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Capacity Choice Counters the Coase Conjecture*

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Abstract: The Coase conjecture (1972) is the proposition that a durable goods monopolist, who sells over time and can quickly reduce prices as sales are made, will price at marginal cost. Subsequent work has shown that in some plausible cases that conjecture does not hold. We show that the Coase conjecture does not hold in *any* plausible case. In particular, we examine that conjecture in a model where there is a vanishingly small cost for production capacity, and the seller may augment capacity in every period. In the "gap case," any positive capacity cost ensures that in the limit, as the size of the gap and the time between sales periods shrink, the monopolist obtains profits identical to those that would prevail when she could commit ex ante to a fixed capacity, given a standard condition on demand. Those profits are at least 29.8% of the full static monopoly optimum.

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1. Introduction

In 1972, Nobel laureate Ronald Coase startled the economics profession with a counterintuitive proposition, which came to be known as the Coase conjecture, concerning the monopoly seller of a durable good. Coase's original example was the hypothetical owner of all land in the United States. The monopolist maximizes profits by identifying the monopoly price and selling the quantity associated with that price. Having sold that quantity, however, the monopolist now faces a residual demand and she is induced to try to sell some additional units to the remaining buyers at a price that is lower than the initial price. Such logic entails a sequence of sales at prices falling toward marginal cost. Rationally anticipating falling prices causes most potential buyers to wait for future lower prices. Provided that the monopolist can make sales and cut price sufficiently rapidly, Coase conjectured that the monopolist's initial offer would be approximately marginal cost, and that the monopoly would replicate the competitive outcome.

Intuitively, the monopolist competes with future incarnations of herself. Even when facing a monopolist, buyers have an alternative supplier: the monopolist in the future. That the power of such a substitution possibility might render the monopoly perfectly competitive remains a captivating idea even for an audience accustomed to the fact that subgame perfection (or time consistency) restricts equilibria in dramatic ways. Arguably, Coase's conjecture remains the most extreme example of the power of subgame perfection.

Not surprisingly, perfect durability (which ensures that high value buyers exit the market) and rapid transactions are not the only assumptions required to prove the Coase conjecture. The first formal proofs of the Coase conjecture are given by Jeremy

Bulow (1982) and Nancy Stokey (1982). The deepest analysis is the challenging 1986 paper by Faruk Gul, Hugo Sonnenschein and Robert Wilson. This paper distinguishes two cases, the so-called "gap" case, where the lowest value buyer has a value strictly exceeding marginal cost, and the "no gap" case, in which demand and marginal cost intersect. The gap case, which is also studied by Drew Fudenberg, David Levine, and Jean Tirole (1985), is more readily analyzed because there is generically a unique subgame perfect equilibrium. Uniqueness arises because, if there are few potential buyers left, it pays to sell to all of them at the lowest willingness-to-pay (which strictly exceeds marginal cost by hypothesis). This conclusion ties down the price in the last stage and ensures that the game is of finite length; backward induction then gives the unique equilibrium. When the stages occur rapidly, prices converge to the final price rapidly, so buyers are unwilling to pay much more than the lowest willingness to pay, which implies that the opening price is the lowest willingness to pay. This is not quite the same as Coase's conjecture, because prices converge on the lowest willingness to pay rather than to marginal cost, but similar in spirit.

In contrast, in the "no gap" case, the Coase conjecture holds in some equilibria, but not in others. There is a "Coasian" equilibrium, which is stationary (at any time, buyers' strategies depend only on the current price, and not upon the prior history of the game) and entails an initial price close to marginal cost. Moreover, this opening price converges to marginal cost as the time between sales periods approaches zero. As Lawrence Ausubel and Raymond Deneckere (1989) demonstrate, this Coasian equilibrium can be used to ensure the existence of other, non-stationary equilibria, by threatening the seller that, should she deviate from the hypothesized equilibrium, buyers' beliefs will revert to the Coasian equilibrium, which involves low profits for the seller. Such a threat guarantees that the seller won't deviate from anything at least as profitable as the low-profit Coasian equilibrium. Ausubel and Deneckere demonstrate that there are many non-stationary equilibria that can be constructed by the threat of reversion to the Coasian equilibrium.

In addition to more profitable equilibria which exist in the no gap case, there are a variety of other means for a durable goods monopolist to escape the grim logic of the Coase conjecture. Leading the list is renting, which is a means identified in Coase's original article. A seller who rents, rather than sells, has no incentive to expand output beyond the monopoly quantity, for such an expansion entails a price cut not just to the new customers, but also to existing customers. By allowing existing customers to renegotiate, rental serves a means of committing to a "most favored customer" clause, in which early buyers are offered terms no worse than later buyers. Renting as a means of commitment has been offered as an explanation for IBM's rental of business machines (Wilson, 1993), although evidence is scant. Other solutions offered in the literature include return policies or money-back guarantees, destroying the production facilities, making the flow costs of staying in the market expensive (e.g. by renting the factory), concealing the marginal cost from buyers to interfere with their expectations about future prices, and planned obsolescence to eliminate the requisite perfect durability. (See Tirole (1988).)

We offer a very different, more general limitation on the Coase conjecture. Focusing on the gap case, we show that the Coase conjecture is not robust to a modification of the game that would be relevant in almost any practical application. In particular, we consider a small cost of capacity, so that selling a given amount over a smaller span of time costs more. We envision a perfectly durable capacity, so that, once bought, the capacity is never purchased again. For example, if the good is produced in a factory, faster sales entail a larger factory, which creates a one-time larger cost. Even Coase's hypothetical seller of all U.S. land, who faces no production cost, still bears a capacity cost: To sell the land rapidly requires a large number of sales agents, so the seller must incur hiring and training costs. Whenever there is an increased cost of increased speed, there is an effective capacity cost.[1]

If a monopolist chose production capacity at the beginning of time and could not augment it later, the monopolist would use capacity as a commitment device, setting a low capacity and dribbling output into the market. That approach has the advantage of ensuring that prices are high early and fall slowly, as high value buyers pay more for early acquisition of the good. Indeed, we will demonstrate that in such a "commitment game" the monopolist obtains at least 29.8% of the static monopoly profits, no matter how fast the stages of the game occur; increasing the speed of the game induces the monopolist to cut capacity in a way that keeps the flow of goods to buyers constant. Note that the monopolist could not achieve anything higher than static monopoly profits even if she could commit ex ante to a sequence of prices. Stokey (1979) shows that she would optimally set the static monopoly price in each period, and thus earn static monopoly profits. That is, the ability to discriminate dynamically does not help the monopolist.

While the specificity of the lower bound of profits is remarkable, the fact that profits don't converge to zero is not; such a model endows the seller with an extraordinary commitment ability – the ability to commit at the beginning of the game

[1] A constant marginal cost of selling does not create a capacity cost, but just an ordinary marginal cost. Potential congestion in sales, so that selling twice as fast incurs more than twice the costs, creates an effective capacity cost.

to restrict future sales. This seller doesn't reach the full static profits because she can't stop herself from selling to all customers eventually; having reached the monopoly quantity, Coasian logic dictates that she continue to sell. However, she can slow herself down, selling slowly enough to ensure that she acquires a significant fraction of the monopoly profits. She loses two ways relative to the static monopoly – she eventually sells too much, and profits are earned slowly and hence discounted, but nevertheless capacity commitment leads to positive profits.

