Sectoral Composition of Government Spending and Macroeconomic (In)stability*

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Abstract

This paper examines the quantitative interrelations between sectoral composition of public spending and equilibrium (in)determinacy in a two-sector real business cycle model with positive productive externalities in investment. When government purchases of consumption and investment goods are set as constant fractions of their respective sectoral output, we show that the public-consumption share plays no role in the model's local dynamics, and that a sufficiently high public-investment share can stabilize the economy against endogenous belief-driven cyclical fluctuations. When each type of government spending is postulated as a constant proportion of the economy's total output, we find that there exists a trade-off between public consumption versus investment expenditures to yield saddle-path stability and equilibrium uniqueness.

Keywords: Government Spending; Equilibrium (In)determinacy; Business Cycles.

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1 Introduction

Starting with the work of Benhabib and Farmer (1994, 1996), considerable progress has been made in exploring indeterminacy and a continuum of stationary rational expectations equilibria within one-sector or multi-sector real business cycle (RBC) models under laissez faire.[1] In this case, since agents' animal spirits can be an independent impulse for generating endogenous business cycles, these "sunspot" models call for Keynesian-type stabilization policies intended to insulate the economy from belief-driven cyclical fluctuations such that saddle-path stability and equilibrium uniqueness are achieved. It turns out that many existing studies have focused on the macroeconomic (in)stability effects of various tax policy rules,[2] and thus left the potential (de)stabilization role of government spending in a representative-agent setting underexplored.[3] Motivated by this gap in the literature, we examine the quantitative interrelations between public expenditures and equilibrium (in)determinacy in a modified Benhabib-Farmer RBC model with two distinct production sectors, consumption and investment. Our analysis is valuable not only for its theoretical insights, but also for its broad implications for the design, implementation and evaluation of the government's spending policies within a macroeconomy.

In this paper, we incorporate government purchases of goods and services into a discrete-time two-sector RBC model, as in Harrison (2001), with positive productive externalities present in the investment sector. The government balances its budget each period by levying lump-sum taxes on the representative household to finance its consumption and investment expenditures. This analytical framework allows us to isolate how sectoral distribution of public spending affects the model's local stability properties, as well as facilitates comparison with previous work that investigates the (de)stabilizing effects of income and/or consumption taxation in a RBC-type economy. In particular, Guo and Harrison (2001, 2011) consider a progressive tax schedule *à la* Guo and Lansing (1998) within the same theoretical setup as ours, and postulate that all the government's tax revenues are returned to households as a lump-sum transfer.[4]

[1] See Benhabib and Farmer (1999) for a survey of this RBC-based indeterminacy literature.

[2] See, for example, Guo and Lansing (1998), Christiano and Harrison (1999), Guo and Harrison (2001, 2011), Greiner (2006), Giannitsarou (2007), Dromel and Pintus (2007, 2008), Mino and Nakamoto (2008), Chen and Guo (2013*a*, 2013*b*), Gokan (2013), and Nourry, Seegmuller and Venditti (2013), among others.

[3] Previous research that studies the (de)stabilizing effects of government spending within dynamic general equilibrium macroeconomic models include Schmitt-Grohé and Uribe (1997, section III), Raurich (2001), Gokan (2006), and Lloyd-Braga, Modesto and Seegmuller (2008), among others.

[4] Guo and Harrison (2011) correct an error in Guo and Harrison's (2001) description of the household's and government's budget constraints, and then show that all of the authors' earlier results are qualitatively unchanged.

We show that the equilibrium dynamics of our model economy depends crucially on how the division of public expenditures between consumption versus investment goods is specified, and the degree of productive externalities in investment. Specifically, when government purchases from the consumption and investment sectors are set as constant fractions of their respective sectoral output, we first find that the public-consumption share plays no role in the model's local dynamics. Although changes in government spending on consumption goods yield a within-sector trade-off between private and public consumption expenditures, they do not influence the relative composition of productive resources across the two sectors that is crucial in validating agents' anticipation about a higher future return on capital.

