Essays on Finite Sample Inference and Financial Econometrics

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OUTLINE

1. Finite Sample Moments of Econometric Estimators with Non-IID Observations

2. The Second-Order Bias and Mean Squared Error of Time Series Estimators*

3. Finite Sample Properties of Maximum Likelihood Estimator in Spatial Models

4. Bias of Value-At-Risk Model

5. Evaluating Predictive Performance of Value-At-Risk Models in Emerging Markets: A Reality Check*

6. A Test for Density Forecast Comparison with Applications to Risk Management

OVERVIEW

Addresses two issues in econometric modeling:

i) a model's in-sample properties of when the sample size is finite (Chapters 1 to 4)

n is finite (small or moderately large), properties of $\hat{\theta}_n$ / properties of a model $f(\hat{\theta}_n)$

ii) a model's out-of-sample predictive ability when the sample size is large (Chapters 5 to 6)

n is infinitely large, predictive ability under proper loss functions

Chapters 1 to 4:

I. INTRODUCTION

Motivation: $\hat{\theta}_n = \hat{\theta} = h(Z)$, properties (moments) of $\hat{\theta}$ or $f(\hat{\theta})$?

Exact results: e.g., $\mathbb{E}(\hat{\theta}) = \mathbb{E}[h(Z)] = \int h(Z)\, f(Z) dZ$

Approximate

- Asymptotic theory (first-order), $n \to \infty$
- finite sample theory (second-order), n is moderately large

Moments (Analytical) of $\hat{\theta} = h(Z)$

Exact	Approximate	Asymptotic Theory
	(Second-order)	(First-order)
(regardless of n)	(moderately large n)	($n \to \infty$)
$E(\hat{\theta}) = E[h(Z)] = \int h(Z) f(Z) dZ$	Bias $= O(n^{-1})$	Consistent
In general, difficult to obtain, or difficult to interpret even if obtainable	MSE $= O(n^{-2})$	$V(\hat{\theta}) = O(n^{-1})$

Problems with the asymptotic theory:

- Can not distinguish a class of estimators of the same asymptotic properties
  - FGLS in SURE models
  - GMM and GEL
- Crucial assumption: $n \to \infty$. Realistic, esp. for macro data? No!
  - JBES, 1996, GMM

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Analytical finite sample theory allows us to

- select the estimator with good finite sample properties from a class of asymptotically equivalent estimators
- measure the magnitude of the loss of asymptotic-theory-based inference in finite samples
- understand the source of the finite sample bias and thereby design a bias-corrected estimator
- check the accuracy of certain Monte Carlo experiments

Extensive literature on the analytical finite sample properties of econometric estimators

- Linear Models: Hurwicz (1950), White (1957, 1958, 1959, 1961), Nagar (1959), Shenton and Johnson (1965), Sawa (1969, 1978), Anderson and Sawa (1973, 1979), Basmann (1974), Sargan (1974, 1976), Phillips (1977,1978, 1979, 1987), Dufour (1984, 1990), Hoque *et al.* (1988), Rothenberg (1984), Kiviet and Phillips (1993, 1997), Dufour and Kiviet (1996), Lieberman (1994), Tsui and Ali (1994), Ullah and Srivastava (1994), Ali (2002), Ullah (2002), among others.

- Nonlinear Models: Amemiya (1980), Cordeiro and Klein (1994), Rilstone *et al.*(1996), Linton (1997), Iglesias and Phillips (2001), Gospodinov (2002), Anatolyev (2003), Bao and Ullah (2003), Newey and Smith (2004), among others.

Issues:

- Generally IID

- Generally Gaussian normal

- Generally linear models

- Generally specific estimators (LS or ML) in specific models

Contribution of this thesis: a unified approach for the second-order bias/MSE of a class of estimators when

- Non-IID (time series, cross section, panel, etc.)
- Nonnormal
- Nonlinear
- Different types of estimators (GMM, LS, QML, and other extremum estimators)

Chapter 1: general results, non-IID and nonnormal

Chapter 2: time series models, normal

Chapter 3: spatial models, normal

Chapter 4: VaR model, nonnormal

II. SECOND-ORDER BIAS AND MSE

Consider a class of $\sqrt{n}$-consistent estimators identified by the moment condition

$$\hat{\theta} = \hat{\theta}_n = \text{arg}\left\{\psi_n(\theta) = 0\right\},$$

where $\psi_n(\theta) = \psi_n(Z;\theta)$ is a known $p \times 1$ vector-valued function of the observable data $Z = \{Z_i\}_{i=1}^n$, and a parameter vector θ of p elements such that $\mathbb{E}\left[\psi_n(\theta)\right] = 0$.

Example 1: LS

$y = X\beta + \varepsilon$

$$\hat{\beta}_{OLS} = \text{arg}\left\{\psi_n(\beta) = 0 \mid \psi_n(\beta) = \frac{1}{n}X'\varepsilon\right\}$$

Example 2: (Q)ML

$\mathcal{L}(\theta) = \frac{1}{n}\sum_{i=1}^n l_i(\theta)$

$$\hat{\theta}_{ML} = \text{arg}\left\{\psi_n(\theta) = 0 \mid \psi_n(\theta) = \frac{1}{n}\sum_{i=1}^n \frac{\partial l_i(\theta)}{\partial \theta}\right\}$$

Example 3: GMM

$g_n(\theta) = \mathbb{E}[g(Z,\theta)] = 0,$

$g_n(\cdot)$ is $m \times 1$, θ is $p \times 1$, $m \geq p$

$Q_n(\theta) = g_n(\theta)' W_n(\theta) g_n(\theta)$

$\hat{\theta}_{GMM} = \arg\{\psi_n(\theta) = 0 \mid \psi_n(\theta) = \partial Q_n(\theta)/\partial\theta\}$

Example 4: GEL

$\min_{P_n \in \mathcal{P}} D_\rho(F_n, P_n)$

s.t. $g_n(\theta) = \mathbb{E}[g(Z,\theta)|P_n] = \int g(Z,\theta)\,dP_n = 0$

$g(\cdot)$ is $m \times 1$, θ is $p \times 1$, $m \geq p$

Define $\Theta = (\theta', \lambda')'$ and

$\mathcal{L}_n(\Theta) = D_\rho(F_n, P_n) + \lambda' g_n(\theta)$

$\hat{\Theta}_{GEL} = \arg\{\psi_n(\Theta) = 0 \mid \psi_n(\Theta) = \partial\mathcal{L}_n(\Theta)/\partial\Theta\}$

Assumption 1: The s-th order derivatives of $\psi_n(\hat{\theta})$ exist in a neighborhood of θ and $\mathbb{E}(||\nabla^s \psi_n(\theta)||^2) < \infty$, where $||A||$ for a matrix A is the usual norm $[\text{tr}\,(AA')]^{1/2}$ and tr is the trace operator on a matrix.

Assumption 2: For some neighborhood of θ, $[\nabla \psi_n(\hat{\theta})]^{-1} = O_p(1)$.

Assumption 3: $\left|\left|\nabla^s \psi_n\left(\hat{\theta}\right) - \nabla^s \psi_n(\theta)\right|\right| \leq \left|\left|\hat{\theta} - \theta\right|\right| M_n$ for some neighborhood of θ, where $\mathbb{E}(|M_n|) < C < \infty$ for some positive constant C.

Given Assumptions 1-3, Taylor expansion

$$0 = \psi_n\left(\hat{\theta}\right)$$

$$= \psi_n(\theta) + \nabla\psi_n(\theta)\left(\hat{\theta} - \theta\right) + \tfrac{1}{2}\nabla^2\psi_n(\theta)\left[\left(\hat{\theta} - \theta\right) \otimes \left(\hat{\theta} - \theta\right)\right]$$

$$+ \tfrac{1}{6}\nabla^3\psi_n(\theta)\left[\left(\hat{\theta} - \theta\right) \otimes \left(\hat{\theta} - \theta\right) \otimes \left(\hat{\theta} - \theta\right)\right]$$

$$+ \tfrac{1}{6}\left[\nabla^3\psi_n\left(\bar{\theta}\right) - \nabla^3\psi_n(\theta)\right]\left[\left(\hat{\theta} - \theta\right) \otimes \left(\hat{\theta} - \theta\right) \otimes \left(\hat{\theta} - \theta\right)\right].$$

Solve for $(\hat{\theta} - \theta)$ from above and use the expansion for $[\nabla\psi_n(\theta)]^{-1}$ as follows

$$\begin{aligned} [\nabla\psi_n(\theta)]^{-1} &= \left[Q^{-1} + V\right]^{-1} \\ &= Q - QVQ + QVQVQ + \cdots, \end{aligned}$$

where $Q = \underbrace{[\mathbb{E}\nabla\psi_n(\theta)]^{-1}}_{O(1)}$ and $V = \underbrace{\nabla\psi_n(\theta) - \mathbb{E}\nabla\psi_n(\theta)}_{O_P(n^{-1/2})}$.

Expansion for $[\nabla \psi_n(\theta)]^{-1}$, an example: OLS

$$y = X\beta + \varepsilon \quad \Rightarrow \psi_n(\beta) = \frac{1}{n} X'\varepsilon = \frac{1}{n} X'(y - X\beta)$$

$$\Rightarrow \nabla \psi_n(\beta) = -\frac{1}{n} X'X$$

$$\Rightarrow$$

$$[\nabla \psi_n(\beta)]^{-1} = [\underbrace{E(-\tfrac{1}{n}X'X)}_{O(1)} + \underbrace{(-\tfrac{1}{n}X'X) - E(-\tfrac{1}{n}X'X)}_{O_P(n^{-1/2})}]^{-1}$$

$$= [E(-\tfrac{1}{n}X'X)]^{-1} \times \{I + \underbrace{[E(-\tfrac{1}{n}X'X)]^{-1}[(-\tfrac{1}{n}X'X) - E(-\tfrac{1}{n}X'X)]}_{O_P(n^{-1/2}) = QV \equiv \Lambda}\}^{-1}$$

$$= [E(-\tfrac{1}{n}X'X)]^{-1} \times \{I - \underbrace{\Lambda}_{O_P(n^{-1/2})} + \underbrace{\Lambda\Lambda}_{O_P(n^{-1})} - \underbrace{\Lambda\Lambda\Lambda}_{O_P(n^{-3/2})} + \ldots\}$$

Therefore

$$\hat{\theta} - \theta = a_{-1/2} + a_{-1} + a_{-3/2} + O_P\left(n^{-2}\right),$$

where $a_{-s/2} = O_P\left(n^{-s/2}\right)$ are functions of $H_i = \nabla^i \psi_n$.

The second-order bias and MSE

$$B\left(\hat{\theta}\right) = \mathbb{E}\left(a_{-1/2}\right) + \mathbb{E}\left(a_{-1}\right) + o\left(n^{-1}\right),$$

$$M(\hat{\theta}) = \mathbb{E}(A_{-1}) + \mathbb{E}(A_{-3/2}) + \mathbb{E}(A_{-2}) + o\left(n^{-2}\right),$$

where the $p \times p$ matrices

$$A_{-1} = a_{-1/2}a'_{-1/2},$$

$$A_{-3/2} = a_{-1/2}a'_{-1} + a_{-1}a'_{-1/2},$$

$$A_{-2} = a_{-1/2}a'_{-3/2} + a_{-3/2}a'_{-1/2} + a_{-1}a'_{-1}.$$

Special case:

- IID observations, Rilstone, Srivastava, Ullah (1996)

III. MODELS

$$y = f(X;\theta) + \varepsilon$$

$\mathbb{E}(\varepsilon_t) = 0, \quad \mathbb{E}\left(\varepsilon_t^2\right) = \sigma^2.$

If nonnormal,

$\mathbb{E}\left(\varepsilon_t^3\right) = \sigma^3\gamma_1,$

$\mathbb{E}\left(\varepsilon_t^4\right) = \sigma^4(\gamma_2 + 3),$

$\mathbb{E}\left(\varepsilon_t^5\right) = \sigma^5(\gamma_3 + 10\gamma_1),$

$\mathbb{E}\left(\varepsilon_t^6\right) = \sigma^6\left(\gamma_4 + 10\gamma_1^2 + 15\gamma_2 + 15\right).$

Time Series Models

1. ARX(1)

$$y_t = \rho y_{t-1} + x_t'\beta + \varepsilon_t,$$

where $|\rho| < 1$ and $X'X = O(n)$.