To add capacity in a more realistic fashion, we consider a monopolist who chooses whether, and by how much, to augment production capacity in each period of the game. After choosing production capacity, the monopolist sets a price and sells the lesser of the demand by buyers and the production capacity. Assume that the monopolist faces s a small cost of capacity. By Coase's logic, she is tempted to cut prices as quickly as possible by shrinking the time between sales periods. The remarkable fact is that when the gap is small the equilibrium to the game with augmentable capacity involves the same capacity and profits as the game in which capacity cannot be increased, as long as the optimal capacity chosen in that game increases with the size of the market. That is, the ability to increase capacity later does not harm the seller even in a Coasian environment, for equilibrium profit levels are the same as those that arise when the option to increase capacity doesn't exist. Consequently, *any* positive cost of capacity prevents the opening price from approaching marginal cost. The presence of a capacity cost permits the seller to behave as if she could commit to capacity initially. In contrast, with a zero cost of capacity, only the Coase equilibrium, with zero profits in the limit, occurs. Thus, there is a discontinuity in seller profits as the cost of capacity goes to zero.

The intuition behind the theorem suggests a defect in Coase's reasoning. The Coasian price path requires a seller to sell all of the demand very rapidly. Because buyers will wait for prices close to marginal cost (since these are coming rapidly), the opening price is close to marginal cost, and most sales take place in the first few minutes. In the limit as the time periods get arbitrarily close, all sales take place immediately. In environments requiring production or some transaction medium, that outcome requires the seller to produce a very large production facility or high-bandwidth transaction facility, so that the flow of sales can be extremely large for a very short period of time. If the cost of capacity is high relative to the size of the market, then the seller will not purchase so much capacity. In fact, for any positive cost, the fact that the seller will not increase capacity near the end of the game, when few buyers remain, allows her to credibly commit to a low level of capacity at the beginning. The logic of backwards induction compels buyers to believe that she will not increase capacity in the future, and thus that prices will fall slowly.

Another way to see that intuition is as follows: Suppose that in the "commitment game," where the seller chooses capacity once and for all at the beginning, the optimal capacity increases with the size of the market. As sales are made, then, the desired commitment capacity falls. Thus, a seller who chooses an initial capacity slightly below the "desired" capacity won't be later tempted to increase it, because the slight reduction will still exceed the subsequent desired capacity. This means that the seller has local commitment ability – she can effectively commit to a slight reduction or increase in capacity around the equilibrium opening level. But the ability to commit to a small change is sufficient to ensure that profits are maximized as a function of capacity because the first order conditions hold; that is, that the level of profits when capacity is augmentable is identical to that when capacity is chosen once and for all. Local is as good as global in ensuring the first-order conditions hold.

Our result that capacity choice in each period delivers the same profits as the commitment version holds in the limit of the gap case, as the gap shrinks to zero. That case, where the gap is positive but small, had been the only setting left where the Coase conjecture had bite, and the monopolist made no profit. In the no gap case, Ausubel and Deneckere (1989) show the existence of equilibria where the seller makes high profits. In the gap case, the monopolist sells at a price near the lowest consumer's valuation, but if the gap between marginal cost and the lowest valuation is large, then that price entails high profits. In this paper, we show that the seller can earn substantial profits even when the gap is vanishingly small. Thus, in any relevant economic situation Coase's conclusion does not hold: A durable goods monopolist *can* make profits.

Alternatively, our result can be interpreted as a way to select from Ausubel and Deneckere's (1989) continuum of equilibria in the no gap case: When the monopolist chooses costly capacity, equilibria where the monopolist makes very low profits are not robust to the introduction of a small discontinuity of buyers' valuations just above marginal cost. (Such a discontinuity could result if, for example, prices can be set only at discrete levels.) Note that without capacity choice, the selected equilibrium is very different. With a very small gap, the unique equilibrium entails very low profits.

The rest of the paper proceeds as follows. In the next section, we set up the model and, as a preliminary, analyze the commitment version of the game. We present the main theorem in the third section, showing that the outcome with capacity choice in each period mirrors the outcome with initial capacity commitment. In the fourth section we consider the robustness of our result, and we conclude in the fifth section.

2. Model

A durable good monopolist faces a market in which a continuum of consumers, indexed by $q \in [0, q_0]$, each demand a single unit. Both consumers and the monopolist live forever and discount the future at the rate δ per unit of time. Sales can occur at discrete, equally spaced intervals. There are N such sales periods per unit of time. Thus, period z occurs at time z / N, and the discount rate per period is $\delta^{1/N}$. In order to produce the good, the monopolist must invest in capacity. The cost of buying the capacity to produce (or to sell) at a constant flow rate of one unit of the good per unit of time is c, which implies that the capacity to produce one unit per *sales period* costs Nc. There is no depreciation: Once purchased, capacity is good forever. Furthermore, there are no other production costs. If the monopolist has capacity K, she can sell at a rate of K units per time period at zero marginal cost forever. Consumers' valuations are determined as follows:

The value of a unit of the good to consumer q is given by $v(q) \equiv p(q) + g$, where the constant g is strictly greater than zero, and p is a decreasing, twice-differentiable function from $[0, q_0]$ to $\boldsymbol{R}_+$ such that $p(q_0) = 0$, $p(q) > 0$ for $q < q_0$, and $p'(q) + qp''(q) < 0$ for all q. (The property that $p'(q) + qp''(q) < 0$ is a standard regularity condition. It is equivalent to log concavity of demand, which ensures, for example, that the best-response function of Cournot duopolists are downward-sloping, and that a monopolist facing a per-unit tax increases his price by less than the amount of the tax.)[2]

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[2] We note that the set of functions satisfying that condition is closed under truncation: Since $p'(q) < 0$, $p'(q) + (q - s)p''(q) < 0$ for all $s \in [0, q_0)$ and all $q \in [s, q_0]$.

Consumers' valuations are bounded above by $v^H \equiv p(0) + g$ and below by g (= $p(q_0) + g$). This is the "gap case," where the lowest valuation among the buyers is strictly greater than the monopolist's marginal cost.

At the beginning of each sales period, the monopolist publicly chooses how much additional capacity to purchase. She then announces a price P for that period, and must sell to any buyer who wants to buy at that price, up to a maximum of K / N units. We will assume that the rationing rule is to serve higher valuation buyers first. Equivalently, we could allow costless resale among the buyers. (We discuss alternative rationing rules in Section 4.)

Assumption 1: If the quantity of consumers who wish to buy in any sales period z is greater than K_z / N, then sales will be made to the subset of size K_z / N of potential buyers with the highest valuations.

The goal of the monopolist is to maximize the discounted value of revenue, minus the discounted value of expenditures on capacity. The consumers seek to maximize their discounted surplus. The surplus to consumer q who buys in sales period z at price P_z is $\delta^{z/N} [v(q) - P_z]$. As is standard in the literature on the Coase conjecture, we will consider only equilibria where deviations by a zero mass set of consumers have no effect on continuation play.

