_t^w = 0$) if the second inequality of (30) is not binding, that is, if:

$$V_2'(c_t^w/B) > \beta_w(r_{t+1}/p_{t+1} + 1 - \delta)V_2'(c_{t+1}^w/B), \tag{31}$$

if one assumes that $c_t^w > 0$ for all t (we will show that this is the case around the steady state). Condition (31) implies that workers choose not to hold capital, and it depends on workers' preferences because of the financial constraint (29).

Moreover, the financial constraint (29) is binding if $\epsilon_t > 0$, that is, if:

$$\omega_t\phi'(\omega_t l_t)V_2'(c_t^w/B)/B > \beta_w V_1'(l_t), \tag{32}$$

if one assumes that $l_t > 0$ (again, we will show that this is the case around the steady state). Condition (32) therefore implies that (29) is binding.

Under conditions (31) and (32), workers spend their money holdings , i.e. $p_t c_t^w = M_{t-1}^w$, and save their wage income in the form of money, i.e. $M_t^w = p_t\phi(\omega_t l_t)$, so as to consume it tomorrow, i.e. $p_{t+1}c_{t+1}^w = M_t^w$. Therefore, workers choose $l_t \geq 0$ and $c_{t+1}^w \geq 0$ as solutions to:

$$\max \{V_2(c_{t+1}^w/B) - V_1(l_t)\} \text{ s.t. } p_{t+1}c_{t+1}^w = p_t\phi(\omega_t l_t). \tag{33}$$

The solutions to (33) are unique under Assumption 2.2 and 2.3 and characterized by the following first-order condition, which is identical to (5) in the main text:

$$v_1(l_t) = \psi(\omega_t l_t)v_2(c_{t+1}^w),\ p_{t+1}c_{t+1}^w = p_t\phi(\omega_t l_t), \tag{34}$$

where $v_2(c) \equiv cV_2'(c/B)/B$, $v_1(l) \equiv lV_1'(l)$.

Finally, it is straightforward to show that, under the assumptions that capitalists discount future less heavily than workers (that is, $\beta_w < \beta$) and that $\beta_w < 1$, conditions (31) and (32) are met at the steady state under study defined in Proposition B.1. $\square$

B Existence of Steady State: Proposition B.1

Proposition B.1 (Existence of a Normalized Steady State)

Under Assumptions 2.1, 2.2 and 2.3, $\lim_{c\to 0} cV_2'(c) < V_1'(1)/\psi(\omega(1)) < \lim_{c\to+\infty} cV_2'(c)$, $(\overline{a}, \overline{k}) = (1,1)$ *is a steady state of the dynamical system in Eqs. (8) if and only if* $A = (1/\beta - 1 + \delta)/f'(1)$ *and* B *is the unique solution of* $\psi(\omega(1))\phi(\omega(1))V_2'(\phi(\omega(1))/B)/B = V_1'(1)$.

Proof: In view of Eqs. (8) and recalling that $a = k/l$, the nonautarkic steady states are the solutions $(\overline{a}, \overline{l})$ in $\mathbf{R}^2_{++}$ of $v_2(\phi(\omega(\overline{a})\overline{l})) = v_1(\overline{l})/\psi(\omega(\overline{a})\overline{l})$ and $\beta R(\overline{a}) = 1$. Equivalently, in view of Eq. (3), the steady states are given by:

$$\begin{cases} v_2(\phi(\omega(\overline{a})\overline{l})) &= v_1(\overline{l})/\psi(\omega(\overline{a})\overline{l}), \\ \rho(\overline{a}) + 1 - \delta &= 1/\beta. \end{cases} \tag{35}$$

We shall solve the existence issue by setting appropriately the scaling parameters A and B, so as to ensure that one stationary solution coincides with, for instance, $(\overline{a}, \overline{l}) = (1,1)$. The second equality of Eqs. (35) is achieved by scaling the parameter A, while the first is achieved by scaling the parameter B. That is, we set $A = (1/\beta - 1 + \delta)/f'(1)$ to ensure that $\overline{a} = 1$. On the other hand, $\psi(\omega(\overline{a})\overline{l})v_2(\overline{c}) = v_1(\overline{l})$ is then equivalent to

$$\psi(\omega(1))\frac{\phi(\omega(1))}{B}V_2'(\frac{\phi(\omega(1))}{B}) = V_1'(1). \tag{36}$$

From Assumption 2.3, v_2 is decreasing in B so the latter condition is satisfied for some unique B if and only if:

$$\lim_{c\to 0} cV_2'(c) < V_1'(1)/\psi(\omega(1)) < \lim_{c\to+\infty} cV_2'(c). \tag{37}$$

□

C Proofs of Propositions C.1 and 2.2

C.1 Proposition C.1: Statement and Proof

We show that the more progressive taxes and transfers (the higher π), the less likely the half-line Δ is to cross the indeterminacy triangle ABC, as shown in Figs. C1-C3, and that there exists a minimal level of fiscal progressivity π_{min} (illustrated in Fig. 2) above which Δ does *not* intersect ABC: the steady state is either a saddle or a source so that there exists a neighborhood in which no local sunspots occur.