Next, in a calibrated version of our baseline specification, we show that the economy is more susceptible to indeterminacy and sunspots when the output fraction of public expenditures in the investment sector is relatively small. To understand this result, start the model from its steady state, and suppose that agents become optimistic about the economy's future. Acting upon this belief, the representative household will consume less and invest more today, hence two opposing effects on the household's intertemporal Euler equation ensue. On the one hand, due to positive productive externalities in the investment sector, the social production possibility frontier that traces out the trade-off between private consumption and investment spending is convex to the origin. As a result, the relative price of investment goods will fall (the price effect) upon agents' optimism that shifts more capital and labor inputs into producing investment goods. On the other hand, an increase in today's private investment expenditures will reduce next period's real interest rate because of diminishing marginal product of capital (the MPK effect). Our analysis finds that for a sufficiently low output fraction of government spending on investment goods, the price effect outweighs the MPK effect such that the household's initial rosy expectation can be justified as a self-fulfilling equilibrium. Moreover, since higher increasing returns-to-scale in the production of investment goods induce agents to move more productive resources out of the consumption sector, the threshold level of the public-investment share needed to stabilize the economy against sunspot-driven cyclical fluctuations will rise as well because of a dominating MPK effect.

For sensitivity analysis, we examine the formulation in which public expenditures of consumption and investment goods are postulated as constant proportions of the economy's total output. In sharp contrast to the benchmark configuration, the GDP fraction of government purchases from the consumption sector is found to appear in the model's Jacobian matrix and thus affects its local stability properties. Intuitively, changes in the public-consumption share not only generate a within-period switch between private and public consumption spending, they also influence the representative household's intertemporal investment decision through its consumption Euler equation. We also find that saddle-path stability is *ceteris paribus* more likely to occur under a sufficiently high level of the public-investment or public-consumption share. It follows that there exists a trade-off between the government's investment versus consumption expenditures to suppress endogenous business cycles within the alternative version of our model economy. In sum, this paper shows that in the context of a two-sector RBC model with positive investment externalities, it is the sectoral composition, rather than the total amount, of public spending that matters for the design of the government's stabilization policies.

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes its equilibrium conditions. Section 3 undertakes a quantitative investigation of local dynamics in a calibrated version of our benchmark specification. Section 4 explores the model economy under a different formulation of the government's spending policy rules. Section 5 concludes.

2 The Economy

We incorporate government spending on goods and services into the discrete-time two-sector real business cycle (RBC) model *à la* Harrison (2001). Households live forever, and derive utility from consumption and leisure. The production side of the economy consists of two distinct sectors, consumption and investment. Based on the empirical findings of Harrison (2003), competitive firms in each sector produce output with identical Cobb-Douglas technologies, but positive productive externalities are limited to the investment sector. The government balances its budget each period by levying lump-sum taxes on the representative household to finance its expenditures. We also postulate that there are no fundamental uncertainties present in the economy.

2.1 Firms

In the consumption sector, output is produced by competitive firms using the following constant returns-to-scale technology:

$$Y_{ct} = K_{ct}^{\alpha} L_{ct}^{1-\alpha}, \ 0 < \alpha < 1, \tag{1}$$

where K_{ct} and L_{ct} are the capital and labor inputs utilized in the production of consumption goods. Under the assumption that factor markets are perfectly competitive, the first-order conditions for profit maximization are

$$\begin{aligned} r_t &= \frac{\alpha Y_{ct}}{K_{ct}}, & (2) \\ w_t &= \frac{(1-\alpha) Y_{ct}}{L_{ct}}, & (3) \end{aligned}$$

where r_t is the capital rental rate, and w_t is the real wage rate. Similarly, investment goods are produced by competitive firms using the technology

$$Y_{It} = X_t K_{It}^{\alpha} L_{It}^{1-\alpha}. \quad (4)$$

Here, K_{It} and L_{It} are physical capital and labor hours in the investment sector, and X_t represents productive externalities that each individual firm takes as given. Moreover, X_t is postulated to take the following form:

$$X_t = (\bar{K}_{It}^{\alpha} \bar{L}_{It}^{1-\alpha})^{\theta}, \quad \theta \geq 0, \quad (5)$$

where $\bar{K}_{It}$ and $\bar{L}_{It}$ denote the within-sector average levels of capital and labor devoted to producing investment goods, and θ measures the degree of sector-specific externalities in the investment sector. In a symmetric equilibrium, all firms in the investment sector take the same actions such that $K_{It} = \bar{K}_{It}$ and $L_{It} = \bar{L}_{It}$, for all t. Our analysis is restricted to cases with $\alpha(1+\theta) < 1$ such that the model economy does not exhibit sustained economic growth.