In the absence of capacity constraints, Fudenberg, Levine, and Tirole (1985) and Gul, Sonnenschein, and Wilson (1986) show that there is generically a unique subgame perfect equilibrium in the gap case. That equilibrium satisfies the Coase conjecture, in the sense that as the number of periods N per unit time goes to infinity, the monopolist earns profits close to gq_0 by setting an initial price close to g and selling to the entire market nearly instantaneously. In that equilibrium, which we will call the Coase equilibrium, all consumers are served in a finite number of sales periods, which implies that prices in each period can be determined by backwards induction.

When there are no capacity constraints, the equilibrium path has the "skimming" property. That is, in any period there is a cutoff valuation $\bar{v}$ such that all consumers with valuations greater than $\bar{v}$ have already bought, and all consumers with valuations less than $\bar{v}$ have yet to buy. (See Fudenberg, Levine, and Tirole's (1985) Lemma 1 and Ausubel and Deneckere's (1989) Lemma 2.1.) The intuition is that if a consumer with valuation v is willing to buy at price P, then so is any consumer with valuation $v' > v$. Both buyers get the same benefit (in the form of lower future prices) from waiting, but the cost of delaying consumption is greater for the high-valuation consumer. Thus, in any period the remaining market can be characterized completely by q, the volume of consumers who have been served so far. Let $S^C(q, g, N)$ and $P^C(q, g, N)$ denote the quantity sold and price offered, respectively, in any period on the Coase path when the quantity served so far is q, the size of the gap is g, and there are N offers per unit time, and let $\pi^C(q, g, N)$ be the remaining Coase profits to the monopolist. Define $S_{\max}^C(q,g,N)$ as the maximum quantity sold in any period along the Coase path when the market served so far is q, the gap is g, and there are N offers per unit time. Note that if K/N is greater than $S_{\max}^C(q,g,N)$, the capacity constraint will never bind along the Coase path. In that case, capacity is no longer relevant, and the only subgame perfect continuation is the Coase equilibrium.

In principle, the skimming property may fail when we introduce capacity constraints, because of the rationing rule that favors high-valuation customers. Even if both types of consumers would prefer to wait and buy at tomorrow's prices, the low-valuation type might still purchase today if he knows that he would be rationed out of the market tomorrow. However, that situation does not arise in our equilibrium analysis – it requires consumers to believe that rationing will occur, but in any period where buyers are rationed the seller could increase revenue by raising the price without affecting the quantity sold. For ease of exposition, therefore, we will continue to let q_z denote the volume of consumers who have already been served at the beginning of period z. Let $G_N(K, q, g, c)$ denote the subgame (with N sales periods per unit of time) in which the monopolist has capacity K in hand, the quantity already served is q, the gap is g, and the cost of additional capacity is c; $G_N(0, 0, g, c)$ is the game itself.

As a preliminary to our main result, in the next section we examine the limiting, continuous-time version of the game, under the assumptions that the monopolist can commit to a capacity level and that she always sells at a rate equal to her capacity. In our analysis in Section 3, we will derive those properties rather than assume them.

2.1 One-Time Capacity Choice

Suppose that the monopolist can only purchase capacity at the beginning of the game. If she purchases none, the game is over. If she chooses a non-zero level of capacity, the equilibrium path of the game is pinned down: The end of the game is on the Coase path, so previous prices and quantities are determined by backwards induction. Let $\pi_N^{com}(K,q,g,c)$ be the profit resulting when the quantity served is q, the gap is g, and the monopolist commits to capacity K, which costs c. Let $R_N^{com}(K,q,g,c)$ be the associated revenue. The quantity sold in a period on the commitment path when the served market is q is given by $S_N^{com}(K,q,g,c)$. Let $K_N^{com}(q,g,c)$ be the capacity level that maximizes the commitment profits $\pi_N^{com}(K,q,g,c)$, and denote by $\Pi_N^{com}(q,g,c)$ the value of the maximized profit.

How large is the maximized commitment profit relative to the static monopoly profits? (Remember that the static monopoly profit is the highest that the monopolist could attain even with the ability to commit to future prices.) We examine the limit of the profits as both the time between offers and the capacity cost c shrink to zero. (For simplicity, we assume in this analysis that $g = 0$ – the no-gap case. The profits for the gap case, where $g > 0$, can be no lower.) Consider the monopolist who sells at a rate K per period and in a continuous time fashion, which is the limit of the discrete case when the intervals get short. Here we simplify the profit expression and prove a global minimum for profits (at least 29.8% of the static monopoly profits). Let MR denote marginal revenue:

$$MR(q) = qp'(q) + p(q) .$$

$MR(q_m) = 0$ defines the static monopoly quantity q_m. Market saturation occurs at quantity q_0. It is permitted in the analysis for $q_0 = \infty$. Given capacity K, the quantity sold through time t is Kt. The market is saturated at $T = q_0/K$; at this point, the price is zero. The value of the buyer who buys at time t is $p(Kt)$. Let $P(Kt)$ be the price charged by the seller. A buyer of type Kt who buys at time s has utility

$$u(Kt) = \max_s e^{-rs}\left(p(Kt) - P(Ks)\right).$$

From the envelope theorem, since this buyer chooses to buy at t,

$$u'(Kt) = e^{-rt} p'(Kt).$$

Thus, since $u(KT) = 0$,

$$e^{-rt}(p(Kt) - P(Kt)) = u(Kt) = u(KT) - \int_t^T u'(Ks)Kds = -\int_t^T e^{-rs} p'(Ks)Kds =$$

$$-\int_{Kt}^{KT} e^{-ry/K} p'(y)dy = e^{-rt} p(Kt) - \int_{Kt}^{KT} \frac{r}{K} e^{-ry/K} p(y)dy.$$

Thus, setting $z = Kt$,

$$P(z) = e^{rt} \int_z^{q_0} \frac{r}{K} e^{-ry/K} p(y)dy.$$

The firm's profits are

$$\pi = \int_0^T e^{-rt} KP(Kt)dt = \int_0^{q_0}\int_z^{q_0} \frac{r}{K} e^{-ry/K} p(y)dydz = \int_0^{q_0} z\frac{r}{K} e^{-rz/K} p(z)dz$$

$$= \int_0^{q_0} \left(zp(z)\right)' e^{-rz/K} dz = \int_0^{q_0} MR(q)e^{-rq/K} dq.$$

Note that the maximized value of profits is independent of the interest rate. Let $a = r/K$, and a^* maximize profits over K.