Insert Figures C1-C3 here.

The key implication of increasing progressivity π from zero can be seen, starting with the benchmark case with linear taxes (see Fig. 1), by focusing on how the two following points vary with π (see Figs. C1-C3). First, direct inspection of Eqs. (11) shows that Λ (the deviation of (T_1, D_1) from (AC)) is negative when σ is small enough (that is, $T_1 < 1 + D_1$ when $\sigma < s$). In fact, the locus of (T_1, D_1) generated when σ increases from zero describes a line Δ_1 which intersects (AC) at point I when $\sigma = +\infty$ (i.e. $\Lambda = 0$). From Eqs. (11), one immediately sees that $D_1(\sigma = +\infty)$ increases with π, so that point I goes north-east when π increases from zero. Second, Eqs. (11) imply that Δ_1 intersects the T-axis of equation $D = 0$ when $\sigma = \theta(1 - s)$ (that is, $D_1 = 0$), and that $\Lambda(\sigma = \theta(1 - s))$ decreases, from zero, with π. An equivalent way of summarizing these two observations is that, when π increases from zero, point I (where Δ_1 intersects (AC)) goes north-east, along (AC), whereas the *slope of* Δ_1 decreases from one, so that three different configurations occur in the (T, D) plane (see Figs. C1-C3).

Proposition C.1 (Local Stability and Bifurcations of the Steady State)

Consider a steady state that is assumed to be set at $(\overline{a}, \overline{k}) = (1, 1)$ *through the procedure in Proposition B.1. If, moreover,* $\theta(1 - s) < s$ *and* $\pi < \pi_{min} \equiv 2[\theta(1 - s) - s + \sqrt{s(s - \theta(1 - s))}]/[\theta(1 - s)]$ *(that is, fiscal* *progressivity is not too large), the following generically holds.*[3]

1. *If* $0 < \pi < 1 - \theta(1-s)/s$*, that is, fiscal progressivity is small enough (see Fig. C1):*
   (a) $0 < \sigma < \sigma_F$*: the steady state is a sink for* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*, where* $\varepsilon_{\gamma H}$ *is the value of* ε_γ *for which* Δ *crosses* $[BC]$*. Then the steady state undergoes a Hopf bifurcation (the complex characteristic roots cross the unit circle) at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$*, and is a source when* $\varepsilon_\gamma > \varepsilon_{\gamma H}$*.*

   (b) $\sigma_F < \sigma < \sigma_H$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*. Then the steady state undergoes a Hopf bifurcation at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$ *and is a source when* $\varepsilon_{\gamma H} < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs (one characteristic root goes through* -1*) at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle when* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

   (c) $\sigma_H < \sigma < \sigma_I$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle if* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

   (d) $\sigma_I < \sigma < s$ *and* $s < \sigma$*: the steady state is a saddle when* $\varepsilon_\gamma > 1$*.*

2. $1 - \theta(1-s)/s < \pi < [s - \theta(1-s)]/[s - \theta(1-s)/2]$ *(see Fig. C2):*
   (a) $0 < \sigma < \sigma_J$*: the steady state is a source when* $\varepsilon_\gamma > 1$*.*

   (b) $\sigma_J < \sigma < \sigma_F$*: the steady state is a sink for* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*. Then the steady state undergoes a Hopf bifurcation at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$*, and is a source when* $\varepsilon_\gamma > \varepsilon_{\gamma H}$*.*

   (c) $\sigma_F < \sigma < \sigma_H$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*. Then the steady state undergoes a Hopf bifurcation at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$ *and is a source when* $\varepsilon_{\gamma H} < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle when* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

   (d) $\sigma_H < \sigma < \sigma_I$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle if* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

   (e) $\sigma_I < \sigma < s$ *and* $s < \sigma$*: the steady state is a saddle when* $\varepsilon_\gamma > 1$*.*

[3] The expressions of σ_F, σ_H, σ_I, σ_J, $\varepsilon_{\gamma H}$ and $\varepsilon_{\gamma F}$ are given in the proof of the proposition in Appendix C.

3. $[s-\theta(1-s)]/[s-\theta(1-s)/2] < \pi < \pi_{min}$ *(see Fig. C3):*

(a) $0 < \sigma < \sigma_J$*: the steady state is a source when* $\varepsilon_\gamma > 1$*.*

(b) $\sigma_J < \sigma < \sigma_H$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*. Then the steady state undergoes a Hopf bifurcation at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$ *and is a source when* $\varepsilon_{\gamma H} < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle when* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

(c) $\sigma_H < \sigma < \sigma_I$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle if* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

(d) $\sigma_I < \sigma < s$ *and* $s < \sigma$*: the steady state is a saddle when* $\varepsilon_\gamma > 1$*.*