The first-order conditions that govern the demand for capital and labor in the investment sector are

$$\begin{aligned} r_t &= p_t \frac{\alpha Y_{It}}{K_{It}}, & (6) \\ w_t &= p_t \frac{(1-\alpha) Y_{It}}{L_{It}}, & (7) \end{aligned}$$

where p_t denotes the relative price of investment to consumption goods at time t. Notice that firms in each sector face the same factor prices in equilibrium since capital and labor inputs are assumed to be perfectly mobile across the two sectors.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households. Each household maximizes its present discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{L_t^{1+\gamma}}{1+\gamma} \right], \quad 0 < \beta < 1, \quad \text{and} \quad \gamma \geq 0, \tag{8}$$

where C_t and L_t are the household's consumption and hours worked, respectively; β is the discount factor, and γ denotes the inverse of the wage elasticity for labor supply.[5] The budget constraint faced by the representative agent is given by

$$C_t + p_t I_t + T_t = Y_t = r_t K_t + w_t L_t, \tag{9}$$

where I_t is gross investment, T_t is a non-distortionary lump-sum tax, Y_t is national income or GDP, and K_t is the household's capital stock that evolves according to the law of motion

$$K_{t+1} = (1-\delta)K_t + I_t, \quad K_0 > 0 \text{ given}, \tag{10}$$

where $\delta \in (0, 1)$ is the capital depreciation rate.

The first-order conditions for the household's dynamic optimization problem are

$$C_t L_t^{\gamma} = w_t, \tag{11}$$

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left[\frac{r_{t+1} + (1-\delta)p_{t+1}}{p_t} \right], \tag{12}$$

$$\lim_{t \to \infty} \beta^t \frac{K_{t+1}}{C_t} = 0, \tag{13}$$

where (11) equates the slope of household's indifference curve to the real wage rate, (12) is the Euler equation for intertemporal choices of private consumption, and (13) is the transversality condition.

2.3 Government

The government spends its (lump-sum) tax revenue T_t on goods and services produced by the consumption and investment sectors, and balances its budget each period. Hence, its period budget constraint is given by

[5] The period utility function in (8) is consistent with balanced long-run growth, a feature that is commonly maintained in the real business cycle literature. Our results, reported in sections 3 and 4 below, are qualitatively robust to a generalized preference formulation that is isoelastic with respect to the household's consumption and additively separable with labor hours.

$$T_t = G_{ct} + p_t G_{It}, \tag{14}$$

where G_{ct} and G_{It} are quantities of consumption and investment goods, respectively, purchased by the government. In order to ensure that the economy exhibits a well-defined competitive equilibrium, we close the benchmark model by postulating

$$\frac{G_{ct}}{Y_{ct}} = \phi_c \tag{15}$$

and

$$\frac{G_{It}}{Y_{It}} = \phi_I, \tag{16}$$

where $0 \leq \phi_c,\ \phi_I < 1$. That is, government purchases from the consumption and investment sectors are set as constant fractions of their respective sectoral output.[6] When $\phi_c = \phi_I = 0$, we recover the laissez-faire model of Harrison (2001). Finally, combining (9) and (14) yields the following aggregate resource constraint for the economy:

$$C_t + p_t I_t + G_t = Y_t, \tag{17}$$

where $G_t = G_{ct} + p_t G_{It}$ denotes total government spending.