The revenue function $R(q)$ is given by $R(q) = qp(q)$. Inverse demand $p(q)$ is decreasing, which implies that for any $q \le q_m$,

$$R(q) \ge \frac{q}{q_m} R(q_m).$$

Note that

$$\pi^* = \max_a \int_0^{q_0} MR(q)e^{-aq} dq = \max_a \int_0^{q_0} R(q)ae^{-aq} dq$$

$$\ge \max_a \int_0^{q_m} R(q)ae^{-aq} dq \ge \max_a \int_0^{q_m} \frac{q}{q_m} R(q_m)ae^{-aq} dq \text{, so}$$

$$\frac{\pi^*}{R(q_m)} \ge \max_a \int_0^{q_m} \frac{q}{q_m} ae^{-aq} dq = \max_a - e^{-aq_m} + \int_0^{q_m} \frac{1}{q_m} e^{-aq} dq =$$

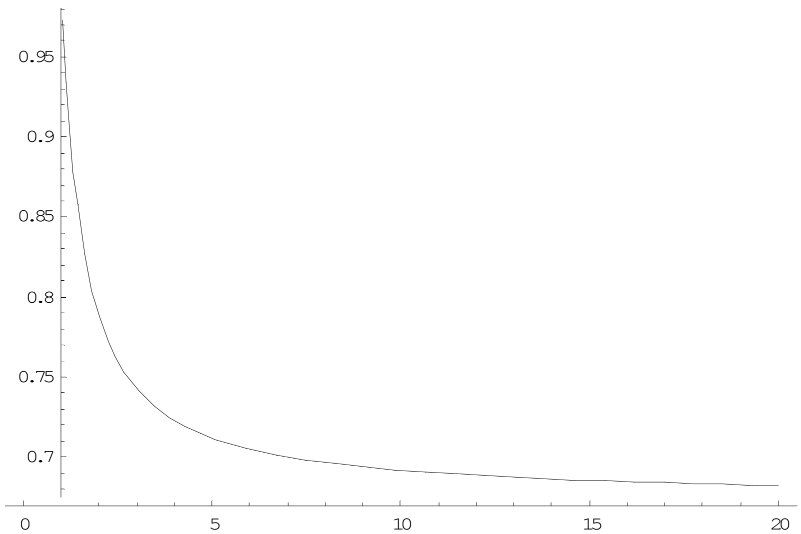
$$= \max_a \frac{1-e^{aq_m}(1+aq_m)}{aq_m} \approx 0.298425 \text{, which is approximately } \frac{57}{191}.$$

Let that lower bound on the ratio $\pi^* / R(q_m)$ be denoted γ:

Definition. Define the constant γ as the value $\max_a \frac{1-e^{aq_m}(1+aq_m)}{aq_m}$.

That bound is tight. Suppose that a mass $1 - \varepsilon$ of consumers have valuation 1, and a mass ε has valuation ε. (A smoothed version of that demand system can satisfy our differentiability assumptions and belong to the set Ψ. The following reasoning will still hold.) For low ε, the monopoly price is 1 and the monopoly quantity q_m is $1 - \varepsilon$. As ε shrinks, the revenue from selling to consumers beyond q_m becomes arbitrarily small. Also, for $q \leq q_m$, $R(q) = (q/q_m)R(q_m)$. Thus, both inequalities in the derivation of the bound γ are arbitrarily close to equalities for small enough values of ε.

The profits with a capacity constraint may be arbitrarily close to the static monopoly profits. Appendix 2 demonstrates that, in the constant elasticity of demand case, in the limit as the elasticity converges to 1, the ratio of the capacity constrained profits to the static monopoly profits converges to 1. This profit ratio is graphed in Figure 1. As the elasticity of demand diverges, the ratio converges to roughly 0.673478, which is also the ratio that prevails when $p(q) = \log(q)$, which arises with $a^* \approx 3.47845$. When demand is linear $p(q) = 1\text{-}q$, $MR = 1\text{-}2q$, and numerical computation shows that $a^* \approx 2.688$ and profits are 55.74% of the static monopoly profits of 0.25. At an annual interest rate of five percent, that value implies that the monopolist sells to approximately 1.86 percent (5 / 2.688) of the market per year.



3. Main Theorem and Proof

Let $\pi^{com}(K,q,g,c)$ and $R^{com}(K,q,g,c)$, respectively, denote the monopolist's profits and revenues in the continuous time, commitment version of the game when the quantity served so far is q, the gap is g, and the monopolist commits to capacity K, which costs c. Let $K^{com}(q,g,c)$ be the level of capacity that maximizes those profits. We can show those functions have the following properties:

Lemma 1: The optimal commitment capacity $K^{com}(q, 0, c)$ is decreasing in the quantity served q, and $R^{com}(K,q,0,c)$ is decreasing in K above $K^{com}(q, 0, c)$ and locally strictly concave in K at $K^{com}(q, 0, c)$.

That is, in the no-gap case the optimal commitment capacity falls as the size of the market shrinks, and commitment revenue is concave in capacity.

Now consider the baseline game, where the monopolist is free to increase capacity in each period. Our main result, which we will state formally in Theorem 1, is that when the gap and the time between offers are small enough, then in fact the monopolist can credibly commit not to increase capacity beyond $K_N^{com}(0,g,c)$ in any period, and thus can earn the same profit as in the commitment version. Intuitively, as the number of sales periods per unit of time N increases, the marginal cost of capacity *per sales period* Nc grows without bound, while the marginal benefit is bounded. For large enough N, then, increasing capacity for a single sales period is prohibitively expensive. The formal proof relies on two more lemmas. Lemma 2 shows that the monopolist can commit not to purchase capacity past $K_N^{\max}(g,c)$, defined as

$$K_N^{\max}(g,c) = \max_{q\in[0,q_0]} K_N^{com}(q,g,c).$$

Capacity $K_N^{\max}(g,c)$ is the maximum optimal commitment capacity over all market sizes no greater than q_0. Intuitively, if the seller chooses that capacity in the first period, Lemma 1 guarantees that she will never be tempted to increase it. Thus, she can earn the profit associated with committing to that capacity forever.

Lemma 2: If Assumption 1 holds, then for any capacity cost $c > 0$ and any gap size $g > 0$, there exists an integer $N(c)$ such that for all $N > N(c)$, any SPE of the game $G_N(0, 0, g, c)$ gives the monopolist a profit of at least $\pi_N^{com}(K_N^{\max}(g,c),g,c)$.

The third lemma demonstrates that as the gap g shrinks and the number of sales periods per unit time N increases, the value of the maximum optimal commitment capacity $K_N^{\max}(g,c)$ converges to $K^{com}(0, 0, c)$, the optimal commitment capacity at the beginning of the (continuous-time) game, given Assumptions 1 and 2.

Lemma 3: If Assumption 1 holds, then for any $\varepsilon > 0$, there exist a real number $c(\varepsilon) > 0$ and real-valued functions $g(c, \varepsilon) > 0$ and $N(c, \varepsilon) > 0$ such that whenever $c < c(\varepsilon)$, $g < g(c, \varepsilon)$, and $N > N(c, \varepsilon)$, then $|K_N^{\max}(g,c) - K^{com}(0, 0, c)| < \varepsilon$.

Now we can state our main result: In the limit, the monopolist earns commitment profits, even when she can augment capacity in every period. Theorem 1 follows immediately from the three lemmas.

Theorem 1: If Assumption 1 holds, then for any $\varepsilon > 0$, there exist a real number $c(\varepsilon) > 0$ and integer-valued functions $g(c, \varepsilon) > 0$ and $N(c, \varepsilon) > 0$ such that whenever $c < c(\varepsilon)$, $g < g(c, \varepsilon)$, and $N > N(c, \varepsilon)$, then any SPE of the game $G_N(0, 0, g, c)$ gives the monopolist a profit of at least $\pi^{com}(0, 0, c) - \varepsilon$.

We showed in Section 2 that revenues in the commitment version of the game are at least a fraction $\gamma (\approx 0.298)$ of static monopoly revenue $R(q_m)$. That result implies the following Corollary of Theorem 1.