Proof: To prove formally the occurrence of three configurations, depending on π, our first task is to show that the point $(T_1(\sigma), D_1(\sigma))$, as a function of σ, indeed describes part of a line Δ_1. From the fact that $T_1(\sigma) = 1 + D_1(\sigma) + \Lambda(\sigma)$ and $D_1(\sigma)$ are fractions of first degree polynomials in σ with the same denominator (see Eq. (11)), we conclude that the ratio of their derivatives $D_1'(\sigma)/T_1'(\sigma)$, or $D_1'(\sigma)/(D_1'(\sigma) + \Lambda'(\sigma))$, is independent of σ. Straightforward computations show that the slope of Δ_1 is:

$$slope_{\Delta_1} = \frac{D_1'(\sigma)}{T_1'(\sigma)} = \frac{s-\theta(1-s)}{s-\theta(1-s)+\pi\theta(1-s)}. \tag{38}$$

From Eq. (11), we conclude that $\Lambda(\sigma)$ vanishes when σ goes to infinity. It follows that Δ_1 intersects the line (AC) at a point I of coordinates $(T_1(+\infty), D_1(+\infty))$, where $D_1(+\infty) = 1/(1-\pi) > 0$ (see Figs. C1-C3). We shall focus throughout on the configuration presented in Figs. C1-C3, where $D_1(+\infty) \geq 1$ and the slope of Δ_1 is smaller than 1 (that is, $\pi \geq 0$). We shall ensure the latter condition by imposing, as in the case of linear (or of no) taxes, that $\theta(1-s) < s$ (that is, the share of capital is large enough). This condition is not very restrictive when $\theta = 1-\beta(1-\delta)$ is small, which is bound to be the case when the period is short since β is then close to one and δ is close to zero. Note that the geometrical method can be applied as well when these conditions are not met.

Then it follows that both $\Lambda(\sigma)$ and $D_1(\sigma)$ are decreasing functions (see Eq. (11)), so that $T_1(\sigma)$ is also a decreasing function, i.e. $T_1'(\sigma) = D_1'(\sigma) + \Lambda'(\sigma) < 0$. Accordingly, the slope of Δ_1 is smaller than one. From the above assumptions, one also gets all the necessary information to appraise the variations of $(T_1(\sigma), D_1(\sigma))$ as well as of the slope of Δ, when σ moves from 0 to $+\infty$. In particular, $T_1(0)$ and $D_1(0) = \theta(1-s)/[s(1-\pi)]$ are positive and the corresponding point is above I on the line Δ_1 when $\pi > 0$ (see Figs. C1-C3). As σ increases from 0, $T_1(\sigma)$ and $D_1(\sigma)$ are decreasing and tend to $-\infty$ when σ tends to s from below. When $\sigma = s$, the function $\omega(a)/a$ of a has a critical point, i.e. its derivative with respect to a vanishes, and the dynamical system derived from Eqs. (8) is not defined. When σ increases from s to $+\infty$, $T_1(\sigma)$ and $D_1(\sigma)$ are still both decreasing, from $+\infty$ to $(T_1(+\infty), D_1(+\infty))$, which is represented by the point I in Figs. C1-C3. On the other hand, the intersection of Δ_1 with $[BC]$ is characterized by $D_1(\sigma) = 1$ which leads, in view of Eq. (11), to $\sigma_J = [\theta(1-s)-s(1-\pi)]/\pi$. In addition, the slope of Δ as a function of σ increases monotonically from $-\infty$ to 1 as σ moves from 0 to $+\infty$, and vanishes when $D_1(\sigma) = 0$. Moreover, the half-line Δ is above Δ_1 when $\sigma < s$, and below it when $\sigma > s$.

Therefore, three configurations arise when π increases from zero. When $\pi < 1 - \theta(1-s)/s$ (case *1*) then $D_1(0) < 1$: the geometric picture is as in Fig. 2 and it is not qualitatively different from the case of linear (or no) taxes (compare Figs. 1 and 2). Second, $D_1(0) > 1$ when $\pi < 1 - \theta(1-s)/s$, and two cases arise depending on whether π is smaller or larger than $[s - \theta(1-s)]/[s - \theta(1-s)/2]$. More precisely, the slope of Δ at $\sigma = \sigma_J$ (that is, when $D_1(\sigma_J) = 1$) is smaller than -1) when $\pi < [s - \theta(1-s)]/[s - \theta(1-s)/2]$ (case *2*). In that case, the steady state is a source when σ is close enough to zero ($0 < \sigma < \sigma_J$) and it undergoes the same sequences of bifurcations as in case *1* when $\sigma > \sigma_J$. Finally, case *3* occurs when the slope of Δ at $\sigma = \sigma_J$ is larger than -1, that is, when $\pi > [s - \theta(1-s)]/[s - \theta(1-s)/2]$ and $\pi < \pi_{min}$. □

C.2 Proposition C.1: Local Bifurcation Values

In this subsection, we derive all bifurcation values as functions of the structural parameters. We define $\theta \stackrel{\text{def}}{=} 1 - \beta(1-\delta)$, and $s_\Delta(\sigma) \stackrel{\text{def}}{=} 1 - \theta(1-s)/\sigma$ as the slope of the half-line Δ.