2.4 Equilibrium

Since firms use identical production technologies and face equal factor prices across the two sectors, the fractions of capital and labor inputs utilized in the consumption sector are the same,

$$\frac{K_{ct}}{K_t} = \frac{L_{ct}}{L_t} \equiv \mu_t. \tag{18}$$

Using (1)-(2), (4)-(6) and (18), the equilibrium relative price of investment goods can be expressed as

$$p_t = \frac{1}{\left[(1-\mu_t)\, K_t^{\alpha} L_t^{1-\alpha}\right]^{\theta}}. \tag{19}$$

[6]Section 4 examines the alternative specification whereby government purchases of consumption and investment goods are postulated as constant proportions of the economy's GDP. When public expenditures of each type are assumed to be a constant fraction of total government spending, our current setup with lump-sum taxation will not have enough equations to pin down the model's equilibrium allocations. It would be a worthwhile topic for future research to investigate this formulation under distortionary income tax policies.

The equalities of demand by households and the government versus supply by firms in the consumption and investment sectors are given by

$$C_t + G_{ct} = Y_{ct}, \tag{20}$$

$$I_t + G_{It} = Y_{It}. \tag{21}$$

Moreover, both the capital and labor markets clear whereby

$$K_{ct} + K_{It} = K_t, \tag{22}$$

$$L_{ct} + L_{It} = L_t. \tag{23}$$

We focus on symmetric perfect-foresight competitive equilibria which consist of a set of prices $\{r_t, w_t, p_t\}_{t=0}^{\infty}$ and allocations $\{C_t, L_t, K_{t+1}, Y_t, Y_{ct}, K_{ct}, L_{ct}, Y_{It}, K_{It}, L_{It}, X_t, I_t, T_t, G_{ct}, G_{It}, \mu_t\}_{t=0}^{\infty}$ that satisfies equations (1)-(7), (9)-(12), (14)-(16), (18), and (20)-(23), together with the given initial aggregate capital stock K_0 and the transversality condition (13).

2.5 Steady State and Local Dynamics

It is straightforward to show that our model possesses a unique interior steady state at which the fraction of factor inputs allocated to producing consumption goods is

$$\mu = 1 - \frac{\alpha\delta}{(1-\phi_I)\left(\frac{1}{\beta} - 1 + \delta\right)}, \tag{24}$$

where time subscript is left out to denote the steady-state value. Given (24), the steady-state expressions of all other endogenous variables can be easily derived.[7]

We then take log-linear approximations to the model's equilibrium conditions in a neighborhood of this steady state to obtain the following dynamical system:

$$\begin{bmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}, \quad \hat{K}_0 \text{ given,} \tag{25}$$

[7] For example, the (aggregate) labor hours and capital stock provided by the household are given by

$$L = \left[\frac{1-\alpha}{\mu(1-\phi_c)}\right]^{\frac{1}{1+\gamma}}, \quad \text{and} \quad K = \left[\frac{(1-\phi_I)(1-\mu)^{1+\theta} L^{(1-\alpha)(1+\theta)}}{\delta}\right]^{\frac{1}{1-\alpha(1+\theta)}}.$$

where hat variables represent percentage deviations from their respective steady-state values, and $\mathbf{J}$ is the Jacobian matrix of partial derivatives of the transformed dynamical system (see Appendix A). The expressions for the trace and determinant of the Jacobian matrix are

$$Tr = \mathbf{J}_{11} + \mathbf{J}_{22}, \tag{26}$$

and

$$Det = \mathbf{J}_{11}\mathbf{J}_{22} - \mathbf{J}_{12}\mathbf{J}_{21}. \tag{27}$$

The model exhibits saddle-path stability and equilibrium uniqueness when one eigenvalue of $\mathbf{J}$ lies inside and the other outside the unit circle. When both eigenvalues are inside the unit circle, the steady state is locally indeterminate and thus a sink. When both eigenvalues are outside the unit circle, the steady state becomes a totally unstable source.

3 Macroeconomic (In)stability

This section examines the quantitative interrelations between sectoral composition of public spending, governed by ϕ_c and ϕ_I, and equilibrium (in)determinacy in the benchmark version of our two-sector RBC model with productive externalities in the investment sector. Our first finding is

Result 1. The output fraction of government purchases in the consumption sector ϕ_c has no effect on the model's local stability properties, whereas the output fraction of government purchases in the investment sector ϕ_I does.