THEOREM. *In the ARX(1) model with $|\rho| < 1$, the second-order bias, up to $O\left(n^{-1}\right)$, of the OLS estimator $\hat{\rho}$ when the errors are nonnormally distributed is*

$$\begin{aligned} B(\hat{\rho}) &= \frac{\sigma^2\left[\operatorname{tr}(MC) - 2\rho\left(1-\rho^2\right)^{-1}\right]}{\bar{D}} \\ &\quad - \frac{2\sigma^2\left[r_D'Cr_D - \rho\left(1-\rho^2\right)^{-1} r_D'r_D\right]}{\bar{D}^2} \\ &\quad + \gamma_1\xi_1, \end{aligned}$$

where

$$\xi_1 = \frac{-\sigma^3\iota'\left\{\left[I \odot (C'MC)\right] r_D + 2\left[I \odot (MC)\right] C'r_D\right\}}{\bar{D}^2}.$$

Bias, AR model, $n = 50$

d.f.	ρ	$\hat{\rho}$	$\widehat{\rho}_{BC,N}$	$\widehat{\rho}_{BC,NN}$	$\widetilde{\rho}_{BC,N}$	$\widetilde{\rho}_{BC,NN}$	Bias_N	Bias_{NN}
5	0.1	0.0772	0.0927	0.0988	0.0916	0.0976	-0.0155	-0.0216
	0.2	0.1735	0.1931	0.1995	0.1919	0.1982	-0.0197	-0.0261
	0.3	0.2702	0.2937	0.3003	0.2924	0.2988	-0.0235	-0.0301
	0.9	0.8818	0.9014	0.9020	0.9021	0.9032	-0.0196	-0.0203

d.f.	degrees of freedom of the non-central t
ρ	true parameter
$\hat{\rho}$	OLS estimate
$\widehat{\rho}_{BC,N}$	bias-corrected estimate using ρ, ignoring nonnormality
$\widehat{\rho}_{BC,NN}$	bias-corrected estimate using ρ
$\widetilde{\rho}_{BC,N}$	feasible bias-corrected estimate using $\hat{\rho}$, ignoring nonnormality
$\widetilde{\rho}_{BC,NN}$	feasible bias-corrected estimate using $\hat{\rho}$
Bias_N	theoretical bias of $\hat{\rho}$, ignoring nonnormality
Bias_{NN}	theoretical bias of $\hat{\rho}$

$n = 50$

d.f.	ρ	$\hat{\rho}$	$\widehat{\rho}_{BC,N}$	$\widehat{\rho}_{BC,NN}$	$\widetilde{\rho}_{BC,N}$	$\widetilde{\rho}_{BC,NN}$	Bias_N	Bias_{NN}
6	0.1	0.0803	0.0959	0.1005	0.0949	0.0994	-0.0155	-0.0202
	0.2	0.1766	0.1963	0.2011	0.1951	0.1999	-0.0197	-0.0246
	0.3	0.2732	0.2967	0.3017	0.2955	0.3004	-0.0235	-0.0286
	0.9	0.8815	0.9011	0.9016	0.9017	0.9026	-0.0196	-0.0201

$n = 80$

d.f.	ρ	$\hat{\rho}$	$\hat{\rho}_{BC,N}$	$\hat{\rho}_{BC,NN}$	$\tilde{\rho}_{BC,N}$	$\tilde{\rho}_{BC,NN}$	Bias_N	Bias_{NN}
5	0.1	0.0847	0.0948	0.0984	0.0943	0.0978	-0.0100	-0.0136
	0.2	0.1823	0.1948	0.1986	0.1943	0.1980	-0.0125	-0.0163
	0.3	0.2801	0.2950	0.2989	0.2944	0.2983	-0.0149	-0.0188
	0.9	0.8869	0.9001	0.9010	0.9005	0.9015	-0.0132	-0.0141

COROLLARY. *In a pure AR(1) model with the autoregressive coefficient $|\rho| < 1$, the second-order bias, up to $O\left(n^{-1}\right)$, of the OLS estimator $\hat{\rho}$* is $-2\rho/n$.

COROLLARY. *If in the ARX(1) model with $|\rho| < 1$, $X = \iota$ and $\beta \neq 0$, the second-order bias, up to $O\left(n^{-1}\right)$, of the OLS estimator $\hat{\rho}$, is given by $B(\hat{\rho}) = -(1 + 3\rho)/n$.*

THEOREM. *The second-order bias of $\hat{\rho}$, up to $O\left(n^{-1}\right)$, and MSE, up to $O\left(n^{-2}\right)$, of the OLS estimator $\hat{\rho}$ in the pure AR(1) model when the errors are normally distributed are*

$$\begin{aligned} B(\hat{\rho}) &= \frac{1}{(n-1)^2} Q^2 \lambda_{11}, \\ M(\hat{\rho}) &= \frac{6Q^2}{(n-1)^2}\lambda_{20} + \frac{2Q^3}{(n-1)^3}\left(1 + 3Q^2\right)\lambda_{21} \\ &\quad + \frac{3Q^4}{(n-1)^4}\lambda_{22}, \end{aligned}$$

where $Q = \left(\frac{\mathrm{tr}(C_1\Sigma)}{n-1}\right)^{-1}$, $\lambda_{rs} = \mathbb{E}\left[(y'Cy)^r \cdot (y'C_1y)^s\right]$ *for* $r, s = 0, 1, 2$.

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Predictive regressions☆

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Abstract

When a rate of return is regressed on a lagged stochastic regressor, such as a dividend yield, the regression disturbance is correlated with the regressor's innovation. The OLS estimator's finite-sample properties, derived here, can depart substantially from the standard regression setting. Bayesian posterior distributions for the regression para-

$$y_t = \alpha + \beta x_{t-1} + u_t,$$
$$x_t = c + \rho x_{t-1} + v_t.$$

Proposition 3. For each integer s, $1 \leqslant s < (T-1)$,

$$m'_s = \mathrm{E}([\hat{\beta} - \beta]^s) = s2^s \sum_i \gamma_s(v_i) \int_0^\infty q^{s-1} |\Delta| \prod_{j=1}^{s} (\mathrm{tr}\, R^j)^{n_{ij}}\, \mathrm{d}q, \tag{10}$$

where the summation is over all vectors $v_i = (n_{i1}, n_{i2}, \ldots, n_{is})$ *whose s elements are non-negative integers satisfying* $\sum_{j=1}^{s} j n_{ij} = s$,

$$\gamma_s(v_i) = \prod_{j=1}^{s} [n_{ij}!(2j)^{n_{ij}}]^{-1}, \tag{11}$$

and where the $2T \times 2T$ *matrices* Δ *and* R *are constructed as follows. Let* P *be a* $2T \times 2T$ *matrix such that* $P'P = I_{2T}$ *and* $P'L'BLP = \Lambda$*, a diagonal matrix. Then* $\Delta = (I_{2T} + 2q\Lambda)^{-1/2}$ *and* $R = \Delta P'L'ALP\Delta$.

	Sample period			
	1927–1996	1927–1951	1952–1996	1977–1996
A. True properties				
Bias	0.07	0.18	0.18	0.42
Standard deviation	0.16	0.33	0.27	0.45
Skewness	0.71	0.83	0.98	1.29
Kurtosis	3.84	4.14	4.62	5.83
p-value for $\beta = 0$	0.17	0.42	0.15	0.64
B. Properties in the standard regression setting				
Bias	0	0	0	0
Standard deviation	0.14	0.27	0.20	0.30
Skewness	0	0	0	0
Kurtosis	3	3	3	3
p-value for $\beta = 0$	0.06	0.22	0.02	0.26
C. Sample characteristics and parameter values				
$\hat{\beta}$	0.21	0.21	0.44	0.19
T	840	300	540	240

Intuition:

$$E(\hat{\beta}-\beta) = \frac{\sigma_{uv}}{\sigma_v^{\ 2}} E(\hat{\rho}-\rho) = -\frac{\sigma_{uv}}{\sigma_v^{\ 2}}\left(\frac{1+3\rho}{T}\right) + O(T^{-2})$$

σ_{uv} is negative, and $\sigma_{uv}/\sigma_v^{\ 2} \in [-22.3, -13.6]$

Even $E(\hat{\rho}-\rho) \approx (1+3\rho)/T$ is quite small for $\rho \in (-1,1)$ for moderately large *T*, the bias of $\hat{\beta}$ is ***scaled up*** substantially due to $-\sigma_{uv}/\sigma_v^{\ 2}$.

Nelson, C.R., Kim, M.J., 1993. Predictable stock returns: the role of small sample bias. ***Journal of Finance*** 48, 641~661.

Mark, N.C., 1995. Exchange rates and fundamentals: evidence on long-horizon predictability. ***American Economic Review*** 85, 201~218.

Bekaert, G., Hodrick, R.J., Marshall, D.A., 1997. On biases in tests of the expectations hypothesis of the term structure of interest rates. ***Journal of Financial Economics*** 44, 309~348.

THEOREM. *In the ARX(1) model with* $|\rho| < 1$, *the second-order bias, up to* $O\left(n^{-1}\right)$, *of the OLS estimator* $\hat{\theta} = (\hat{\rho}, \hat{\beta}')'$ *is*

$$B\left(\hat{\theta}\right) = -\bar{D}^{-1}[\sigma^2 \bar{Z}' C \bar{Z} \bar{D}^{-1} e_1 + \sigma^2 e_1 \text{tr}\left(\bar{Z}' C \bar{Z} \bar{D}^{-1}\right) + 2\sigma^4 e_1 \left(e_1' \bar{D}^{-1} e_1\right) \text{tr}\left(CC'C\right)] + \gamma_1 \xi_1,$$

where $\xi_1 = -\sigma^3 \bar{D}^{-1} e_1 e_1' \bar{D}^{-1} \bar{Z}' S \iota$.

THEOREM. *In the ARX(1) model with* $|\rho| < 1$, *the second-order bias, up to* $O\left(n^{-1}\right)$, *of* $\hat{\sigma}^2 = (y - \hat{\rho} y_{-1} - X\hat{\beta})'(y - \hat{\rho} y_{-1} - X\hat{\beta})/n$ *is*

$$B\left(\hat{\sigma}^2\right) = -\frac{2}{n}\left(\xi_0 + \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_3 + \gamma_4 \xi_4 + \gamma_1^2 \xi_{11}\right).$$

2. MA(1)

$$y_t = \varepsilon_t - \phi \varepsilon_{t-1}, \quad |\phi| < 1.$$

THEOREM. *In the MA(1) model with* $\varepsilon_0 = 0$, *the second-order bias, up to* $O\left(n^{-1}\right)$, *of the conditional QMLE* $\hat{\phi}$ *is*

$$B\left(\hat{\phi}\right) = \frac{\text{tr}\left(N\right)\text{tr}\left(N_1\right) + \text{tr}\left(N^* N_1\right)}{\left[\text{tr}\left(N_1\right)\right]^2} - \frac{2\text{tr}\left(N\right)}{\text{tr}\left(N_1\right)} - \frac{\text{tr}\left(N_2\right)\left\{\left[\text{tr}\left(N\right)\right]^2 + \text{tr}\left(N^* N\right)\right\}}{2\left[\text{tr}\left(N_1\right)\right]^3} + \gamma_2 \xi_2,$$

where

$$\xi_2 = \{\text{tr}(N_1)\text{tr}(N_1 \odot N) - \text{tr}(N_2)\text{tr}(N \odot N)/2\} / \left[\text{tr}\left(N_1\right)\right]^3.$$

3. ARCH(1)

$$\begin{aligned} y_t &= \varepsilon_t, \\ \varepsilon_t &= z_t\sqrt{h_t}, \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2, \\ z_t &\sim \text{IID } (0,1). \end{aligned}$$