An eigenvalue of -1: the flip bifurcation.

The equality $s_\Delta(\sigma) = -1$ allows one to derive $\sigma_F = \theta(1-s)/2$, so that $s_\Delta(\sigma) < -1$ when $\sigma < \sigma_F$.

Equation $1 + T(\varepsilon_\gamma) + D(\varepsilon_\gamma) = 0$ yields $\varepsilon_{\gamma F} = (1-\pi)(2s + \theta(1-s) - 2\sigma)/[2\sigma - \theta(1-s)]$.

The condition that $1 + T_1(\sigma) + D_1(\sigma) = 0$ or, equivalently $\varepsilon_{\gamma F} = 1$, gives the last flip bifurcation value $\sigma_I = [\theta(1-s)(2-\pi) + 2s(1-\pi)]/[2(2-\pi)]$ so that $\varepsilon_{\gamma F} > 1$ when $\sigma < \sigma_I$.

A pair of eigenvalues of modulus 1: the Hopf bifurcation.

The condition that $T(\varepsilon_{\gamma H}) = -2$ when $D = 1$, i.e. when $\varepsilon_\gamma = \varepsilon_{\gamma H} = 1/D_1$, is rewritten as $Q_H(\sigma) \stackrel{\text{def}}{=} a\sigma^2 + b\sigma + c$, the roots of which contain the bifurcation value σ_H. The coefficients of $Q_H(\sigma)$ are:

$$\begin{aligned} a &= 4, \\ b &= -4[s + \theta(1-s)], \\ c &= \theta(1-s)[\theta(1-s) + 3s]. \end{aligned}$$

It is easily shown that there must exist two distinct real roots, and that $\sigma_H = s[1+\theta(1-s)/s - \sqrt{1-\theta(1-s)/s}]/2$ is the lowest.

The condition $D_1(\sigma) = 1$ yields, in view of Eqs. (11), $\sigma_J = [\theta(1-s) - s(1-\pi)]/\pi$. Morever, the bifurcation value $\varepsilon_{\gamma H} = (1-\pi)(s-\sigma)/[\theta(1-s) - \sigma)]$ follows from $D = \varepsilon_\gamma D_1 = 1$, i.e. $\varepsilon_{\gamma H} = 1/D_1$. □

C.3 Proof of Proposition 2.2

Proving Proposition 2.2 relies on the geometrical configuration and local bifurcation values of Proposition C.1, as exposed in Subsections C.1 and C.2. In particular, there exists a critical value π_{min} such that Δ_1 goes through point B (see Fig. 2). The expression π_{min} is the value of π that solves $\sigma_I = \sigma_J$, that is, $\pi \equiv 2[\theta(1-s) - s + \sqrt{s(s-\theta(1-s))}]/[\theta(1-s)]$. This implies that Δ does not intersect the ABC triangle when $\pi > \pi_{min}$. Therefore, the steady state is either a saddle or a source for all $\varepsilon_\gamma > 1$. □

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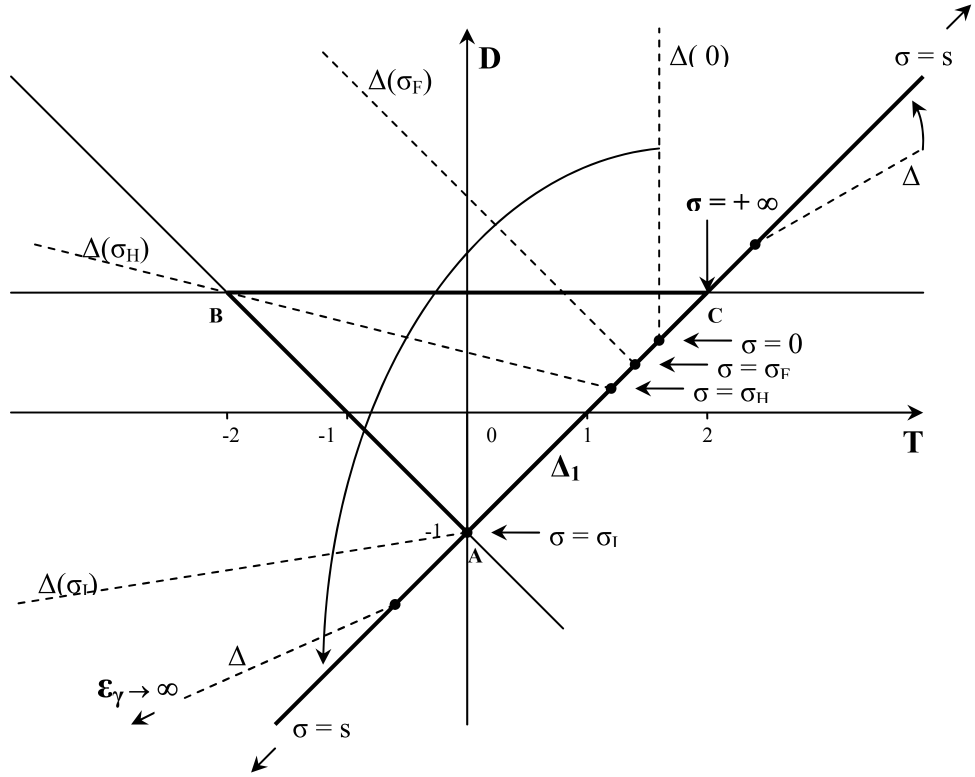