The intuition for the above result can be understood as follows. Start the economy from its steady state, and consider an anticipated increase in the rate of return on capital caused by agents' optimistic expectation about an expansion of future economic activities. Acting upon this belief, the representative household will consume less and invest more today, which in turn reduces the amount of capital and labor inputs allocated to the production of consumption goods. As it turns out, although changes in government spending on consumption goods generate a within-sector trade-off between private and public consumption expenditures, they do not affect the relative composition of productive resources across the two sectors that is crucial in validating agents' initial optimism. Specifically, the public-consumption share ϕ_c does not enter (24) which governs the steady-state intensity of factor inputs μ; or the elements that make up the Jacobian matrix $\mathbf{J}$ shown in Appendix A.[8] This implies that the government's purchases of consumption goods play no role in the equilibrium dynamics of our benchmark model.

On the other hand, equations (24) together with (A.1)-(A.6) reported in Appendix A show that sectoral distribution of productive resources at the model's steady state, as well as the resulting local stability properties, are influenced by the output fraction of government purchases in the investment sector ϕ_I. For example, an increase in public spending on investment goods not only crowds out private investment expenditures, it also raises the steady-state proportion of factor inputs that will be shifted toward the investment sector $\left(\frac{\partial(1-\mu)}{\partial\phi_I} > 0\right)$ upon agents' optimistic expectation. It follows that the public-investment share may (de)stabilize the macroeconomy since ϕ_I affects the accumulation of physical capital, and thus the intertemporal trade-off between consumption choices at different time periods.

Next, we investigate how the government's spending policies affect equilibrium dynamics of competitive equilibria for combinations of calibrated parameters that are consistent with post Korean-war U.S. data. Each period in the model is taken to be one quarter. As in Benhabib and Farmer (1996), Harrison (2001) and many previous studies in the real business cycle literature, the labor share of national income, $1-\alpha$, is chosen to be 0.7; the discount factor, β, is set to be $\frac{1}{1.01}$; the labor supply elasticity, γ, is equal to 0 (*i.e.* indivisible labor *à la* Hansen [1985] and Rogerson [1988]); and the capital depreciation rate, δ, is fixed at 0.025. In addition, Result 1 allows us to ignore ϕ_c within the benchmark version of our model.

Given the above parameterization, Figure 1 depicts the local stability properties of our baseline model as a function of the output fraction of government purchases in the investment sector and the degree of productive externalities in investment. In particular, the $\phi_I - \theta$ space is divided into regions of "Saddle", "Sink" and "Source". When $\phi_I \to 0.7857$, the steady-state fraction of capital and labor inputs utilized in the consumption sector μ will approach zero (see equation 24), and hence an equilibrium is no longer feasible. Using durables as a proxy for the investment goods in our model, we follow Guo and Harrison (2010, p. 298) and set the upper bound of investment externalities θ to be 0.44, which is the highest possible value that is regarded as empirically plausible, in Figure 1. This is one standard deviation above Basu and Fernald's (1997, Table III) aggregation-corrected point estimate for returns-to-scale in the U.S. durables manufacturing industry.

[8] It is straightforward to show that the household's consumption expenditures can be expressed as $C_t = (1-\phi_c)\,\mu_t Y_t$. As a result, the public-consumption share ϕ_c drops out from the Jacobian matrix when we log-linearize the model's equilibrium conditions.

Result 2. When $\phi_I = 0$ as shown in the vertical axis of Figure 1, the model's steady state is a saddle point for $0 \leq \theta \leq 0.0773$; or a sink for $\theta \geq 0.0774$.

This result illustrates that irrespective of the value of ϕ_c, our baseline model without public spending on investment goods ($\phi_I = 0$) displays exactly the same local dynamics as those in Harrison (2001, section 4) under laissez faire. Intuitively, when agents anticipate a higher future return on today's investment, they need incentive to give up current consumption in exchange for more capital accumulation. As long as they are rewarded with productive investment, in the form of sufficiently high increasing returns-to-scale within that sector, it will be worthwhile for them to do so. Specifically in our calibrated economy, the rate of return on capital will rise to fulfill agents' rosy expectation provided the external effects in the investment firms' production processes are strong enough with $\theta \geq 0.0774$.