Corollary 1: If Assumptions 1 and 2 hold, then for any $\varepsilon > 0$, there exist a real number $c(\varepsilon) > 0$ and integer-valued functions $g(c, \varepsilon) > 0$ and $N(c, \varepsilon) > 0$ such that if $c < c(\varepsilon)$, $g < g(c, \varepsilon)$, and $N > N(c, \varepsilon)$, then any SPE of the game $G_N(0, 0, g, c)$ gives the monopolist a profit of at least $\gamma R(q_m) - \varepsilon$.

Thus, rather than making the zero profit predicted by the Coase conjecture, the monopolist attains a substantial fraction of the profit that she could make if she could commit to a schedule of prices. Note that throughout our analysis we take the limit as the capacity cost shrinks of the limit of profits given the cost as the time between sales periods shrinks. As discussed in the introduction, that is the appropriate order of limits: the cost is exogenous to the monopolist, while Coasian logic compels her to sell as quickly as possible.

4. Robustness

In this section, we briefly examine the impact of relaxing Assumption 1 (efficient rationing) and of allowing the monopolist the option to reduce as well as to augment capacity in each period.

Removing Assumption 1 fundamentally changes the nature of the monopolist's problem. First, consider a change to the rationing rule that gives priority to high-value consumers. If, in contrast, the low-value buyers are served first, then the monopolist could set very low prices at the beginning of the game, clearing out the bottom of the market and thus eliminating her commitment problem when the time comes to serve the high-value customers. Suppose, for example, that there is a mass $q \in [0, 1]$ of consumers with valuation H and a mass $1 - q$ with valuation $L < H$. Even if $1 - q$ is very small, in the absence of capacity constraints the Coase outcome is that the monopolist sells to the entire market at price L. But if the monopolist chooses capacity, then even if capacity is costless her equilibrium profit cannot be less than $(1 - q)L + \delta^{1/N} qH$. She can obtain that profit by choosing capacity $K_1 = 1 - q$ in the first period, setting a price of L, and selling to all the low-value consumers. (The low-value consumers will buy, since the monopolist will never set a lower price in equilibrium.) In period 2, she increases capacity to q and sells to the remaining consumers at price H (which is the Coase outcome once the low-value types have been served).

Thus, the rationing rule that gives low-value buyers priority, when combined with capacity choice, effectively allows the monopolist to trace out the demand curve from the bottom and appropriate nearly all the surplus. That result is in stark contrast to the Coase conjecture, where in the limit the consumers get all of the surplus. In a different way, a rationing rule that assigns items randomly among demanders may also benefit the seller. The more high-value customers there are who remain unserved after being rationed out of the market, the smaller is the temptation for the seller to lower prices. In that way, random rationing may slow the fall of prices in equilibrium relative to efficient rationing and thereby increase the monopolist's profits.

Next, suppose that in each sales period, the monopolist can either increase or decrease capacity, at a symmetric cost of c per unit. (For example, there may be a cost to closing down a factory or laying off workers.) In such an environment, the monopolist may be able to credibly commit to a sales path where the volume of sales per period decreases over time. Such a path could yield revenues even higher than the commitment profits in Section 2, because the monopolist sells to high value consumers quickly. Those consumers are still willing to pay a high price, because after they buy prices will fall only very slowly as the monopolist reduces capacity.

5. Conclusion

This paper demonstrates that capacity costs of arbitrarily small degree can eliminate the zero profit conclusion of Ronald Coase's 1972 conjecture. Coasian dynamics – prices falling over time and quantities eventually exceeding the static monopoly quantity – prevail, but capacity choice provides a strong means of slowing the sales, thereby slowing the fall in prices, and thus permitting initial prices well in excess of marginal costs.

Whenever capacity is a choice, Coase's conjecture requires a monopolist to act in a manner not in her best interest. In order to implement the Coase conjecture path, the monopolist must invest in the resources to sell to all of the buyers instantly. Usually this investment will require some outlay; our result shows that even an arbitrarily small outlay serves as a strong commitment device for the monopolist. The monopolist cannot be compelled by the rational expectations of buyers to expand capacity beyond the profit-maximizing level. That is, Coase's reasoning asks the monopolist to create the capacity which makes rapid sales and hence low prices a rational belief on the part of buyers, an action not in her best interest. When she fails to buy a large capacity, backwards induction forces the buyers to conclude that she won't expand capacity in the future, which makes the decision to choose low capacity rational.

Thus, we find that the Coase conjecture is not robust to a very reasonable change in the specification of the environment. The logic of subgame perfection dictates that the monopolist continue to sell beyond the static monopoly level (which maximizes monopoly profits even in the dynamic game), but the ability to slow these sales by a smaller capacity choice, even in the limit when capacity is free, ensures that the monopolist makes a significant fraction of the static monopoly profits. Provided that the optimal commitment capacity increases with market size, the monopolist makes at least 29.8% of the monopoly profits, which is a far cry from zero.

The analysis of Joel Sobel (1991) suggests that entry of potential buyers (quite reasonable in light of finite human lifespans), which is isomorphic to imperfect durability, will create a price cycle. Prices tend to fall until it pays to sell to low value consumers because sales to high value consumers have made them relatively rare; once existing low value consumers are satisfied, prices rise and sales are made only to newly born high value buyers. This analysis was enhanced by Wolfgang Pesendorfer (1995) for goods with a network diseconomy. A logical conjecture is that the presence of capacity choice will enhance the seller's ability to dynamically price discriminate and lengthen the price cycle, but we have not investigated this formally. It is conceivable that the seller might choose such a low capacity as to sell only to high value buyers, thereby eliminating cycles altogether.

Charles Kahn (1986) offers a model in which more rapid sales cost more, recognizing the restrictions on the seller emphasized by this paper. Kahn's model features a quadratic cost of the rate of sales. The increasing cost of faster selling ensures that the equilibrium involves positive profits. Our result makes two contributions relative to Kahn's work. First, we endogenize the seller's capacity; Kahn treats it as an exogenous parameter. Second, in the limit as the cost shrinks, Kahn's seller is again making zero profits (the Coase outcome), while in our model the seller continues to make a large fraction of static monopoly revenue, a fraction which remains positive (and greater than .298) even in the limit. We conjecture, however, that a version of our results (in particular, that ex ante commitment to selling capacity produces the same profits as augmentable selling capacity) holds in his model with endogenized capacity.

Moreover, there is no obvious impediment to employing an analogous proof, where we interpret capacity as an input that reduces the slope of the quadratic cost, but further investigation is needed. The thought experiment of the present paper is very natural in Kahn's elegant framework. Intuitively, Kahn's cost function is a smoothed version of ours – in our model, the production cost in a sales period is zero for quantities below capacity and infinite beyond it.

The Coase conjecture has been investigated in two finite versions. Mark Bagnoli, Stephen Salant, and Joseph Swierzbinski (1989) demonstrate that, if there are finitely many potential buyers of known types, there is a subgame perfect equilibrium with the seller extracting almost all of the surplus. Essentially, she offers the good at a price near the maximum value; when that transacts, she offers at the next highest value, and so on. This model requires an unreasonable level of detailed knowledge on the part of the seller, but is interesting because it has such a different outcome from the continuous model. A bridge between the continuous and discrete demand cases was developed by David Levine and Pesendorfer (1995). Fehr and Kuhn (1995) show, on the other hand, that if it is the seller who can set prices only in a discrete set, then her profits shrink to zero. The form of discreteness in our model (namely that within a sales periods marginal cost is zero up to capacity and infinite above it) is qualititatively different, and it yields a very different outcome. McAfee and Daniel Vincent (1997) consider the Coasian auction problem, where the seller has one unit to sell to finitely many buyers who privately know their willingness to pay. They show that the opening reserve price exceeds marginal cost (even in the limit), but that the likelihood that the reserve price binds converges to zero as the periods come faster. Consequently, auction profits converge to the same level of profits arising from holding an auction with efficient reserve price. This model suggests a very different setting in which to consider capacity choice.