Figure 1
The heterogeneous agents model with linear taxes and transfers

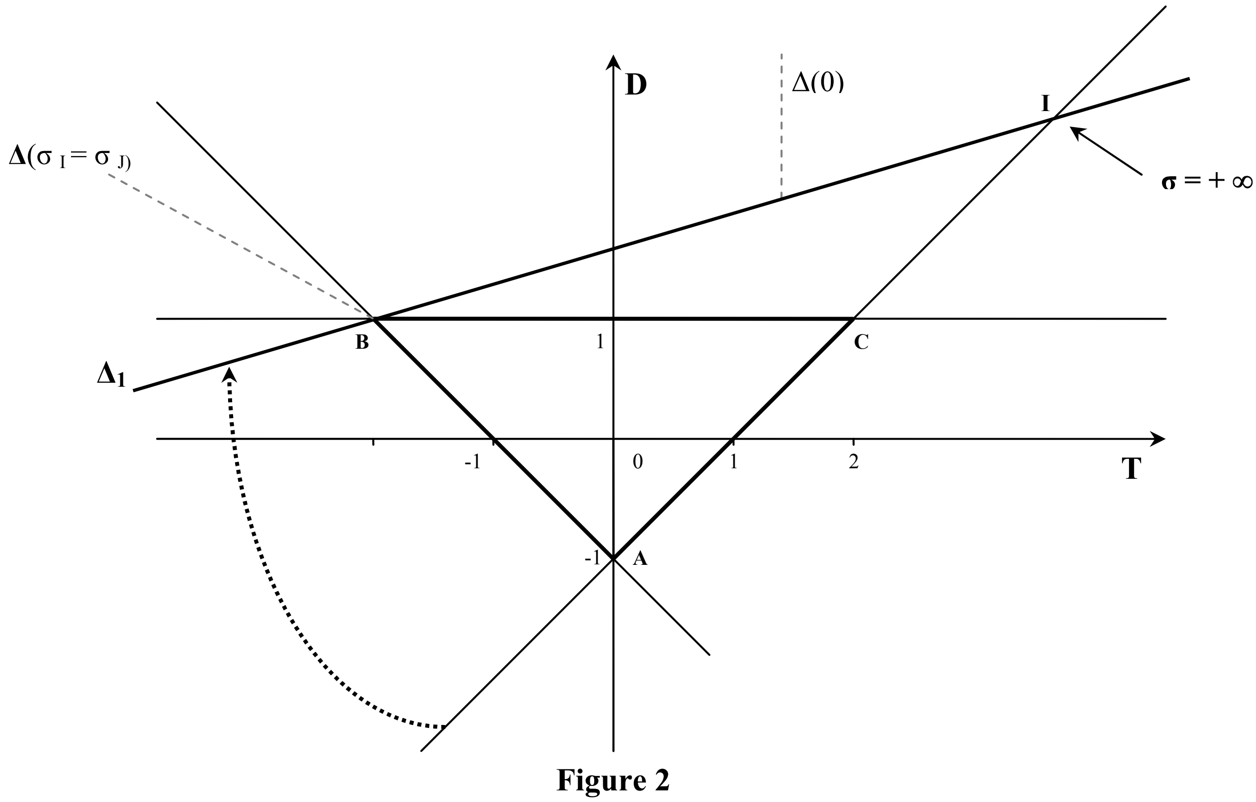

Figure 2
Threshold level of progressive taxes and transfers on labor income π_{min}
in the heterogeneous agents model