Define

$$\varphi = A_{02}A_{22} - A_{12}^2,$$

$$A_{ij} = \mathbb{E}\left(\varepsilon_{t-1}^{2i}/h_t^j\right),$$

$$C_1 = \sum_{i=1}^n \mathbb{E}\left(\frac{1}{h_t^2 h_{t-i}} - \frac{\varepsilon_{t-i}^2}{h_t^2 h_{t-i}^2}\right),$$

$$C_2 = \sum_{i=1}^n \mathbb{E}\left(\frac{\varepsilon_{t-i-1}^2}{h_t^2 h_{t-i}} - \frac{\varepsilon_{t-i}^2 \varepsilon_{t-i-1}^2}{h_t^2 h_{t-i}^2}\right),$$

$$C_3 = \sum_{i=1}^n \mathbb{E}\left(\frac{\varepsilon_{t-1}^2}{h_t^2 h_{t-i}} - \frac{\varepsilon_{t-1}^2 \varepsilon_{t-i}^2}{h_t^2 h_{t-i}^2}\right),$$

$$C_4 = \sum_{i=1}^n \mathbb{E}\left(\frac{\varepsilon_{t-1}^2 \varepsilon_{t-i-1}^2}{h_t^2 h_{t-i}} - \frac{\varepsilon_{t-1}^2 \varepsilon_{t-i}^2 \varepsilon_{t-i-1}^2}{h_t^2 h_{t-i}^2}\right),$$

$$C_5 = \sum_{i=1}^n \mathbb{E}\left(\frac{\varepsilon_{t-1}^4}{h_t^2 h_{t-i}} - \frac{\varepsilon_{t-1}^4 \varepsilon_{t-i}^2}{h_t^2 h_{t-i}^2}\right),$$

$$C_6 = \sum_{i=1}^n \mathbb{E}\left(\frac{\varepsilon_{t-1}^4 \varepsilon_{t-i-1}^2}{h_t^2 h_{t-i}} - \frac{\varepsilon_{t-1}^4 \varepsilon_{t-i}^2 \varepsilon_{t-i-1}^2}{h_t^2 h_{t-i}^2}\right).$$

THEOREM. *The second-order bias, up to* $O\left(n^{-1}\right)$, *of the QMLE for the ARCH(1) model is given by*

$$B\left(\hat{\theta}\right)=\frac{1}{n\varphi^2}\left(\begin{array}{c}\delta_0\\ \delta_1\end{array}\right),$$

where

$\delta_0 = A_{22}^2 C_1 - A_{12}A_{22}\left(C_2+2C_3\right) + A_{02}A_{22}C_4 + A_{12}^2\left(C_4+C_5\right) - A_{02}A_{12}C_6,$

$\delta_1 = -A_{12}A_{22}C_1 + A_{12}^2\left(C_2+C_4\right) + A_{02}A_{22}C_3 - 2A_{02}A_{12}C_4 - A_{02}A_{12}C_5 + A_{02}^2 C_6,$

Special case: $\alpha_1 = 0$,

$$B\left(\hat{\alpha}_0\right)=-\frac{\alpha_0}{n},\ B\left(\hat{\alpha}_1\right)=-\frac{1}{n},$$

also see Engle *et al.* (1985).

THEOREM. *The second-order bias of the Value-at-Risk estimated by the method of QML, where the conditional volatility is specified by an ARCH(1) model, is*

$$\mathbb{E}\left[\widehat{\mathsf{VaR}}_{n+1|n}(\alpha)-\mathsf{VaR}_{n+1|n}(\alpha)\right]=Bias_1+Bias_2,$$

where Bias$_1 = \cdots$, *is due to misspecification of the conditional distribution, and Bias*$_2 = \cdots$, *is due to the parameter estimation error.*

Berkowitz, J., and J. O'Brien. (2002). "How Accurate Are Value-at-Risk Models at Commercial Banks?" *Journal of Finance* 57, 1093-1111.

Bias of the 5% VaR, n = 1000

$\alpha_0 = 1 - \alpha_1$, Student t

α_1	v	Bias	Bias_1	Bias_2
0.1	5	-0.1509	-0.0836	-0.0672
	6	-0.1005	-0.0580	-0.0424
	10	-0.0495	-0.0237	-0.0258
	50	-0.0216	-0.0028	-0.0188
	∞	-0.0169	0.0000	-0.0169
0.5	5	-0.2105	-0.0768	-0.1337
	6	-0.1433	-0.0537	-0.0896
	10	-0.0740	-0.0221	-0.0519
	50	-0.0379	-0.0026	-0.0353
	∞	-0.0331	0.0000	-0.0331
0.9	5	-0.2625	-0.0456	-0.2170
	6	-0.1688	-0.0323	-0.1365
	10	-0.0957	-0.0136	-0.0821
	50	-0.0589	-0.0017	-0.0572
	∞	-0.0528	0.0000	-0.0528

4. SPATIAL MODEL

$$y = \rho W y + \varepsilon,$$

where $|\rho| < 1$ and W is the spatial weights matrix, assumed to be known *a priori*.

Example 1: crime rates

$$\begin{pmatrix} \text{Riverside} \\ \text{San Diego} \\ \text{Irvine} \end{pmatrix} = \rho \begin{pmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{pmatrix} \begin{pmatrix} \text{Riverside} \\ \text{San Diego} \\ \text{Irvine} \end{pmatrix} + \varepsilon$$

Example 2: starting salaries for new assistant professors

$$\begin{pmatrix} \text{UCR} \\ \text{UCSD} \\ \text{UCI} \end{pmatrix} = \rho \begin{pmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{pmatrix} \begin{pmatrix} \text{UCR} \\ \text{UCSD} \\ \text{UCI} \end{pmatrix} + \varepsilon$$

Properties of $\hat{\rho}$?

- Asymptotic theory? no...until Lee (2001a, 2001b)

- Finite sample? Monte Carlo results, Anselin (1980, 1982), Kelejian and Prucha (1999), Das (2000), Das, Kelejian, and Prucha (2001), Gress (2003).

THEOREM. *The QMLE $\hat{\rho}$ in the SAR(1) model has the second-order bias, up to $O\left(n^{-1}\right)$,*

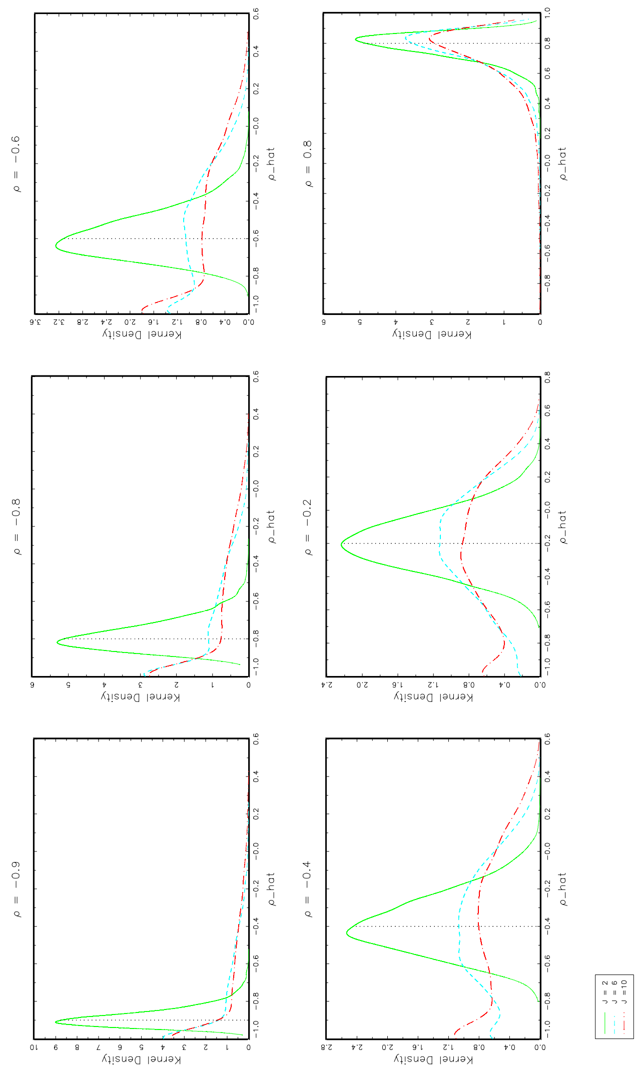
$$\begin{aligned} B(\hat{\rho}) &= \frac{4B_1B_2 - 2B_2\text{tr}(M_1) - 2B_1\text{tr}(M_2)}{[2B_2 - \text{tr}(M_2)]^2} \\ &+ \frac{\text{tr}(M_1)\,\text{tr}(M_2) + 2\text{tr}(M_1M_2)}{[2B_2 - \text{tr}(M_2)]^2} \\ &- \frac{B_3\{4B_1^2 - 4B_1\text{tr}(M_2) + [\text{tr}(M_1)]^2 + 2\text{tr}(M_1^2)\}}{[2B_2 - \text{tr}(M_2)]^3} \\ &+ \gamma_2\xi_2, \end{aligned}$$

where

$$\xi_2 = \{[2B_2 - \text{tr}(M_2)]\,\text{tr}(M_1 \odot M_2) - B_3\text{tr}(M_1 \odot M_1)\} \div [2B_2 - \text{tr}(M_2)]^3 .$$

	ρ	$\hat{\rho}$	$\hat{\rho}_{BC}$	$\tilde{\rho}_{BC}$
$J = 2$	-0.9	-0.882	-0.900	-0.900
	-0.8	-0.776	-0.799	-0.799
	-0.6	-0.574	-0.598	-0.597
	-0.4	-0.382	-0.401	-0.399
	-0.2	-0.188	-0.198	-0.197
	0	-0.001	-0.001	-0.001
	0.2	0.189	0.200	0.199
	0.4	0.382	0.402	0.400
	0.6	0.577	0.602	0.600
	0.8	0.776	0.798	0.798
	0.9	0.881	0.899	0.899
$J = 6$	-0.9	-0.810	-0.812	-0.804
	-0.8	-0.741	-0.733	-0.728
	-0.6	-0.596	-0.568	-0.570
	-0.4	-0.434	-0.388	-0.394
	-0.2	-0.253	-0.191	-0.199
	0	-0.071	0.003	-0.007
	0.2	0.124	0.205	0.196
	0.4	0.322	0.402	0.397
	0.6	0.527	0.599	0.598
	0.8	0.736	0.790	0.794
	0.9	0.853	0.892	0.897

Figure 7: Distribution of MLE of ρ in Spatial Model $y = \rho Wy + \varepsilon$, $n = 30$



✓ MSE of MLE

✓ SAR(1)+X

✓ Spatial Autoregressive Error Model:

$$
\begin{aligned}
y &= X\beta + \varepsilon, \\
\varepsilon &= \rho W \varepsilon + u, \\
u &\sim IIDN\left(0, \sigma^2 I\right),
\end{aligned}
$$

✓ SAR(1)+X+Spatial Autoregressive Error

✓ Bias and MSE of $\hat{\beta}_{ML}$ and $\hat{\sigma}^2_{ML}$

Chapters 5 to 6:

I. INTRODUCTION

Out-of-sample predictive ability of a model?

Traditionally, point forecast under MSE loss.

Point forecast is of decreasing relevance for risk management since it does not account take into many other distribution aspects of what is forecasted!

i) Quantile / Interval Forecast: Value-at-Risk

$(-\infty, VaR(\alpha)]$

ii) Density Forecast

All aspects of what is forecasted

II. VAR FORECAST

Consider a financial return series $\{r_t\}_{t=1}^{T}$, generated by the probability law $\Pr(r_t \leq r|\mathcal{F}_{t-1}) \equiv F_t(r)$ conditional on the information set $\mathcal{F}_{t-1}$ (σ-field) at time $t-1$

$$r_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t,$$

where $\mu_t = \mathbb{E}(r_t|\mathcal{F}_{t-1})$, $\sigma_t^2 = \mathbb{E}(\varepsilon_t^2|\mathcal{F}_{t-1})$, and $\{z_t\} \equiv \{\varepsilon_t/\sigma_t\}$ has the conditional distribution function $G_t(z) \equiv \Pr(z_t \leq z|\mathcal{F}_{t-1})$.