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Appendix 1. Proofs.

Proof of Lemma 1:

Notation

K = capacity
x = units already sold
$P(t)$ = price at time t
$p(q)$ = marginal value of q^{th} unit
$u(q)$ = utility of person buying q^{th} unit

$$u(x+Kt) = \max_{s} e^{-rs}(p(x+Kt) - P(s))$$

Thus,

$$P(t) = p(x+Kt) - e^{rt}u(x+Kt)$$

The seller's revenue is

$$\pi = \int_0^{\frac{q_0-x}{K}} e^{-rt} KP(t)dt$$

$$= \int_0^{\frac{q_0-x}{K}} e^{-rt} Kp(x+Kt)dt - \int_0^{\frac{q_0-x}{K}} Ku(x+Kt)dt$$

$$= \int_x^{q_0} e^{-r\frac{q-x}{K}} p(q)dq - Ktu(x+Kt)\Big|_{t=0}^{t=\frac{q_0-x}{K}} + \int_0^{\frac{q_0-x}{K}} K^2tu'(x+Kt)dt \qquad (q=x+Kt)$$

$$= \int_x^{q_0} e^{-a(q-x)} p(q)dq + \int_0^{\frac{q_0-x}{K}} e^{-rt} K^2tp'(x+Kt)dt \qquad (a=r/K)$$

$$= \int_x^{q_0} e^{-a(q-x)} p(q)dq + \int_x^{q_0} e^{-a(q-x)} (q-x)p'(q)dq$$

$$\left(= \int_x^{q_0} e^{-a(q-x)} (p(q) + (q-x) p'(q)) dq \right)$$

$$= \int_x^{q_0} e^{-a(q-x)} p(q) dq + e^{-a(q-x)} (q-x) p(q) \Big|_x^{q_0} - \int_0^{q_0} e^{-a(q-x)} \big(1 - a(q-x)\big) p(q) dq$$

$$= \int_x^{q_0} e^{-a(q-x)} a(q-x) p(q) dq$$

$$= \int_0^{a(q_0-x)} z e^{-z} p(x + z/a) \frac{dz}{a} \qquad z=a(q-x)$$

$$= \int_0^{(q_0-x)/b} z e^{-z} p(x+bz) b\, dz \qquad b = 1/a = K/r$$

$$\frac{\partial \pi}{\partial b} = \int_0^{(q_0-x)/b} z e^{-z} \big(p(x+bz) + bz\, p'(x+bz)\big) dz$$

$$\frac{\partial^2 \pi}{\partial x \partial b} = \int_0^{(q_0-x)/b} z e^{-z} \big(p'(x+bz) + bz\, p''(x+bz)\big) dz$$

$$= \int_x^{(q_0-x)/b} a(q-x) e^{-a(q-x)} \big(p'(q) + (q-x)\, p''(q)\big) dq$$

Thus a sufficient condition for K^* to decrease in x is $p' + qp'' \le 0$. That this is sufficient follows from the following logic. If $p'' \le 0$, then $p' + (q-x)p'' \le 0$. If $p'' > 0$, then $p' + (q-x)p'' \le p' + qp'' \le 0$. Either way, the term is non-positive. Moreover,

$$\frac{\partial^2 \pi}{(\partial b)^2} = \int_0^{(q_0-x)/b} z e^{-z} \big(2p'(x+bz) + bz\, p''(x+bz)\big) dz < 0,$$

so the FOCs uniquely characterize a maximum. *Q.E.D.*

Next, we turn to Lemma 2. The intuition for the lemma is as follows: Suppose that play is one period away from the Coase path. That is, if K / N units are sold in the current period, then in all subsequent periods (along the Coase path) the capacity constraint will not bind. In that case, any additional capacity that the monopolist purchases will be used in at most one sales period, so its marginal benefit is bounded by v^H / N, which shrinks to zero as the number of sales periods per unit time increases. The marginal cost c of capacity, on the other hand, does not vary with N, so for large enough N the monopolist will not increase capacity.

Now suppose that the firm has capacity K at least as great as $K_N^{\max}(g,c)$, and suppose that the size $1 - q$ of the remaining market is such that play is one sales period away from a subgame where the unique equilibrium entails never increasing capacity. In that case, this period's capacity will be the capacity for the rest of the game. The firm, then, would like to choose capacity $K_N^{com}(q,g,c)$. By the definition of $K_N^{\max}(g,c)$, though, existing capacity K is already greater than or equal to optimal capacity $K_N^{com}(q,g,c)$, so Lemma 1 implies that increasing capacity cannot raise revenue. Therefore, the firm will not purchase any additional capacity. Thus, by induction, if the monopolist chooses capacity $K_N^{\max}(g,c)$ in period 1, she will never increase it, and so will earn the profits from committing to that capacity.

Proof of Lemma 2: The proof is inductive. First, we say that play is one sales period away from the Coase path if i) current capacity is less than the Coase quantity, and ii) the market that remains after the monopolist sells her capacity in the current period is such that the quantities sold in each period in the Coase equilibrium are no greater than current capacity – that is, if

$$S^C(q_z, g, N) > \frac{K_{z-1}}{N} \geq S^C_{\max}(q_z + \frac{K_{z-1}}{N}, g, N).$$

We will show that if play is one sales period away from the Coase path, then there is a unique subgame perfect continuation, in which the monopolist will never increase capacity. Note that the capacity constraint cannot bind in the last period of sales, so eventually play must be on the Coase path.[3] Second, we say that play is one sales period away from a unique continuation with constant capacity if $G_N(K, q, g, c)$ has a unique SPE for all markets $q \geq q_z + S_N^{com}(K_{z-1}, q_z, g, c)$ and all $K \geq K_{z-1}$, and that in that SPE capacity is never increased. We will show that if $K_{z-1} \geq K_N^{\max}(g, c)$, and play is one sales period away from a unique continuation with constant capacity, again there is a unique subgame perfect continuation, in which the monopolist will never increase capacity. Finally, we show that by choosing K_1 equal to $K_N^{\max}(g, c)$ the monopolist can guarantee profits of at least $\pi_N^{com}(K_N^{\max}(g, c), g, c)$.

<u>Step 1:</u> *One sales period from the Coase path.*

First, note that the monopolist, having chosen $K_z \geq K_{z-1}$, will set a price so as to sell either K_z / N or the Coase quantity $S^C(q_z, g, N)$, if K_z / N exceeds $S^C(q_z, g, N)$. Let q^*

[3] An argument similar to Fudenberg, Levine, and Tirole's (1985) Lemma 3 shows that the monopolist will sell to all consumers in finite time.

equal $q_z + \min\{K_z / N, S^C(q_z, g, N)\}$. Fudenberg, Levine, and Tirole (1985) show that the optimal action (subject to subgame perfection) for the monopolist when i) q lies in the interval $[q^* - S^C(q_z, g, N), q^*]$, and ii) the continuation from q^* is on the Coase path, is to sell volume $q^* - q_z$ at the Coase price. Thus, the quantity sold S in period z is equal to the smaller of K_z / N and $S^C(q_z, g, N)\}$.