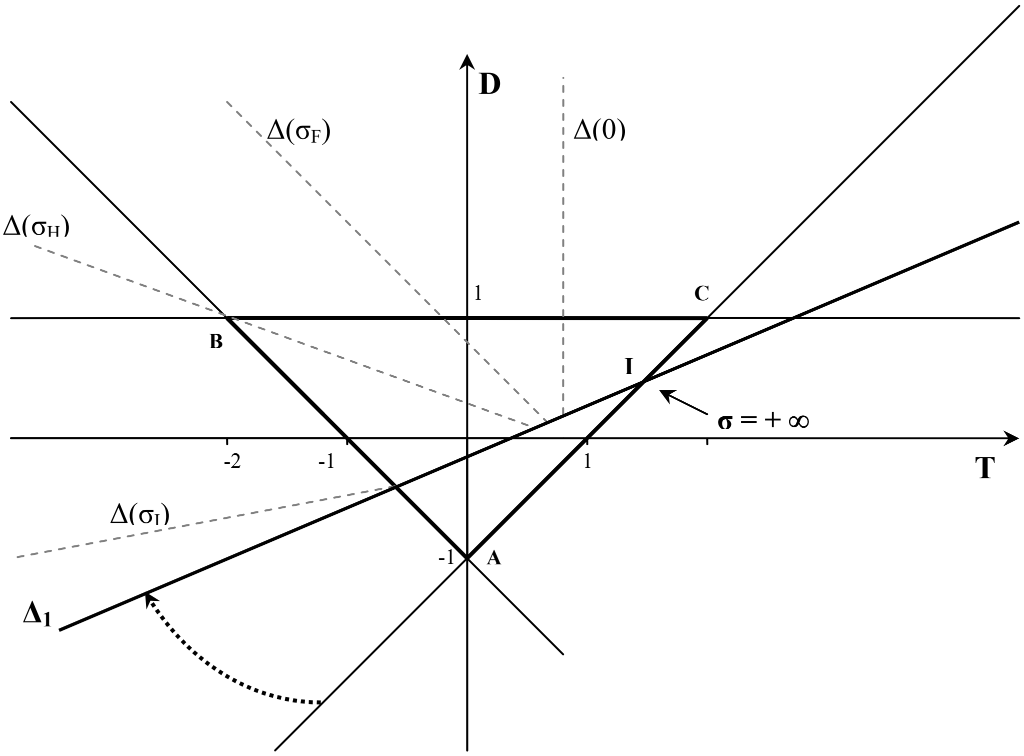

Figure 3
Progressive taxes and transfers on capital income
in the heterogeneous agents model