The VaR with a given tail probability $\alpha \in (0,1)$, denoted by $q_t(\alpha)$, is defined as the conditional quantile

$$F_t(q_t(\alpha)) = \alpha,$$

which can be estimated by inverting the distribution function:

$$q_t(\alpha) = F_t^{-1}(\alpha) = \mu_t + \sigma_t G_t^{-1}(\alpha).$$

VaR forecast: $\hat{q}_t(\alpha) = \hat{F}_t(\alpha)$ or $\hat{q}_t(\alpha) = \hat{\mu}_t + G^{-1}(\alpha)\hat{\sigma}_t$,

	Whether filtered or not	
Which distribution	Unfiltered	Filtered
Parametric Distribution		Normal* t(6)*
Historical Distribution	HS	HS*
Monte Carlo Distribution	MC	MC*
NP Distribution	NP	NP*
EVT Distributions	GP GEV HILL	GP* GEV* HILL*
No Distribution	CaViaR_S CaViaR_A	

1. Parametric Distribution: $G(\cdot) = \Phi(\cdot)$ or t_6
2. Historical Distribution: EDF
3. Monte Carlo Distribution:

$$\begin{aligned} S_t &= S_{t-1} \exp\left([\mu_t - \frac{1}{2}\sigma_t^2] + \sigma_t z_t \right), \\ r_t &\equiv 100 \log (S_t / S_{t-1}) \end{aligned}$$

4. Nonparametrically Estimated Distribution: weighted Nadaraya-Watson (NW) estimator

$$F(y|x_t) = \frac{\sum_{i=1}^{n} p_i K_h(x_i - x_t) \mathbf{1}(Y_i \leq y)}{\sum_{i=1}^{n} p_i K_h(x_i - x_t)}$$

5. Extreme Value Distributions

   (a) Generalized Extreme Value Distribution, based on sample minima

   (b) Generalized Pareto Distribution, based on exceedances over threshold

   (c) Hill Estimator, based on ordered statistics

6. Conditional Autoregressive VaR

   (a) Symmetric CaViaR (CaViaR$_S$)

$$q_t(\alpha) = a_0 + a_1 q_{t-1}(\alpha) + a_2 |r_{t-1}|,$$

   (b) Asymmetric CaViaR (CaViaR$_A$)

$$q_t(\alpha) = a_0 + a_1 q_{t-1}(\alpha) + a_2 |r_{t-1}| + a_3 |r_{t-1}| \mathbf{1}(r_{t-1} < 0).$$

Loss functions:

- Predictive Likelihood for Quantile Forecasts

$$\hat{Q}_P(\alpha) = \frac{1}{P} \sum_{t=R+1}^{T} [\alpha - \hat{d}_t(\alpha)][y_t - \hat{q}_t(\alpha)],$$
$$\hat{d}_t(\alpha) \equiv \mathbf{1}(y_t < \hat{q}_t(\alpha))$$

- Predictive Likelihood for Interval Forecasts

$$\hat{C}_P(\alpha) = -\frac{1}{P} \sum_{t=R+1}^{T} \log\left(\hat{p}_t(\alpha)^{\hat{d}_t(\alpha)} [1 - \hat{p}_t(\alpha)]^{[1-\hat{d}_t(\alpha)]}\right)$$

III. DENSITY FORECAST

Extensive literature on evaluating density forecast models: e.g. Diebold *et al.* (1998), Diebold *et al.* (1999), Clements and Smith (2000), Berkowitz (2001), Hong (2002), etc.

Compare alternative density forecast models? Why important?

Recent evidence on volatility clustering, return asymmetry, and tail-fatness in financial time series $\Longrightarrow$ so many models arising from different specification of volatilities and/or distributions. Which one to use?

Problem: each model can be possibly misspecified and we do not know the DGP

Aim: Propose a test for comparing various density forecast models; hence assess which volatility and/or distribution are statistically more appropriate to mimic the time series

Criteria: "distances" of these models to the true, unknown model

Minimum Kullback-Leibler Information Criterion (KLIC) divergence measure to define the distance between the candidate model and the true model

It can be tailored for the tails

Multiple comparison based on the KLIC distance: reality check of White (2000)

Note: the test is not designed for comparing *density* models *per se*; it can be a test for comparing competing models (in the mean, volatility, etc.) in terms of density forecast (e.g., Diebold *et al.*, Clements and Smith, Corradi and Swanson)

DGP:

$$Y_t = \mu_t + \varepsilon_t \equiv \mu_t + Z_t \sigma_t,$$
$$\mu_t = \mathbb{E}(Y_t | F_{t-1}), \ \sigma_t^2 = \mathbb{E}(\varepsilon_t^2 | F_{t-1}), \ Z_t \equiv \varepsilon_t / \sigma_t$$

True density: $\varphi_t(y) \equiv \varphi_t(y|F_{t-1})$

Density forecast model: $\psi_t(y;\boldsymbol{\theta}) \equiv \psi_t(y|F_{t-1};\boldsymbol{\theta})$

Define the minimum KLIC distance measure

$$I(\varphi : \psi, \boldsymbol{\theta}^*) = \mathbb{E}\left[\ln \varphi_t(y_t) - \ln \psi_t(y_t;\boldsymbol{\theta}^*)\right],$$

where $\boldsymbol{\theta}^*$ is the pseudo-true value of $\boldsymbol{\theta}$, the parameter value that gives the minimum $I(\varphi : \psi, \boldsymbol{\theta}) \equiv \mathbb{E}[\ln \varphi_t(y_t) - \ln \psi_t(y_t;\boldsymbol{\theta})]$ for all $\boldsymbol{\theta} \in \Theta$ (e.g., Sawa, 1978; White, 1982)

$\mathbb{E}\left[\ln \varphi_t(y_t) - \ln \psi_t(y_t;\boldsymbol{\theta}^*)\right]$ can be consistently estimated by

$$\hat{I}(\varphi : \psi, \boldsymbol{\theta}^*) = \frac{1}{T}\sum_{t=1}^{T}[\ln \varphi_t(y_t) - \ln \psi_t(y_t;\boldsymbol{\theta}^*)],$$

where $\boldsymbol{\theta}^*$ can be consistently estimated by $\hat{\boldsymbol{\theta}}_T$ that maximizes $\frac{1}{T}\sum_{t=1}^{T} \ln \psi_t(y_t;\boldsymbol{\theta})$

But we still do not know $\varphi_t(\cdot)$. Way out?

We utilize a inverse normal transform of the probability integral transform (PIT) of the actual realizations of the process with respect to the models's density forecast. The equivalence between $\ln[\varphi_t(y_t)/\psi_t(y_t;\hat{\boldsymbol{\theta}}_T)]$ and the log likelihood ratio of the transformed PITs enables us to consistently estimate $I(\varphi : \psi, \boldsymbol{\theta}^*)$ and hence to compare possibly misspecified models in terms of their distance to the true model.

PIT: $u_t = \int_{-\infty}^{y_t} \hat{\psi}_t(y)dy$, where $\hat{\psi}_t(y) = \psi_t\left(y;\hat{\boldsymbol{\theta}}_T\right)$

Inverse normal transform of PIT: $x_t = \Phi^{-1}(u_t)$

Remark: Checking IID U[0,1] of $\{u_t\}$ or IID $N(0,1)$ of $\{x_t\}$ provides a powerful approach to evaluating the quality of a density forecast model: Berkowitz, Diebold *et al.*, Hong, Duan, etc.

However, out aim is to compare density forecast models; for this, we utilize the following mapping:

$$\ln\left[\varphi_t(y_t)/\hat{\psi}_t(y_t)\right] = \ln\left[p_t(x_t)/\phi(x_t)\right],$$

where $p_t(\cdot)$ is the density of x_t and $\phi(\cdot)$ is the standard normal density.

Therefore, the distance of a density forecast model to the unknown true model can be equivalently estimated by the departure of $\{x_t\}_{t=1}^{T}$ from IID N(0,1),

$$\tilde{I}\left(\varphi : \psi, \hat{\boldsymbol{\theta}}_T\right) = \frac{1}{T}\sum_{t=1}^{T}\left[\ln p_t(x_t) - \ln\phi(x_t)\right]$$

A loop? Do not know $\varphi_t(\cdot)$, make use of the transformed PITs, but do not know $p_t(\cdot)$ either?

$p_t(\cdot)$ should be able to accommodate heterogeneity, dependency, and nonnormality, possibly existing in the transformed PITs due to some misspecification of the density forecast model

However, measuring departure of the unknown $p_t(\cdot)$ from IID N(0,1) is more straightforward than measuring departure of the postulated $\psi_t(\cdot;\boldsymbol{\theta})$ from something unknown in the sense that we can at least specify a flexible $p_t(\cdot)$ to include IID N(0,1) as a special case, but we when we specify $\psi_t(\cdot;\boldsymbol{\theta})$ there is no guarantee that the postulated $\psi_t(\cdot;\boldsymbol{\theta})$ will accommodate the complicated $\varphi_t(\cdot)$, which is unknown at all *a priori*

We follow Berkowitz (2001) by specifying $\{x_t\}_{t=1}^T$ as an AR(L) process

$$x_t = \boldsymbol{\rho}' X_{t-1} + \sigma \eta_t,$$

but with IID η_t admitting the SNP density of Gallant and Nychka (1987)

$$p(\eta_t; \boldsymbol{\vartheta}_\eta) = \frac{\left(\sum_{k=0}^K r_k \eta_t^k\right)^2 \phi(\eta_t)}{\int_{-\infty}^{+\infty} \left(\sum_{k=0}^K r_k u^k\right)^2 \phi(u)\, du},$$

where $r_0 = 1$, $\boldsymbol{\vartheta}_\eta = (r_1, \cdots, r_K)'$. Setting $r_k = 0$, $k = 1, \cdots, K$, $p(\eta_t) = \phi(\eta_t)$.

Hence we estimate $I(\varphi : \psi, \boldsymbol{\theta}^*)$ by

$$\bar{I}\left(\varphi : \psi, \hat{\boldsymbol{\theta}}_T\right) = \frac{1}{T}\sum_{t=1}^T \left[\ln p\left(x_t; \hat{\boldsymbol{\vartheta}}_T\right) - \ln \phi(x_t)\right]$$

$$= \frac{1}{T}\sum_{t=1}^T \left[\ln \frac{p\left[\left(x_t - \hat{\boldsymbol{\rho}}_T' X_{t-1}\right)/\hat{\sigma}_T; \hat{\boldsymbol{\vartheta}}_{\eta T}\right]}{\hat{\sigma}_T} - \ln \phi(x_t)\right],$$

where $\hat{\boldsymbol{\vartheta}}_T = (\hat{\boldsymbol{\rho}}', \hat{\sigma}, \hat{\boldsymbol{\vartheta}}_{\eta T}')'$ is the MLE.

Therefore, we can use the KLIC distance as a loss function for a given density forecast model!

Related works:

Testing IID $N(0,1)$ *per se:* Jarque and Bera (1980), Pearson distribution; Hall (1990), SNP; Kiefer and Salmon (1983), Smith (1989), Gram-Charlier/Edgeworth-Sargan.

Comparing density forecast models: Corradi and Swanson (2003a, 2003b), Kolmogorov-Smirnov type statistics: mean square error of the CDF and the EDF, integrated integrated out.

Our KLIC-PIT approach answers the questions raised in Corradi and Swanson (2003a, 2003b)

1. Applicability of PIT approach: Yes

2. Applicability of the KLIC discrepancy measure over some specific regions: Yes, next slide

Define the censored PIT

$$x_t^c = \begin{cases} \Phi^{-1}(\alpha) \equiv c & \text{if } x_t \geq c \\ x_t & \text{if } x_t < c. \end{cases}$$

and hence the censored likelihood

$$p^c(x_t^c; \boldsymbol{\vartheta}) = \left[1 - P\left(\frac{c - b_0 - \mathbf{b}_1' X_{t-1}}{\sigma}; \boldsymbol{\vartheta}_\eta\right)\right]^{\mathbf{1}(x_t \geq c)}$$

$$\times \left[\frac{\phi[(x_t - b_0 - \mathbf{b}_1' X_{t-1})/\sigma]}{\sigma}\right]^{\mathbf{1}(x_t < c)}.$$

Accordingly, the minimum tail KLIC distance

$$\bar{I}^c\left(\varphi : \psi, \hat{\boldsymbol{\theta}}_T\right) = \frac{1}{T}\sum_{t=1}^{T}\left[\ln p^c\left(x_t; \hat{\boldsymbol{\vartheta}}_T\right) - \ln \phi^c(x_t)\right]$$

IV. MODEL COMPARISON

Benchmark model: 0; competing models: $k = 1, \cdots, l$

Define the loss differential: $f_{k,t} = L_{0,t} - L_{k,t}$

- Pairwise comparison: model k is no better than the benchmark

  $$\mathbb{H}_1 : \mathbb{E}(f_{k,t}) \leq 0$$

  Diebold and Mariano (1995), West (1996)

- Multiple comparison: can any one of the competing models beat the benchmark model?

  $$\mathbb{H}_2 : \max_{1 \leq k \leq l} \mathbb{E}(f_{k,t}) \leq 0$$

  – In practice bootstrap the following statistics to get the "reality check p-value"

  $$\bar{V}_n = \max_{1 \leq k \leq l} n^{1/2}[\bar{f}_{k,n} - \mathbb{E}(f_{k,t})],$$

  where $\mathbb{E}(f_{k,t})$ is set to be zero.