Whether or not the monopolist increases capacity above K_{z-1}, then, play starting next period will be on the Coase path, where the capacity constraint never binds. Therefore, any additional capacity the monopolist purchases in period z will be used at most once. Furthermore, because Coase profits $\pi^C(q, g, N)$ are decreasing in q and $S = \min\{K_z / N, S^C(q, g, N)\}$, continuation profits are no greater than $\pi^C(q_Z + K_{Z-1} / N, g, N)$, the level that results from not increasing capacity. By similar reasoning, the price charged in period z is no higher than the price if capacity is not increased. Thus, the marginal revenue of increasing capacity in period z is bounded above by the highest consumer valuation, v^H, times the amount of the additional capacity used in one period, $1 / N$. That bound, v^H / N, shrinks to zero as N increases. The marginal cost of raising capacity, on the other hand, is the constant c. Thus, if N is large enough, the only subgame perfect action for the monopolist is to choose $K_z = K_{z-1}$.

Step 2: *One sales period from a unique continuation with constant capacity and* $K_{z-1} \geq K_N^{\max}(g,c)$.

First, note that since $K_z \geq K_{z-1} \geq K_N^{\max}(g,c) \geq S_N^{com}(K_{Z-1}, q_Z, g, c)$, if the monopolist does not increase capacity in period z, she will set a price so as to sell quantity $S_N^{com}(K_{z-1}, q_z, g, c)$, by definition of $S_N^{com}(K_{z-1}, q_z, g, c)$. Note also that the monopolist's maximal revenue consistent with subgame perfection if she does increase capacity to $K_z > K_{z-1}$ is bounded above by $R_N^{com}(K_z, q, g, c)$, the revenue if the monopolist can commit to never again increasing capacity. (The only way that the maximal revenue without commitment could be higher than the commitment revenue is if the monopolist would optimally increase capacity after the first period. In the present case, however, the induction hypothesis ensures that capacity will not be increased when the served market q rises to $q_z + S_N^{com}(K_{z-1}, q_z, g, c)$ or higher, and the monopolist already has sufficient capacity to reach that level in ones sales period.) Since current capacity K_{z-1} already weakly exceeds $K_N^{\max}(g,c)$, and above that level Lemma 1 implies that commitment profits are decreasing in *K*, augmenting capacity cannot lead to revenue greater than $R_N^{com}(K_z, q_{z-1}, g, c)$. But revenue $R_N^{com}(K_z, q, g, c)$ is what the monopolist will earn if she does not augment capacity. In order to maximize profits, then, the monopolist will not increase capacity when play is one period away from a unique continuation with constant capacity and $K_{z-1} \geq K_N^{\max}(g,c)$.

That ends the induction. Thus, the monopolist can guarantee herself a profit of $\pi_N^{com}(K_N^{\max}(g,c), g, c)$ by purchasing capacity $K_N^{\max}(g,c)$ in the first period. Any SPE, therefore, must give the monopolist at least that level of profits. *Q.E.D.*

Finally, we prove Lemma 3:

Proof of Lemma 3: First, we will construct values $\underline{K} > 0$ and $\overline{K}$ such that in the limit the initial optimal capacity $K_N^{com}(q,g,c)$ must lie in the interval $[\underline{K},\overline{K}]$. Next, we will show that for all $(K, q) \in [\underline{K},\overline{K}] \times [0, q_0]$, $\pi_N^{com}(K,q,g,c)$ converges to $\pi^{com}(K,q,0,c)$ uniformly as N increases and g shrinks (when c is small). That will establish the result: Since commitment profits converge uniformly in the relevant range, and Lemma 1 ensures both that $K^{com}(0, 0, c)$ is the unique optimum and that $K^{com}(q, 0, c) \leq K^{com}(0, 0, c)$, $K_N^{\max}(g,c)$ converges to $K^{com}(0, 0, c)$.

To find the lower bound on capacity $\underline{K}$, recall that we established in the previous section that $R_N^{com}(K,0,g,c)$ is at least a fraction γ of monopoly revenue $R(q_m)$. Choose $\underline{K}$ such that

$$\int_0^{q_0/\underline{K}} \delta^t \underline{K} p(0) dt < \gamma R(q_m) / 2.$$

For small c and g and large N, then, the optimal initial commitment capacity cannot be lower than $\underline{K}$, since choosing such a low capacity generates a discounted value of revenue strictly less than $\gamma R(q_m)$ (even if every consumer pays $p(0)$). Define $\overline{K}$ as $2q_0 p(0) / c$. For small enough g, the cost of capacity greater than $\overline{K}$ exceeds the revenue from selling to all consumers instantly at the highest valuation.

To show uniform convergence at $K \in [\underline{K}, \overline{K}]$, first fix the value of c. We now bound the marginal effect on $\pi_N^{com}(K,q,g,c)$ and $\pi^{com}(K, q, g, c)$ of small changes in q and K. The marginal increase in profits from increasing the size of the remaining market is no greater than v^H (= $p(0) + g$). Thus, if $|q - q'| < \varepsilon/5p(0)$, then $|\pi_N^{com}(K,q,g,c) - \pi_N^{com}(K,q',g,c)|$ and $|\pi^{com}(K, q, g, c) - \pi^{com}(K, q', g, c)|$ are both less than $\varepsilon/5$ for small enough g.

An increase in capacity may either increase or decrease commitment profits. The increase comes from allowing sales to be made more quickly. If the capacity constraint binds in every period, then the waiting time until the last buyer is served is q_0 / K, rounded up to the nearest $1/N$. (Call that value T.) Thus, an increase in K that reduces the value of q_0 / K by d might decrease that waiting time by up to $d + 1/N$. The resulting increase in the discounted value of revenue, then, is no more than

$$\delta^{T-(d+1/N)} v^H q_0 - \delta^T v^H q_0 \qquad \leq \qquad (\delta^{-(d+1/N)} - 1) v^H q_0 ,$$

which bounds the increase in case i) all sales take place in the last period, ii) all buyers pay v^H, and iii) the monopolist does not have to charge lower prices in order to sell more quickly.

Raising capacity can reduce revenue because buyers anticipating faster falls in prices are willing to pay less. Since consumer q is indifferent between waiting $d + 1/N$ units of time to pay the last price g and paying $\delta^{(d+1/N)} g + (1 - \delta^{(d+1/N)}) v(q)$ now, the reduction in revenue that results from lowering the value of q_0 / K by d (and thus decreasing the waiting time by up to $d + 1/N$) is no greater than

$$[\delta^T g + (1-\delta^T)v^H]q_0 - [\delta^{T-(d+1/N)} g + (1-\delta^{T-(d+1/N)})v^H]q_0$$

$$\leq \quad (\delta^{-(d+1/N)} - 1)[v^H - g]q_0 .$$

That bound applies even if all consumers have the highest valuation v^H, and the reduction in revenue is not discounted. Because $g > 0$, the magnitude of the bound is strictly less than that of the bound in the previous paragraph.