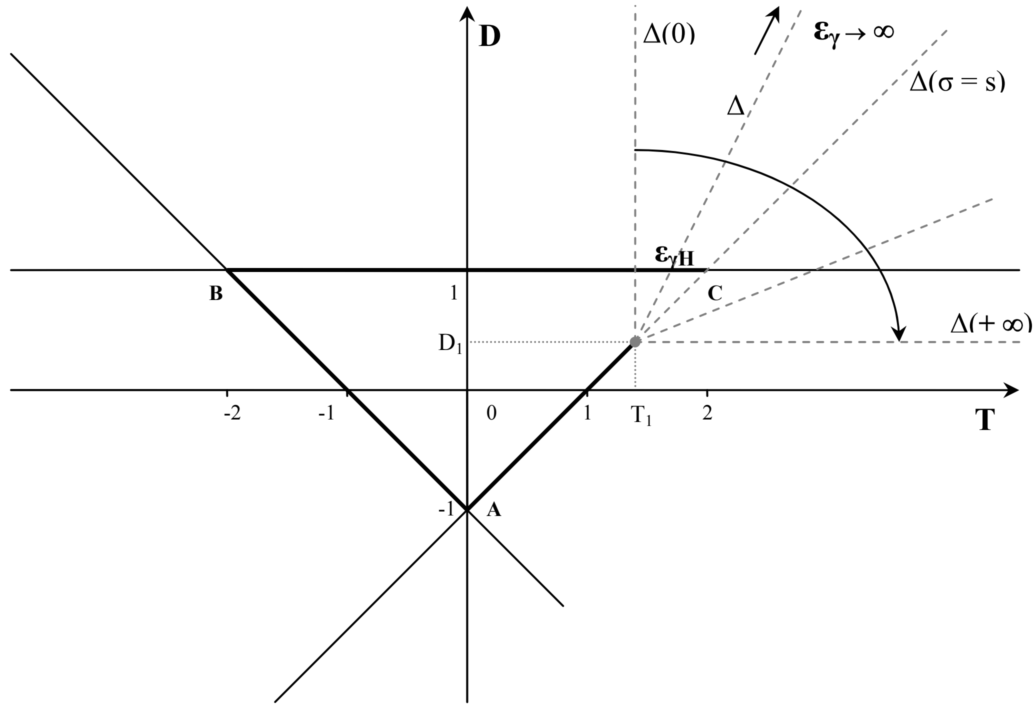

Figure 4
The OLG model with linear taxes and transfers

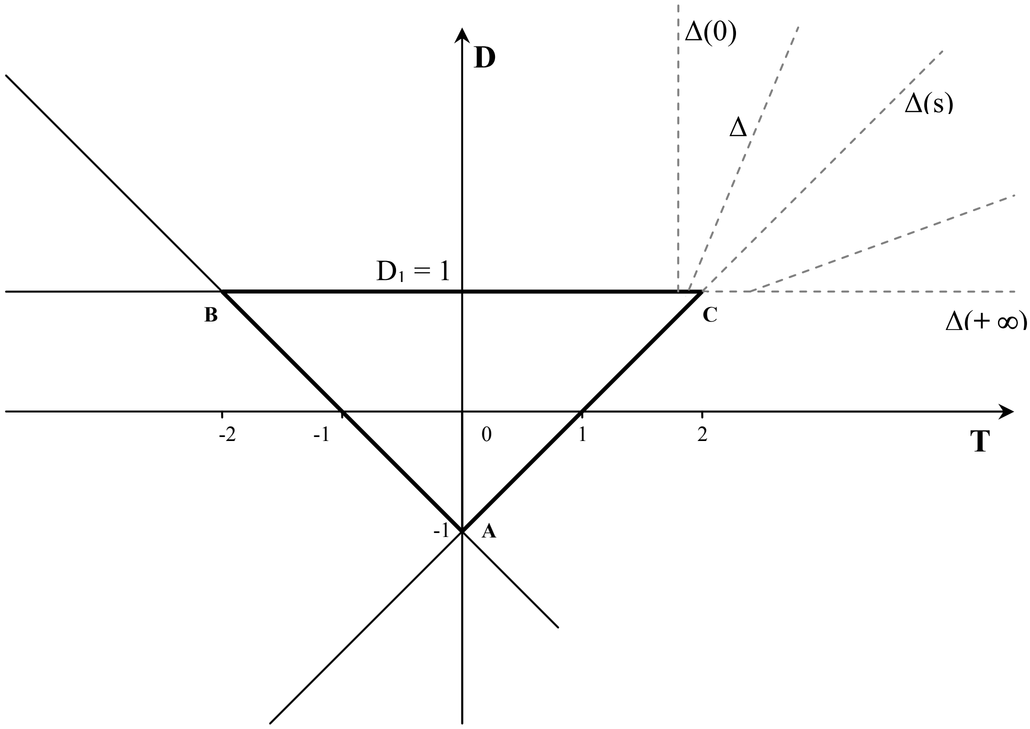

Figure 5
Threshold level of progressive taxes and transfers on capital income π_{min}
in the OLG model