  – Also see Hansen's (2001) p-value that depends on the variance of $\bar{d}_{k,n}$.

V. EMPIRICAL FINDINGS

Compare VaR Models:

- Filtered models dominate most unfiltered models

- Most of the unfiltered models are dominated by the Riskmetrics EWMA model while many of the filtered models dominate the Riskmetrics EWMA model

- The filtered EVT models generally produce the best risk forecasts, especially for the 1% tail

- Among the filtered EVT models, HILL* and GP* perform the best, especially for the 1% tail for turmoil economies

- Filtered nonparametric models, HS* and NP*, perform quite well

- The $t(6)$* model works better than the Normal* model for the extreme 1% tail, while Normal* is better than $t(6)$* for the 5% tail. However, both $t(6)$* and Normal* are inferior to the EVT-based filtered models at both $\alpha = 0.01$ and $\alpha = 0.05$

- Filtered EVT models mostly do better with $\alpha = 0.01$ than with $\alpha = 0.05$. Other models (EWMA, Normal*, HS*, MC*, NP*) tend to perform better with $\alpha = 0.05$ than with $\alpha = 0.01$

- The asymmetric CaViaR model does better than the symmetric one, particularly at $\alpha = 0.01$

Table 6: Unfiltered vs Filtered VaR Models (Pair-wise Comparison)

Panel A. GW Test

Loss Function	Model	Indonesia *GW*	Indonesia *I*	Korea *GW*	Korea *I*	Malaysia *GW*	Malaysia *I*	Taiwan *GW*	Taiwan *I*	Thailand *GW*	Thailand *I*
$\hat{Q}_P(0.05)$	HS vs HS*	0.005	0.000	0.000	0.107	0.002	0.000	0.029	0.153	0.040	0.287
	MC vs MC*	0.284	0.287	0.002	0.019	0.017	0.008	0.010	0.199	0.054	0.318
	NP vs NP*	0.005	0.000	0.000	0.107	0.002	0.000	0.030	0.149	0.041	0.287
	GEV vs GEV*	0.000	0.004	0.000	0.046	0.000	0.000	0.040	0.678	0.077	0.050
	GPD vs GPD*	0.000	0.720	0.075	0.073	0.290	0.789	0.000	0.100	0.020	0.751
	Hill vs Hill*	0.153	0.100	0.058	0.065	0.414	0.054	0.000	0.035	0.006	0.678
$\hat{Q}_P(0.01)$	HS vs HS*	0.032	0.184	0.017	0.046	0.110	0.038	0.000	0.169	0.000	0.207
	MC vs MC*	0.112	0.253	0.002	0.031	0.018	0.008	0.830	0.456	0.006	0.069
	NP vs NP*	0.033	0.184	0.017	0.046	0.113	0.038	0.000	0.169	0.006	0.199
	GEV vs GEV*	0.031	0.084	0.002	0.035	0.054	0.027	0.656	0.107	0.017	0.123
	GPD vs GPD*	0.000	0.996	0.200	0.510	0.199	0.513	0.000	0.195	0.000	0.506
	Hill vs Hill*	0.000	0.494	0.076	0.100	0.427	0.107	0.000	0.157	0.000	0.487
$\hat{C}_P(0.05)$	HS vs HS*	0.003	0.084	0.003	0.012	0.002	0.000	0.101	0.996	0.017	0.000
	MC vs MC*	0.028	0.061	0.003	0.019	0.000	0.000	0.075	0.996	0.030	0.027
	NP vs NP*	0.003	0.084	0.003	0.012	0.002	0.000	0.101	0.996	0.017	0.000
	GEV vs GEV*	0.048	0.123	0.028	0.027	0.006	0.004	0.109	0.923	0.001	0.000
	GPD vs GPD*	0.117	0.096	0.090	0.042	0.155	0.029	0.563	0.989	0.151	0.946
	Hill vs Hill*	0.015	0.126	0.001	0.004	0.005	0.000	0.641	0.954	0.285	0.958
$\hat{C}_P(0.01)$	HS vs HS*	0.079	0.000	0.008	0.015	0.000	0.008	0.700	0.950	0.680	0.054
	MC vs MC*	0.058	0.012	0.001	0.000	0.014	0.004	0.777	0.843	0.125	0.012
	NP vs NP*	0.079	0.000	0.008	0.015	0.005	0.004	0.700	0.946	0.680	0.054
	GEV vs GEV*	0.005	0.004	0.000	0.008	0.001	0.004	0.687	0.839	0.329	0.027
	GPD vs GPD*	0.849	0.035	0.508	0.149	0.558	0.015	0.700	0.958	0.849	0.387
	Hill vs Hill*	0.103	0.035	0.007	0.004	0.508	0.157	0.564	0.889	0.849	0.103

Table 6: Unfiltered vs Filtered VaR Models (Pair-wise Comparison)

Panel B. Reality Check

Loss Fn	Benchmark	Indonesia $\bar{f}_1$	Indonesia *White*	Indonesia *Hansen*	Korea $\bar{f}_1$	Korea *White*	Korea *Hansen*	Malaysia $\bar{f}_1$	Malaysia *White*	Malaysia *Hansen*	Taiwan $\bar{f}_1$	Taiwan *White*	Taiwan *Hansen*	Thailand $\bar{f}_1$	Thailand *White*	Thailand *Hansen*
$\hat{Q}_P(0.05)$	HS	0.178	0.000	0.000	0.277	0.000	0.000	0.168	0.001	0.001	0.017	0.066	0.066	0.032	0.091	0.091
	MC	0.046	0.120	0.120	0.224	0.001	0.001	0.131	0.003	0.003	0.010	0.140	0.140	0.014	0.274	0.274
	NP	0.179	0.000	0.000	0.277	0.000	0.000	0.168	0.001	0.001	0.017	0.066	0.066	0.033	0.090	0.090
	GEV	0.237	0.000	0.000	0.323	0.000	0.000	0.252	0.000	0.000	-0.008	0.819	0.819	0.096	0.003	0.003
	GPD	-0.030	0.756	0.756	0.110	0.022	0.022	-0.024	0.799	0.799	0.050	0.000	0.000	-0.031	0.954	0.449
	Hill	0.088	0.010	0.010	0.137	0.009	0.009	0.062	0.068	0.068	0.084	0.000	0.000	-0.033	0.961	0.447
$\hat{Q}_P(0.01)$	HS	0.092	0.027	0.027	0.139	0.003	0.003	0.067	0.042	0.042	0.008	0.092	0.092	0.005	0.289	0.289
	MC	0.066	0.059	0.059	0.205	0.001	0.001	0.110	0.005	0.005	0.001	0.449	0.449	0.032	0.100	0.100
	NP	0.094	0.027	0.027	0.140	0.003	0.003	0.068	0.037	0.037	0.008	0.086	0.086	0.005	0.279	0.279
	GEV	0.141	0.012	0.012	0.199	0.000	0.000	0.095	0.011	0.011	0.004	0.211	0.211	0.014	0.214	0.214
	GPD	-0.046	1.000	0.503	0.027	0.138	0.138	-0.003	0.503	0.503	0.007	0.029	0.029	-0.001	0.492	0.492
	Hill	0.022	0.185	0.185	0.074	0.020	0.020	0.029	0.130	0.130	0.012	0.000	0.000	-0.007	0.652	0.652
$\hat{C}_P(0.05)$	HS	0.247	0.001	0.001	0.328	0.000	0.000	0.304	0.000	0.000	-0.091	0.986	0.516	0.208	0.000	0.000
	MC	0.225	0.001	0.001	0.309	0.002	0.002	0.439	0.000	0.000	-0.096	0.990	0.515	0.202	0.003	0.003
	NP	0.248	0.001	0.001	0.329	0.000	0.000	0.304	0.000	0.000	-0.091	0.986	0.516	0.208	0.000	0.000
	GEV	0.165	0.014	0.014	0.239	0.003	0.003	0.277	0.000	0.000	-0.052	0.978	0.539	0.280	0.000	0.000
	GPD	0.165	0.026	0.026	0.167	0.044	0.044	0.159	0.029	0.029	-0.041	0.944	0.530	-0.046	0.843	0.843
	Hill	0.175	0.015	0.015	0.376	0.000	0.000	0.321	0.000	0.000	-0.046	0.954	0.537	-0.053	0.928	0.512
$\hat{C}_P(0.01)$	HS	0.203	0.008	0.008	0.326	0.006	0.006	0.299	0.000	0.000	-0.021	0.868	0.481	0.054	0.034	0.034
	MC	0.184	0.017	0.017	0.413	0.000	0.000	0.295	0.000	0.000	-0.020	0.867	0.513	0.117	0.003	0.003
	NP	0.203	0.008	0.008	0.326	0.006	0.006	0.317	0.000	0.000	-0.021	0.868	0.481	0.054	0.034	0.034
	GEV	0.314	0.000	0.000	0.433	0.000	0.000	0.379	0.000	0.000	-0.008	0.616	0.616	0.097	0.015	0.015
	GPD	0.001	0.064	0.064	0.098	0.137	0.137	0.092	0.051	0.051	-0.024	0.870	0.480	0.001	0.352	0.352
	Hill	0.186	0.019	0.019	0.328	0.000	0.000	0.083	0.059	0.059	-0.028	0.888	0.450	0.022	0.086	0.086

Table 7: Reality Check, Unfiltered VaR Models

		Indonesia			Korea			Malaysia			Taiwan			Thailand		
Loss Fn	Benchmark	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*
	Riskmetrics	0.386	0.470	0.297	0.342	0.985	0.504	0.312	0.510	0.288	0.205	0.373	0.246	0.269	0.366	0.254
	HS	0.574	0.000	0.000	0.636	0.000	0.000	0.498	0.000	0.000	0.217	0.127	0.052	0.307	0.017	0.017
	MC	0.419	0.150	0.057	0.571	0.001	0.001	0.449	0.001	0.001	0.209	0.264	0.115	0.285	0.097	0.088
$\hat{Q}_P(0.05)$	NP	0.575	0.000	0.000	0.636	0.000	0.000	0.498	0.000	0.000	0.216	0.127	0.052	0.307	0.016	0.016
	GEV	0.666	0.000	0.000	0.690	0.000	0.000	0.600	0.000	0.000	0.193	1.000	0.707	0.375	0.000	0.000
	GPD	0.361	0.983	0.543	0.454	0.017	0.016	0.295	0.972	0.712	0.252	0.000	0.000	0.249	1.000	0.632
	Hill	0.468	0.006	0.006	0.470	0.012	0.012	0.374	0.041	0.032	0.304	0.000	0.000	0.256	0.573	0.332
	Riskmetrics	0.141	0.374	0.120	0.103	0.900	0.523	0.097	0.686	0.423	0.064	0.494	0.494	0.072	0.770	0.754
	HS	0.225	0.028	0.028	0.243	0.002	0.002	0.166	0.043	0.043	0.066	0.487	0.465	0.077	0.674	0.384
	MC	0.203	0.042	0.042	0.316	0.000	0.000	0.218	0.008	0.008	0.061	0.847	0.847	0.102	0.130	0.130
$\hat{Q}_P(0.01)$	NP	0.226	0.027	0.027	0.244	0.002	0.002	0.166	0.042	0.042	0.066	0.487	0.465	0.078	0.670	0.381
	GEV	0.270	0.010	0.010	0.305	0.001	0.001	0.192	0.015	0.015	0.062	0.893	0.893	0.086	0.309	0.301
	GPD	0.109	0.942	0.596	0.129	0.425	0.130	0.096	0.876	0.630	0.064	0.615	0.537	0.084	0.454	0.454
	Hill	0.150	0.213	0.104	0.172	0.025	0.016	0.121	0.314	0.127	0.069	0.326	0.326	0.076	0.718	0.677
	Riskmetrics	0.372	0.643	0.323	0.352	0.999	0.531	0.302	0.931	0.776	0.165	0.000	0.000	0.220	0.080	0.014
	HS	0.700	0.000	0.000	0.792	0.000	0.000	0.803	0.000	0.000	0.057	0.144	0.041	0.505	0.000	0.000
	MC	0.560	0.005	0.003	0.745	0.001	0.001	0.796	0.000	0.000	0.055	0.169	0.044	0.475	0.000	0.000
$\hat{C}_P(0.05)$	NP	0.700	0.000	0.000	0.792	0.000	0.000	0.803	0.000	0.000	0.057	0.144	0.041	0.505	0.000	0.000
	GEV	0.742	0.000	0.000	0.738	0.000	0.000	0.821	0.000	0.000	0.108	0.006	0.006	0.590	0.000	0.000
	GPD	0.349	0.920	0.677	0.609	0.011	0.011	0.353	0.496	0.224	0.053	0.225	0.063	0.129	0.589	0.127
	Hill	0.605	0.003	0.003	0.620	0.009	0.009	0.606	0.000	0.000	0.016	0.985	0.560	0.100	0.994	0.524
	Riskmetrics	0.254	0.010	0.005	0.185	0.848	0.695	0.149	0.585	0.384	0.078	0.018	0.018	0.066	0.329	0.197
	HS	0.387	0.000	0.000	0.475	0.004	0.004	0.387	0.000	0.000	0.018	0.370	0.080	0.088	0.106	0.077
	MC	0.384	0.001	0.001	0.606	0.000	0.000	0.519	0.000	0.000	0.041	0.082	0.077	0.199	0.001	0.001
$\hat{C}_P(0.01)$	NP	0.387	0.000	0.000	0.475	0.004	0.004	0.405	0.000	0.000	0.018	0.370	0.080	0.088	0.106	0.077
	GEV	0.460	0.000	0.000	0.582	0.001	0.001	0.486	0.000	0.000	0.030	0.217	0.123	0.132	0.020	0.020
	GPD	0.048	0.997	0.602	0.219	0.652	0.305	0.116	0.970	0.763	0.017	0.383	0.080	0.022	0.943	0.783
	Hill	0.225	0.032	0.030	0.419	0.010	0.010	0.179	0.440	0.195	0.000	0.919	0.503	0.041	0.625	0.453