Choose a d^* satisfying $(\delta^{-d^*} - 1)v^H q_0 < \varepsilon/5$. If $|K - K'| < (\underline{K}^2/q_0)d^*$, then

$$\left| q_0/K - q_0/K' \right| < d^*.$$

Thus, if N is large enough and $|K - K'| < (\underline{K}^2/q_0)d^*$, then the values of the distances $|\pi_N^{com}(K,q,g,c) - \pi_N^{com}(K',q,g,c)|$ and $|\pi^{com}(K, q, g, c) - \pi^{com}(K', q, g, c)|$ are both less than $\varepsilon/5$ for all $K, K' \in [\underline{K}, \overline{K}]$.

Finally, note that $\pi_N^{com}(K,q,g,c)$ converges to $\pi^{com}(K,q,0,c)$ pointwise. Choose a finite subset $F \in [\underline{K}, \overline{K}] \times [0, q_0]$ such that every point in $[\underline{K}, \overline{K}] \times [0, q_0]$ is within $\min\{\varepsilon/5p(0), d^*\}$ of an element of F. Choose N large enough and g small enough that, in addition to the conditions above, $|\pi_N^{com}(K,q,g,c) - \pi^{com}(K,q,0,c)| <$

$\varepsilon/5$ for every $(K, q) \in F$. Now pick any point $(K, q) \in [\underline{K}, \overline{K}] \times [0, q_0]$, and let (K_F, q_F) be the nearest element of F. By construction, $|\pi_N^{com}(K,q,g,c) - \pi^{com}(K,q,0,c)| < \varepsilon$. ($\pi_N^{com}(K,q,g,c)$ is within $\varepsilon/5$ of $\pi_N^{com}(K_F,q,g,c)$, which is within $\varepsilon/5$ of $\pi_N^{com}(K_F,q_F,g,c)$, which is within $\varepsilon/5$ of $\pi^{com}(K_F,q_F,0,c)$, which is within $\varepsilon/5$ of $\pi^{com}(K_F,q,0,c)$, which is within $\varepsilon/5$ of $\pi^{com}(K,q,0,c)$.)

Q.E.D.

Appendix 2: The Case of Constant Elasticity of Demand (Not Intended for Publication)

The case of constant elasticity (with a positive marginal cost) can be embedded into this framework using:

$$p(q) = q^{\frac{-1}{\varepsilon}} - c,$$

where ε is the elasticity of demand. Note $q_0 = c^{-\varepsilon}$. (Remark: prices have been translated by marginal cost; the case where actual marginal cost is zero would need to be handled separately.)

Monopoly profits are determined by

$$0 = \pi'(q_m) = (1 - 1/\varepsilon) q_m^{-1/\varepsilon} - c,$$

and $q_m = \left(\frac{c}{1-\varepsilon^{-1}}\right)^{-\varepsilon}$.

Monopoly profits are

$$\pi_m = (q_m^{-\varepsilon^{-1}} - c)q_m = \left(\frac{c}{1-\varepsilon^{-1}}\right)^{-\varepsilon}\left(\frac{c}{1-\varepsilon^{-1}} - c\right) = \frac{1}{\varepsilon}\left(\frac{c}{1-\varepsilon^{-1}}\right)^{1-\varepsilon}.$$

Monopoly profits are finite only when demand is elastic and attention is restricted to this case. The profits associated with the Coase path are

$$\pi^* = \max_{\hat{a}} \int_0^{q_0} e^{-\hat{a}y} MR(q)dq$$

$$= \max_{\hat{a}} \int_0^{c^{-\varepsilon}} e^{-\hat{a}y}\left((1-\varepsilon^{-1})q^{-\varepsilon^{-1}} - c\right)dq$$

$$= \max_{a} \int_0^{1} e^{-ay}\left((1-\varepsilon^{-1})(c^{-\varepsilon}y)^{-\varepsilon^{-1}} - c\right)c^{-\varepsilon}dy$$

$$= c^{1-\varepsilon} \max_{a} \int_0^{1} e^{-ay}\left((1-\varepsilon^{-1})y^{-\varepsilon^{-1}} - 1\right)dy$$

Thus,

$$\frac{\pi^*}{\pi_m} = \frac{\max_{a} \int_0^{1} e^{-ay}\left((1-\varepsilon^{-1})y^{-\varepsilon^{-1}} - 1\right)dy}{\varepsilon^{-1}(1-\varepsilon^{-1})^{\varepsilon-1}}.$$

Note $\lim_{\varepsilon\to\infty} (1-\varepsilon^{-1})^{\varepsilon-1} = e^{-1}$. and

$$\lim_{\varepsilon\to\infty} \frac{(1-\varepsilon^{-1})y^{-\varepsilon^{-1}} - 1}{\varepsilon^{-1}} = \lim_{\varepsilon\to\infty} -y^{-\varepsilon^{-1}} + \frac{y^{-\varepsilon^{-1}} - 1}{\varepsilon^{-1}}$$

$$= -1 + \lim_{x\to 0} \frac{y^{-x} - 1}{x} = -1 - \log(y).$$

Thus,

$$\lim_{\varepsilon\to\infty}\frac{\pi^*}{\pi_m}=\lim_{\varepsilon\to\infty}\frac{\max\limits_a\int_0^1 e^{-ay}\left((1-\varepsilon^{-1})y^{-\varepsilon^{-1}}-1\right)dy}{\varepsilon^{-1}(1-\varepsilon^{-1})^{\varepsilon-1}}=\lim_{\varepsilon\to\infty}\frac{-\max\limits_a\int_0^1 e^{-ay}\left(1+\log(y)\right)dy}{e^{-1}}=0.673478$$

Note $\lim\limits_{\varepsilon\to1}(1-\varepsilon^{-1})^{\varepsilon-1}=1$. Thus,

$$\lim_{\varepsilon\to1}\frac{\pi^*}{\pi_m}=\lim_{\varepsilon\to1}\max_a\int_0^1 e^{-ay}\left((1-\varepsilon^{-1})y^{-\varepsilon^{-1}}-1\right)dy=\lim_{x\uparrow1}\max_a\int_0^1 e^{-ay}\left((1-x)y^{-x}-1\right)dy$$

$$=\lim_{x\uparrow1}\max_a\int_0^1 ae^{-ay}\left(y^{1-x}-y\right)dy=\lim_{x\uparrow1}\max_a a\int_0^1\sum_{n=0}\frac{(-ay)^n}{n!}\left(y^{1-x}-y\right)dy$$

$$=\lim_{x\uparrow1}\max_a a\sum_{n=0}\frac{(-a)^n}{n!}\int_0^1 y^n\left(y^{1-x}-y\right)dy=\lim_{x\uparrow1}\max_a a\sum_{n=0}\frac{(-a)^n}{n!}\left(\frac{1}{n+2-x}-\frac{1}{n+2}\right)$$

$$=\max_a a\sum_{n=0}\frac{(-a)^n}{n!}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\max_a\frac{1}{a}\sum_{n=0}\frac{(-a)^{n+2}}{(n+2)!}$$

$$=\max_a\frac{1}{a}\sum_{n=2}\frac{(-a)^n}{n!}=\max_a\frac{1}{a}\left(e^{-a}-1+a\right)=1.$$

Thus, as $\varepsilon\to1$, the Coase path and commitment path converge to the same profits (which are obtained selling arbitrarily little at a very high price).