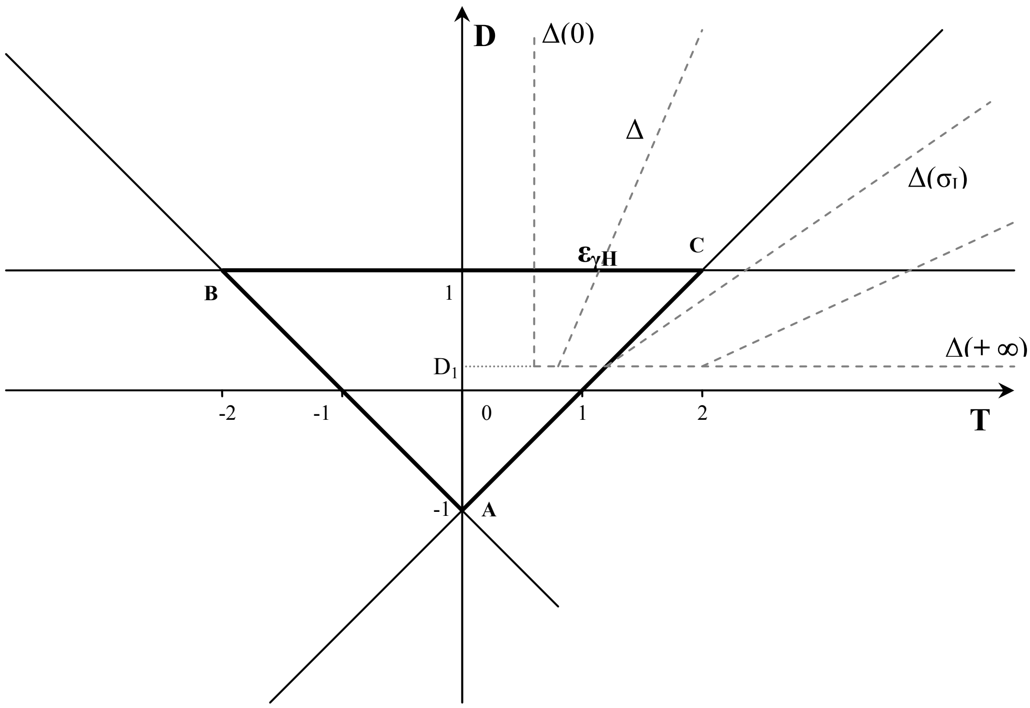

Figure 6
Progressive taxes and transfers on labor income
in the OLG model

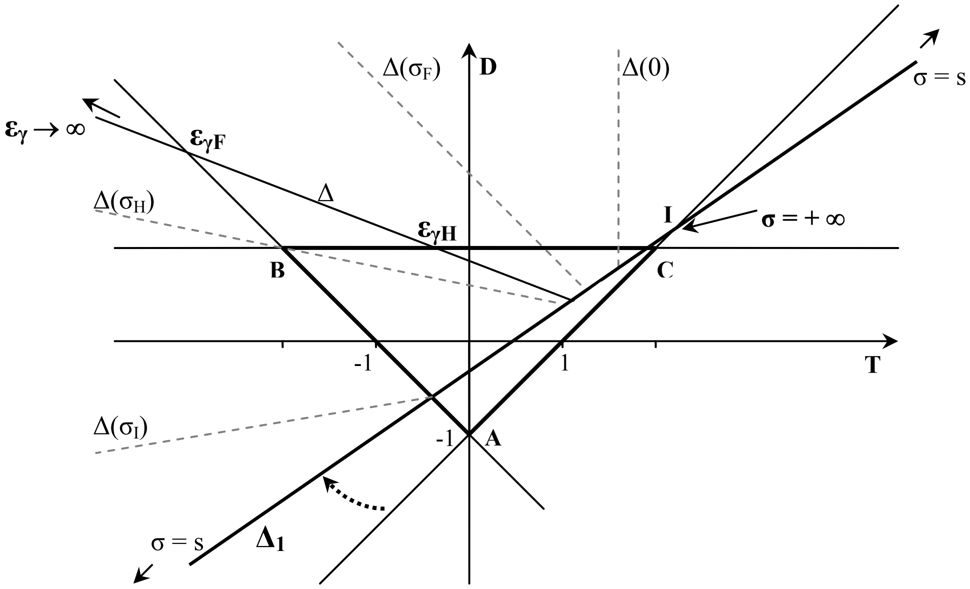

Figure C1

Progressive taxes and transfers on labor income
in the heterogeneous agents model – Case 1

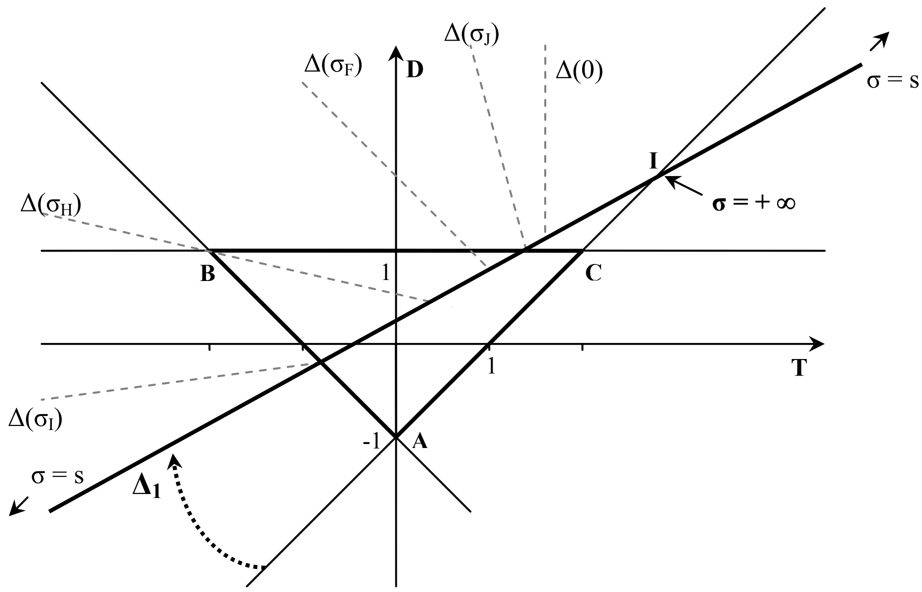

Figure C2

Progressive taxes and transfers on labor income
in the heterogeneous agents model – Case 2

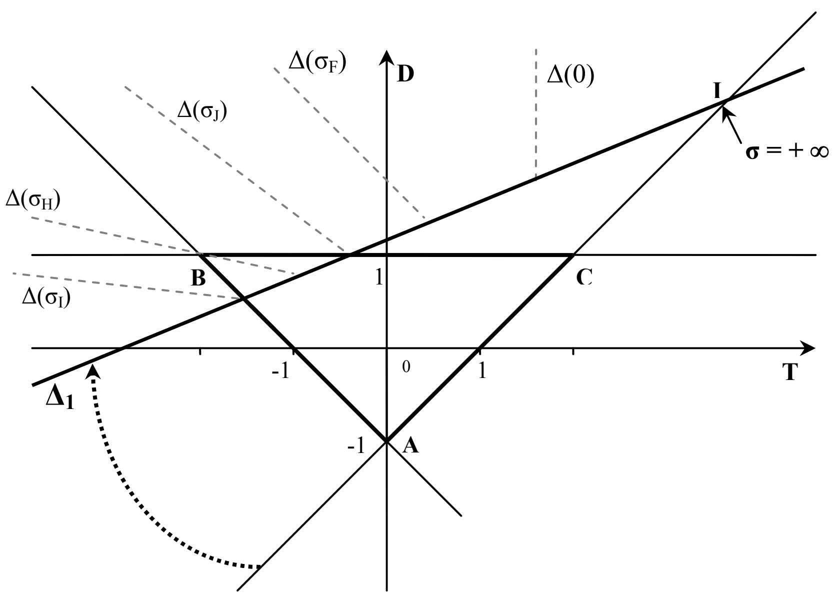

Figure C3

Progressive taxes and transfers on labor income
in the heterogeneous agents model – Case 3