Table 8: Reality Check, Filtered VaR Models

		Indonesia			Korea			Malaysia			Taiwan			Thailand		
Loss Fn	Benchmark	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*
	Riskmetrics	0.386	0.634	0.529	0.342	0.457	0.418	0.312	0.856	0.829	0.205	0.360	0.360	0.269	0.853	0.817
	Normal*	0.371	1.000	0.993	0.350	0.241	0.241	0.317	0.789	0.482	0.200	0.893	0.798	0.275	0.578	0.503
	t(6)*	0.409	0.242	0.242	0.359	0.138	0.138	0.326	0.379	0.315	0.203	0.456	0.392	0.283	0.254	0.254
	HS*	0.396	0.359	0.324	0.359	0.128	0.128	0.330	0.251	0.250	0.199	0.918	0.872	0.275	0.504	0.418
$\hat{Q}_P(0.05)$	MC*	0.373	0.966	0.910	0.348	0.317	0.293	0.319	0.696	0.438	0.199	0.996	0.996	0.271	0.831	0.782
	NP*	0.396	0.355	0.322	0.359	0.126	0.126	0.330	0.251	0.250	0.199	0.919	0.878	0.275	0.506	0.420
	GEV*	0.429	0.146	0.146	0.367	0.090	0.090	0.347	0.131	0.131	0.201	0.644	0.424	0.279	0.328	0.328
	GPD*	0.391	0.468	0.468	0.345	0.430	0.363	0.319	0.534	0.534	0.202	0.724	0.447	0.280	0.321	0.321
	Hill*	0.380	0.836	0.530	0.333	0.842	0.832	0.313	0.884	0.705	0.220	0.094	0.094	0.289	0.185	0.185
	Riskmetrics	0.141	0.488	0.488	0.103	0.432	0.432	0.097	0.663	0.655	0.064	0.269	0.269	0.072	0.462	0.354
	Normal*	0.135	0.706	0.522	0.112	0.158	0.158	0.107	0.275	0.275	0.062	0.334	0.334	0.072	0.458	0.398
	t(6)*	0.157	0.194	0.194	0.105	0.326	0.293	0.101	0.385	0.352	0.059	0.696	0.696	0.069	0.998	0.940
	HS*	0.132	0.799	0.556	0.104	0.355	0.322	0.098	0.603	0.426	0.058	0.941	0.941	0.072	0.573	0.338
$\hat{Q}_P(0.01)$	MC*	0.137	0.628	0.461	0.111	0.162	0.162	0.109	0.260	0.260	0.061	0.486	0.486	0.070	0.759	0.711
	NP*	0.132	0.792	0.553	0.104	0.354	0.322	0.098	0.596	0.423	0.058	0.938	0.938	0.072	0.607	0.363
	GEV*	0.129	0.941	0.709	0.105	0.300	0.296	0.097	0.651	0.435	0.058	0.877	0.877	0.072	0.621	0.411
	GPD*	0.154	0.237	0.237	0.101	0.649	0.424	0.099	0.500	0.500	0.057	0.987	0.987	0.085	0.023	0.023
	Hill*	0.128	0.949	0.826	0.098	0.961	0.961	0.092	1.000	1.000	0.058	0.620	0.620	0.082	0.033	0.033

Table 8 (Continued): Reality Check, Filtered VaR Models

		Indonesia			Korea			Malaysia			Taiwan			Thailand		
Loss Fn	Benchmark	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*
	Riskmetrics	0.372	0.018	0.018	0.352	0.057	0.030	0.302	0.108	0.034	0.165	0.006	0.006	0.220	0.121	0.043
	Normal*	0.337	0.026	0.013	0.456	0.000	0.000	0.359	0.003	0.003	0.155	0.014	0.014	0.300	0.001	0.001
	t(6)*	0.526	0.000	0.000	0.463	0.000	0.000	0.422	0.001	0.001	0.167	0.006	0.006	0.337	0.001	0.001
	HS*	0.453	0.001	0.001	0.463	0.000	0.000	0.499	0.000	0.000	0.148	0.018	0.018	0.297	0.001	0.001
$\hat{C}_p(0.05)$	MC*	0.334	0.031	0.015	0.436	0.001	0.001	0.357	0.004	0.004	0.151	0.017	0.017	0.273	0.006	0.006
	NP*	0.452	0.001	0.001	0.463	0.000	0.000	0.499	0.000	0.000	0.148	0.018	0.018	0.297	0.001	0.001
	GEV*	0.577	0.000	0.000	0.500	0.001	0.001	0.544	0.000	0.000	0.160	0.012	0.012	0.310	0.004	0.004
	GPD*	0.185	0.996	0.540	0.443	0.001	0.001	0.194	0.998	0.628	0.094	0.214	0.048	0.175	0.568	0.144
	Hill*	0.430	0.001	0.001	0.244	0.996	0.540	0.285	0.155	0.027	0.062	0.970	0.539	0.153	0.959	0.535
	Riskm's	0.254	0.007	0.007	0.185	0.078	0.078	0.149	0.018	0.016	0.078	0.064	0.064	0.066	0.250	0.195
	Normal*	0.181	0.014	0.014	0.214	0.017	0.017	0.211	0.003	0.003	0.060	0.099	0.099	0.131	0.020	0.020
	t(6)*	0.201	0.004	0.004	0.133	0.351	0.085	0.178	0.005	0.005	0.047	0.300	0.148	0.067	0.203	0.170
	HS*	0.184	0.010	0.010	0.149	0.164	0.129	0.089	0.180	0.054	0.039	0.578	0.329	0.034	0.741	0.228
$\hat{C}_p(0.01)$	MC*	0.200	0.009	0.009	0.193	0.030	0.030	0.224	0.002	0.002	0.061	0.096	0.096	0.082	0.093	0.093
	NP*	0.184	0.010	0.010	0.149	0.164	0.129	0.089	0.180	0.054	0.039	0.584	0.349	0.034	0.741	0.228
	GEV*	0.146	0.054	0.019	0.149	0.171	0.138	0.107	0.087	0.032	0.038	0.632	0.466	0.035	0.724	0.228
	GPD*	0.046	0.754	0.101	0.120	0.553	0.120	0.024	0.984	0.525	0.041	0.532	0.429	0.020	0.776	0.450
	Hill*	0.039	0.992	0.496	0.091	0.964	0.563	0.096	0.151	0.054	0.028	0.903	0.747	0.018	0.977	0.939

Table 9: Reality Check, CaViaR Models

		Indonesia			Korea			Malaysia			Taiwan			Thailand		
Loss Fn	Benchmark	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*	Loss	*White*	*Hansen*
	Riskmetrics	0.386	0.993	0.513	0.342	0.574	0.574	0.312	0.713	0.713	0.205	0.078	0.078	0.269	0.676	0.676
$\hat{Q}_p(0.05)$	$CaViaR_S$	0.464	0.011	0.011	0.348	0.261	0.261	0.324	0.263	0.263	0.198	0.527	0.112	0.275	0.376	0.364
	$CaViaR_A$	0.544	0.000	0.000	0.342	0.766	0.502	0.322	0.303	0.303	0.198	0.968	0.508	0.282	0.204	0.204
	Riskmetrics	0.141	0.999	0.546	0.103	0.194	0.194	0.097	0.103	0.103	0.064	0.152	0.152	0.072	0.693	0.693
$\hat{Q}_p(0.01)$	$CaViaR_S$	0.191	0.030	0.030	0.106	0.121	0.121	0.094	0.171	0.171	0.060	0.457	0.457	0.074	0.373	0.373
	$CaViaR_A$	0.167	0.133	0.037	0.096	0.916	0.845	0.086	0.927	0.585	0.058	0.897	0.549	0.075	0.359	0.359
	Riskmetrics	0.372	0.982	0.491	0.352	0.540	0.540	0.302	0.995	0.534	0.165	0.134	0.134	0.220	0.983	0.560
$\hat{C}_p(0.05)$	$CaViaR_S$	0.477	0.051	0.044	0.369	0.401	0.401	0.499	0.005	0.005	0.148	0.803	0.615	0.330	0.014	0.014
	$CaViaR_A$	0.598	0.001	0.001	0.347	0.804	0.615	0.485	0.006	0.006	0.148	0.743	0.522	0.316	0.036	0.036
	Riskmetrics	0.254	0.795	0.640	0.185	0.635	0.635	0.149	0.113	0.113	0.078	0.129	0.129	0.066	0.227	0.227
$\hat{C}_p(0.01)$	$CaViaR_S$	0.314	0.172	0.172	0.206	0.322	0.322	0.130	0.213	0.213	0.063	0.972	0.610	0.034	0.927	0.816
	$CaViaR_A$	0.260	0.613	0.413	0.178	0.685	0.685	0.088	0.921	0.573	0.064	0.520	0.226	0.052	0.296	0.296

Compare Density Forecast Models:

Distribution Specification: $f_t(z) = f_t(z|F_{t-1};\boldsymbol{\theta}_d)$, normal and nonnormal

- Nonnormal
  - Symmetric
    - Student t
    - GED
    - Double Exponential
    - Double Weibull
  - Skewed
    - Skewed t
    - Hansen t
    - Skewed GED
    - IHS
    - Mixture
    - Double Gamma
    - Edgeworth
  - Historical

Volatility Specification: $\sigma_t^2 = \sigma_t^2(F_{t-1};\boldsymbol{\theta}_v)$, GARCH-family

EWMA: Riskmetrics

GARCH: symmetric

GJR: asymmetric

APARCH: asymmetric

EGARCH: asymmetric

HYGARCH: symmetric and long memory

HYAPARCH: asymmetric and long memory

CGARCH: symmetric and long memory

- The choice of conditional distributions may be more important than the choice of volatility
- A model that provides superior density forecasts does not necessary meet the needs of risk managers who care much more about the tails
- Nonnormality and long memory in the second moments exist for both the S&P 500 and Nasdaq return series, but there are clear differences between the stochastic processes to generate the two series
- The Hansen t, skewed t, normal and historical distributions generally appear to be the worst distributions
- In the tails, however, Skewed t systematically dominates other distributions and in every case it fares better than the Hansen t

Table 10: Reality Check, S&P 500 Data (DIEBOLD)

Panel A: Whole Distribution

	EWMA	GARCH	GJR	APARCH	EGARCH	STGARCH	HYGARCH	HYAPARCH	CGARCH
HS	0.0142	0.0093	0.0827	0.0801	0.0767	0.0096	0.0093	0.0807	0.0090
	0.889	0.628	0.217	0.248	0.278	0.623	0.628	0.239	0.652
	0.888	0.590	0.217	0.248	0.278	0.587	0.590	0.239	0.612
NM	0.0146	0.0166	0.0234	0.0314	0.0251	0.0185	0.0166	0.0307	0.0136
	0.805	0.799	0.766	0.705	0.750	0.793	0.799	0.710	0.805
	0.784	0.784	0.760	0.703	0.746	0.784	0.784	0.708	0.783
St *t*	0.0163	0.0163	0.0212	0.0270	0.0221	0.0172	0.0163	0.0328	0.0154
	0.791	0.792	0.765	0.702	0.756	0.790	0.792	0.644	0.792
	0.783	0.783	0.764	0.701	0.755	0.783	0.783	0.644	0.783
GED	0.0389	0.0435	0.0540	0.1259	0.0584	0.0456	0.0435	0.1247	0.0153
	0.595	0.545	0.545	0.168	0.545	0.545	0.545	0.169	0.792
	0.595	0.545	0.545	0.168	0.545	0.545	0.545	0.169	0.783
LP	0.0259	0.0218	0.0263	0.0263	0.0283	0.0229	0.0206	0.0260	0.0214
	0.694	0.754	0.685	0.688	0.651	0.739	0.768	0.690	0.758
	0.694	0.754	0.685	0.688	0.651	0.739	0.768	0.690	0.758
DW	0.0113	0.0075	0.0144	0.0120	0.0127	0.0126	0.0039	0.0301	0.0038
	0.817	0.881	0.801	0.824	0.813	0.811	0.971	0.669	0.974
	0.787	0.800	0.783	0.787	0.783	0.784	0.872	0.669	0.881
Sk *t*	0.0289	0.0263	0.0186	0.0181	0.0164	0.0237	0.0263	0.0182	0.0300
	0.647	0.652	0.779	0.783	0.785	0.686	0.652	0.782	0.646
	0.647	0.652	0.779	0.783	0.783	0.686	0.652	0.782	0.646
Hn *t*	0.0393	0.1001	0.1441	0.1368	0.1371	0.1003	0.1516	0.1407	0.1015
	0.636	0.137	0.094	0.112	0.110	0.136	0.065	0.104	0.135
	0.636	0.137	0.094	0.112	0.110	0.136	0.065	0.104	0.135
SGED	0.0159	0.0431	0.0535	0.0659	0.0318	0.0468	0.0431	0.0650	0.0150
	0.793	0.545	0.545	0.542	0.650	0.545	0.545	0.543	0.794
	0.783	0.545	0.545	0.542	0.650	0.545	0.545	0.543	0.783
IHS	0.0160	0.0160	0.0211	0.0268	0.0218	0.0170	0.0160	0.0263	0.0152
	0.791	0.793	0.765	0.706	0.759	0.791	0.793	0.713	0.792
	0.783	0.783	0.764	0.705	0.758	0.783	0.783	0.712	0.783
MX	0.0137	0.0169	0.0233	0.0275	0.0118	0.0167	0.0169	0.0289	0.0150
	0.792	0.793	0.756	0.702	0.811	0.794	0.793	0.711	0.795
	0.785	0.782	0.755	0.701	0.785	0.782	0.782	0.710	0.783
DG	0.0014	0.0053	0.0019	0.0025	0.0037	0.0015	0.0016	0.0074	0.0016
	1.000	0.928	0.998	0.992	0.965	0.999	0.998	0.887	0.998
	0.994	0.825	0.941	0.915	0.864	0.980	0.953	0.810	0.960
SGN	0.0148	0.0158	0.0200	0.0240	0.0206	0.0164	0.0158	0.0236	0.0152
	0.798	0.798	0.781	0.744	0.781	0.797	0.798	0.749	0.796
	0.783	0.783	0.777	0.740	0.776	0.783	0.783	0.745	0.783

Table 10: Reality Check, S&P 500 Data (DIEBOLD)

Panel B: 5% Tail

	EWMA	GARCH	GJR	APARCH	EGARCH	STGARCH	HYGARCH	HYAPARCH	CGARCH
HS	0.3187	0.1624	0.0219	0.0203	0.0197	0.1651	0.1624	0.0200	0.1615
	0.127	0.236	0.599	0.599	0.599	0.170	0.236	0.599	0.272
	0.127	0.236	0.599	0.599	0.599	0.170	0.236	0.599	0.272
NM	0.0266	0.0266	0.0271	0.0331	0.0293	0.0286	0.0266	0.0328	0.0229
	0.696	0.680	0.657	0.648	0.645	0.670	0.680	0.649	0.687
	0.696	0.680	0.657	0.648	0.645	0.670	0.680	0.649	0.687
St *t*	0.0028	0.0038	0.0069	0.0094	0.0093	0.0040	0.0038	0.0115	0.0026
	0.960	0.919	0.758	0.704	0.701	0.909	0.924	0.609	0.973
	0.924	0.873	0.732	0.689	0.682	0.867	0.881	0.609	0.917
GED	0.1334	0.1359	0.1357	0.2825	0.1398	0.1359	0.1359	0.2816	0.0081
	0.549	0.545	0.545	0.196	0.544	0.545	0.545	0.197	0.795
	0.549	0.545	0.545	0.196	0.544	0.545	0.545	0.197	0.778
LP	0.0001	0.0010	0.0014	0.0015	0.0019	0.0007	0.0019	0.0015	0.0014
	1.000	0.998	0.973	0.972	0.970	1.000	0.985	0.970	0.994
	1.000	0.934	0.802	0.796	0.862	0.977	0.842	0.792	0.879
DW	0.0032	0.0035	0.0038	0.0060	0.0051	0.0036	0.0049	0.0062	0.0035
	0.972	0.958	0.929	0.834	0.852	0.959	0.904	0.807	0.949
	0.946	0.922	0.884	0.799	0.811	0.925	0.856	0.762	0.911
Sk *t*	0.0089	0.0080	0.0044	0.0044	0.0049	0.0073	0.0080	0.0048	0.0093
	0.641	0.660	0.867	0.892	0.850	0.686	0.660	0.867	0.628
	0.631	0.644	0.786	0.838	0.798	0.668	0.644	0.807	0.623
Hn *t*	0.0090	0.0261	0.0365	0.0359	0.0368	0.0260	0.0367	0.0366	0.0267
	0.647	0.599	0.599	0.599	0.599	0.599	0.599	0.599	0.599
	0.638	0.599	0.599	0.599	0.599	0.599	0.599	0.599	0.599
SGED	0.0065	0.1339	0.1320	0.1372	0.0075	0.1341	0.1339	0.1366	0.0071
	0.866	0.545	0.545	0.545	0.739	0.545	0.545	0.545	0.831
	0.846	0.545	0.545	0.545	0.714	0.545	0.545	0.545	0.793
IHS	0.0018	0.0027	0.0050	0.0071	0.0065	0.0029	0.0027	0.0067	0.0020
	0.995	0.976	0.851	0.763	0.778	0.970	0.976	0.778	0.993
	0.952	0.927	0.812	0.740	0.748	0.929	0.927	0.749	0.931
MX	0.0061	0.0154	0.0169	0.0120	0.0192	0.0161	0.0154	0.0191	0.0123
	0.878	0.728	0.707	0.688	0.642	0.726	0.728	0.697	0.754
	0.842	0.728	0.707	0.687	0.642	0.726	0.728	0.697	0.752
DG	0.0010	0.0024	0.0006	0.0016	0.0016	0.0010	0.0011	0.0023	0.0011
	1.000	0.966	1.000	0.992	0.983	0.998	0.998	0.957	0.998
	0.993	0.844	0.993	0.930	0.899	0.981	0.974	0.835	0.971
SGN	0.0018	0.0010	0.0018	0.0022	0.0026	0.0007	0.0010	0.0020	0.0015
	0.989	1.000	0.968	0.951	0.926	1.000	1.000	0.962	0.994
	0.831	0.924	0.816	0.803	0.784	0.959	0.924	0.811	0.862

Table 11: Reality Check, S&P 500 Data (SP)

Panel A: Whole Distribution

	EWMA	GARCH	GJR	APARCH	EGARCH	STGARCH	HYGARCH	HYAPARCH	CGARCH
HS	0.0333	0.0096	0.0118	0.0130	0.0107	0.0054	0.0096	0.0136	0.0115
	0.001	0.298	0.173	0.124	0.225	0.686	0.295	0.111	0.201
	0.001	0.098	0.043	0.034	0.066	0.279	0.102	0.033	0.065
NM	0.0032	0.0046	0.0075	0.0056	0.0076	0.0084	0.0051	0.0052	0.0022
	0.814	0.580	0.392	0.524	0.392	0.355	0.530	0.551	0.951
	0.474	0.332	0.178	0.268	0.170	0.174	0.298	0.301	0.665
St *t*	0.0022	0.0029	0.0043	0.0030	0.0043	0.0063	0.0030	0.0028	0.0014
	0.964	0.897	0.725	0.906	0.722	0.449	0.890	0.918	0.992
	0.764	0.618	0.419	0.662	0.428	0.182	0.590	0.657	0.901
GED	0.0020	0.0027	0.0043	0.0029	0.0043	0.0059	0.0029	0.0027	0.0012
	0.973	0.909	0.712	0.895	0.712	0.494	0.895	0.916	0.996
	0.812	0.642	0.426	0.658	0.422	0.224	0.588	0.632	0.935
LP	0.0054	0.0018	0.0021	0.0006	0.0011	0.0052	0.0009	0.0006	0.0011
	0.592	0.978	0.964	0.998	0.997	0.610	0.998	0.998	0.998
	0.286	0.836	0.799	0.961	0.919	0.300	0.957	0.969	0.930
DW	0.0022	0.0043	0.0049	0.0053	0.0035	0.0059	0.0025	0.0022	0.0020
	0.961	0.753	0.662	0.625	0.823	0.517	0.942	0.969	0.978
	0.758	0.424	0.333	0.304	0.550	0.233	0.683	0.765	0.821
Sk *t*	0.0438	0.0581	0.0375	0.0394	0.0360	0.0405	0.0584	0.0396	0.0664
	0.001	0.000	0.001	0.001	0.001	0.001	0.000	0.001	0.000
	0.001	0.000	0.001	0.001	0.001	0.001	0.000	0.001	0.000
Hn *t*	0.0386	0.0447	0.1270	0.1303	0.1218	0.0446	0.1340	0.1274	0.0611
	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SGED	0.0019	0.0025	0.0038	0.0026	0.0040	0.0057	0.0026	0.0024	0.0011
	0.968	0.924	0.777	0.920	0.748	0.517	0.908	0.944	0.997
	0.813	0.698	0.499	0.650	0.476	0.236	0.656	0.697	0.943
IHS	0.0021	0.0026	0.0037	0.0025	0.0038	0.0060	0.0026	0.0024	0.0012
	0.960	0.934	0.804	0.941	0.788	0.485	0.927	0.949	0.996
	0.776	0.705	0.525	0.711	0.514	0.210	0.679	0.738	0.939
MX	0.0001	0.0067	0.0043	0.0031	0.0045	0.0130	0.0030	0.0030	0.0045
	1.000	0.428	0.697	0.861	0.675	0.228	0.876	0.881	0.656
	0.979	0.188	0.391	0.612	0.367	0.040	0.545	0.622	0.377
DG	0.0020	0.0019	0.0040	0.0037	0.0028	0.0047	0.0031	0.0029	0.0026
	0.966	0.977	0.797	0.827	0.909	0.720	0.861	0.893	0.914
	0.814	0.821	0.478	0.532	0.652	0.416	0.590	0.612	0.678
SGN	0.0008	0.0007	0.0017	0.0008	0.0021	0.0027	0.0041	0.0052	0.0006
	1.000	0.999	0.960	0.999	0.932	0.868	0.636	0.558	1.000
	0.963	0.951	0.779	0.953	0.711	0.611	0.361	0.303	0.965

Table 11: Reality Check, S&P 500 Data (SP)

Panel B: 5% Tail

	EWMA	GARCH	GJR	APARCH	EGARCH	STGARCH	HYGARCH	HYAPARCH	CGARCH
HS	0.0162	0.0049	0.0071	0.0062	0.0050	0.0040	0.0047	0.0064	0.0056
	0.169	0.743	0.541	0.586	0.720	0.840	0.769	0.565	0.649
	0.169	0.513	0.428	0.454	0.502	0.529	0.521	0.444	0.468
NM	0.0181	0.0219	0.0169	0.0135	0.0143	0.0225	0.0231	0.0134	0.0199
	0.124	0.107	0.151	0.204	0.189	0.101	0.099	0.205	0.119
	0.120	0.107	0.146	0.186	0.175	0.101	0.099	0.187	0.119
St *t*	0.0038	0.0032	0.0028	0.0021	0.0022	0.0042	0.0035	0.0021	0.0031
	0.898	0.929	0.956	0.982	0.976	0.869	0.920	0.983	0.946
	0.718	0.728	0.778	0.853	0.838	0.699	0.732	0.855	0.753
GED	0.0046	0.0048	0.0039	0.0032	0.0031	0.0052	0.0056	0.0032	0.0052
	0.855	0.846	0.890	0.948	0.951	0.815	0.767	0.947	0.793
	0.681	0.677	0.699	0.785	0.764	0.641	0.597	0.778	0.624
LP	0.0039	0.0036	0.0025	0.0029	0.0029	0.0038	0.0034	0.0029	0.0033
	0.885	0.915	0.968	0.945	0.951	0.905	0.914	0.947	0.925
	0.680	0.676	0.757	0.663	0.656	0.711	0.695	0.669	0.693
DW	0.0041	0.0025	0.0020	0.0010	0.0018	0.0050	0.0039	0.0022	0.0036
	0.888	0.973	0.982	0.999	0.988	0.811	0.895	0.985	0.928
	0.701	0.805	0.884	0.967	0.903	0.640	0.720	0.878	0.770
Sk *t*	0.0140	0.0211	0.0169	0.0176	0.0174	0.0158	0.0208	0.0176	0.0246
	0.217	0.078	0.151	0.134	0.139	0.174	0.083	0.135	0.042
	0.217	0.078	0.151	0.134	0.139	0.174	0.083	0.135	0.042
Hn *t*	0.0183	0.0306	0.0401	0.0381	0.0371	0.0306	0.0396	0.0380	0.0326
	0.129	0.009	0.000	0.000	0.000	0.009	0.000	0.000	0.005
	0.126	0.009	0.000	0.000	0.000	0.009	0.000	0.000	0.005
SGED	0.0059	0.0049	0.0031	0.0034	0.0022	0.0058	0.0051	0.0033	0.0045
	0.735	0.827	0.949	0.938	0.980	0.742	0.810	0.939	0.858
	0.573	0.648	0.798	0.808	0.858	0.583	0.636	0.816	0.677
IHS	0.0036	0.0036	0.0022	0.0020	0.0019	0.0032	0.0037	0.0023	0.0036
	0.906	0.920	0.985	0.989	0.988	0.939	0.917	0.981	0.918
	0.736	0.706	0.821	0.869	0.841	0.738	0.704	0.820	0.701
MX	0.0045	0.0032	0.0023	0.0018	0.0017	0.0041	0.0036	0.0019	0.0040
	0.834	0.943	0.972	0.990	0.988	0.887	0.925	0.989	0.901
	0.584	0.748	0.791	0.865	0.873	0.703	0.722	0.834	0.711
DG	0.0021	0.0004	0.0001	0.0003	0.0002	0.0010	0.0015	0.0003	0.0008
	0.978	1.000	1.000	0.999	1.000	0.996	0.988	1.000	0.997
	0.880	0.992	0.986	0.992	0.955	0.964	0.873	0.871	0.945
SGN	0.0089	0.0155	0.0106	0.0080	0.0087	0.0135	0.0187	0.0131	0.0030
	0.436	0.137	0.313	0.497	0.463	0.196	0.134	0.212	0.938
	0.344	0.136	0.287	0.373	0.364	0.186	0.134	0.195	0.705

Table 12: Reality Check, Nasdaq Data

Panel A: Whole Distribution

	EWMA	GARCH	GJR	APARCH	EGARCH	STGARCH	HYGARCH	HYAPARCH	CGARCH
HS	0.0055	0.0266	0.0265	0.0245	0.0202	0.0301	0.0234	0.0238	0.0177
	0.980	0.351	0.342	0.379	0.443	0.308	0.387	0.387	0.511
	0.588	0.041	0.021	0.029	0.026	0.028	0.044	0.033	0.097
NM	0.0066	0.0126	0.0200	0.0190	0.0185	0.0151	0.0131	0.0188	0.0065
	0.952	0.709	0.473	0.501	0.521	0.613	0.692	0.506	0.961
	0.474	0.205	0.067	0.083	0.100	0.163	0.193	0.090	0.462
St *t*	0.0064	0.0102	0.0158	0.0147	0.0137	0.0121	0.0101	0.0144	0.0068
	0.966	0.820	0.594	0.633	0.677	0.729	0.822	0.640	0.955
	0.417	0.232	0.085	0.128	0.145	0.164	0.232	0.141	0.382
GED	0.0118	0.0108	0.0165	0.0157	0.0149	0.0127	0.0109	0.0154	0.0066
	0.720	0.790	0.568	0.593	0.628	0.705	0.788	0.603	0.959
	0.255	0.213	0.085	0.116	0.141	0.175	0.218	0.123	0.400
LP	0.0121	0.0104	0.0133	0.0093	0.0087	0.0123	0.0073	0.0092	0.0077
	0.726	0.822	0.679	0.865	0.893	0.720	0.941	0.868	0.928
	0.138	0.195	0.107	0.306	0.262	0.116	0.355	0.310	0.349
DW	0.0149	0.0208	0.0246	0.0185	0.0113	0.0120	0.0098	0.0123	0.0051
	0.592	0.426	0.334	0.488	0.764	0.722	0.839	0.727	0.988
	0.147	0.049	0.015	0.071	0.196	0.168	0.227	0.164	0.554
Sk *t*	0.0234	0.0218	0.0154	0.0144	0.0137	0.0172	0.0227	0.0146	0.0240
	0.389	0.423	0.596	0.638	0.661	0.539	0.400	0.633	0.373
	0.032	0.042	0.154	0.185	0.208	0.119	0.036	0.180	0.029
Hn *t*	0.0784	0.4247	0.2229	0.7250	0.1234	0.0242	0.1322	0.1209	0.0437
	0.047	0.000	0.000	0.000	0.002	0.350	0.000	0.002	0.187
	0.000	0.000	0.000	0.000	0.000	0.018	0.000	0.000	0.000
SGED	0.0096	0.0149	0.0118	0.0115	0.0128	0.0123	0.0093	0.0131	0.0059
	0.831	0.661	0.797	0.805	0.704	0.716	0.850	0.696	0.972
	0.341	0.186	0.253	0.264	0.172	0.198	0.271	0.161	0.457
IHS	0.0107	0.0121	0.0098	0.0096	0.0089	0.0132	0.0127	0.0102	0.0063
	0.783	0.790	0.865	0.880	0.908	0.744	0.762	0.848	0.969
	0.264	0.230	0.285	0.304	0.324	0.216	0.217	0.270	0.414
MX	0.0033	0.0129	0.0139	0.0132	0.0118	0.0158	0.0084	0.0101	0.0099
	1.000	0.699	0.654	0.695	0.743	0.588	0.892	0.867	0.828
	0.810	0.174	0.137	0.156	0.191	0.114	0.325	0.286	0.248
DG	0.0036	0.0072	0.0129	0.0102	0.0026	0.0016	0.0021	0.0026	0.0075
	0.996	0.934	0.715	0.830	1.000	1.000	1.000	1.000	0.916
	0.712	0.476	0.192	0.293	0.824	0.995	0.870	0.837	0.422
SGN	0.0103	0.0200	0.0152	0.0148	0.0067	0.0072	0.0052	0.0071	0.0034
	0.821	0.455	0.624	0.642	0.955	0.940	0.978	0.940	0.997
	0.268	0.055	0.157	0.173	0.438	0.419	0.539	0.437	0.781

Table 12: Reality Check, Nasdaq Data

Panel B: 5% Tail

	EWMA	GARCH	GJR	APARCH	EGARCH	STGARCH	HYGARCH	HYAPARCH	CGARCH
HS	0.0111	0.0136	0.0137	0.0148	0.0169	0.0117	0.0120	0.0133	0.0064
	0.321	0.220	0.212	0.174	0.119	0.298	0.289	0.230	0.678
	0.237	0.160	0.156	0.170	0.116	0.207	0.200	0.177	0.494
NM	0.0060	0.0103	0.0148	0.0148	0.0160	0.0116	0.0108	0.0151	0.0059
	0.592	0.364	0.237	0.236	0.207	0.330	0.351	0.232	0.629
	0.515	0.303	0.231	0.232	0.201	0.276	0.294	0.229	0.527
St *t*	0.0008	0.0039	0.0061	0.0054	0.0050	0.0048	0.0041	0.0051	0.0018
	1.000	0.926	0.760	0.841	0.874	0.863	0.907	0.867	0.994
	1.000	0.734	0.613	0.707	0.729	0.668	0.729	0.725	0.968
GED	0.0026	0.0028	0.0058	0.0062	0.0067	0.0039	0.0029	0.0061	0.0009
	0.988	0.992	0.783	0.733	0.649	0.953	0.987	0.740	1.000
	0.879	0.926	0.573	0.530	0.478	0.820	0.914	0.540	1.000
LP	0.0034	0.0035	0.0030	0.0033	0.0031	0.0042	0.0029	0.0032	0.0033
	0.896	0.906	0.967	0.946	0.963	0.860	0.951	0.960	0.911
	0.650	0.641	0.781	0.716	0.752	0.584	0.694	0.748	0.637
DW	0.0037	0.0032	0.0011	0.0007	0.0038	0.0034	0.0028	0.0031	0.0016
	0.948	0.963	0.998	1.000	0.940	0.958	0.984	0.979	0.997
	0.780	0.802	0.990	0.997	0.714	0.810	0.865	0.824	0.981
Sk *t*	0.0222	0.0214	0.0154	0.0144	0.0121	0.0177	0.0230	0.0155	0.0231
	0.028	0.031	0.130	0.161	0.253	0.085	0.026	0.132	0.015
	0.028	0.031	0.098	0.121	0.197	0.084	0.026	0.098	0.015
Hn *t*	0.0310	0.0444	0.0423	0.0438	0.0395	0.0211	0.0410	0.0398	0.0227
	0.008	0.000	0.000	0.000	0.000	0.031	0.000	0.000	0.017
	0.008	0.000	0.000	0.000	0.000	0.031	0.000	0.000	0.017
SGED	0.0045	0.0006	0.0002	0.0003	0.0014	0.0010	0.0011	0.0005	0.0010
	0.883	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
	0.689	1.000	0.999	0.997	0.988	0.993	0.986	0.998	0.964
IHS	0.0054	0.0005	0.0005	0.0007	0.0009	0.0005	0.0007	0.0004	0.0025
	0.829	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.984
	0.632	1.000	0.999	0.995	0.986	1.000	1.000	1.000	0.783
MX	0.0035	0.0009	0.0003	0.0006	0.0012	0.0014	0.0013	0.0002	0.0008
	0.917	1.000	1.000	1.000	1.000	0.998	0.997	1.000	1.000
	0.645	0.994	1.000	0.996	0.996	0.973	0.967	1.000	0.990
DG	0.0017	0.0027	0.0034	0.0025	0.0018	0.0027	0.0022	0.0010	0.0037
	0.995	0.964	0.939	0.978	0.995	0.973	0.984	1.000	0.852
	0.907	0.748	0.704	0.826	0.954	0.852	0.890	0.992	0.578
SGN	0.0042	0.0014	0.0001	0.0001	0.0179	0.0235	0.0252	0.0155	0.0241
	0.916	0.998	1.000	1.000	0.046	0.004	0.003	0.094	0.003
	0.732	0.985	1.000	1.000	0.046	0.004	0.003	0.055	0.003