Result 3. When $\phi_I > 0$ and $0 \leq \theta \leq 0.0773$, the economy always exhibits saddle-path stability and equilibrium uniqueness.

Substituting $\theta = 0$ into (19) shows that the relative price of investment to consumption goods becomes $p_t = 1$ for all t. As a result, our model without any investment externality collapses to a canonical one-sector RBC formulation with aggregate constant returns-to-scale in production, except that we have added a government sector which does not change its saddle-path stability. It follows that equilibrium indeterminacy and belief-driven cyclical fluctuations can never arise within this setting. We also obtain the same finding under a "low" degree of investment externality ($0 < \theta \leq 0.0773$) because it is not sufficiently strong to raise the equilibrium rate of return on capital, regardless of how public expenditures are divided between the two sectors.[9]

Result 4. For a given level of $\theta \geq 0.0841$, the model's local stability property switches from being a sink to a source, and then to a saddle point as the public-investment share ϕ_I increases.[10]

As an example, when the investment externality takes on an empirically realistic value of $\theta_{US} = 0.108$ (see Harrison, 2003), local indeterminacy occurs for public-investment shares in the range $0 \leq \phi_I \leq 0.0728$. The steady state turns into a source when ϕ_I is raised to the interval of $[0.0729, 0.2756]$. In this case, any trajectory that diverges from this completely

[9] By contrast, Guo and Harrison (2001, 2011) show that a regressive income tax policy may destabilize the economy by inducing endogenous cyclical fluctuation in an otherwise identical two-sector RBC model with zero or low investment externality.

[10] When the investment externality θ lies within the interim interval $[0.0774, 0.084]$, the steady state changes from being a sink to a saddle point as ϕ_I rises.

unstable steady state may settle down to a limit cycle or to some complicated attracting sets. The model exhibits a locally unique equilibrium (a saddle path) for $\phi_I \geq 0.2757$. The policy implication of this result is that the economy is more susceptible to indeterminacy and endogenous business cycles when the output fraction of government purchases in the investment sector is zero (see Result 2) or "relatively small".

Result 5. When $\phi_I > 0$ and $\theta \geq 0.0774$, the threshold level for the output fraction of public spending in the investment sector, denoted as ϕ_I^{mim}, that leads to saddle-path stability is monotonically increasing with respect to the degree of investment externalities, *i.e.* $\frac{\partial \phi_I^{mim}}{\partial \theta} > 0$.

To understand the intuition for the previous results, we rewrite the intertemporal Euler equation for private consumption (12) as

$$\frac{C_{t+1}}{C_t} = \beta \left[\frac{r_{t+1} + (1-\delta) p_{t+1}}{p_t} \right]. \tag{28}$$

Start from the model's steady state at period t, and suppose that households become optimistic about the economy's future. In response to this change in non-fundamental expectations, they sacrifice consumption (C_t falls) for higher investment today (raising K_{t+1}), which in turn raises next period's hours worked via firms' labor demand, thereby producing more output and higher consumption in period $t+1$ (C_{t+1} rises). It follows that the left-hand side of (28) will rise. For this alternative dynamic path to be justified as a self-fulfilling equilibrium, the (price-weighted) rate of return on K_{t+1} net of depreciation, *i.e.* the right-hand side of (28), needs to increase as well.

As it turns out, the quantitative interdependence between θ and ϕ_I that governs our model's local stability properties depends crucially on the relative strength of two opposing forces. On the one hand, due to the presence of positive externalities in producing investment goods, the social production possibility frontier which traces out the trade-off between private consumption and investment expenditures is convex to the origin. As a result, the relative price of investment p_t falls (the price effect) upon agents' optimism that shifts more capital and labor inputs into the investment sector – this effect causes right-hand side of (28) to rise. On the other hand, an increase in today's private investment expenditures that raises K_{t+1} will result in a lower real interest rate r_{t+1} because of diminishing marginal product of capital (the MPK effect) – this effect causes the right-hand side of (28) to fall.

Results 2 and 4, together with footnote 10, illustrate that for a given degree of investment externalities $\theta \geq 0.0774$, the price effect outweighs the MPK effect provided the output fraction of government spending on investment goods is zero or relatively low. In either formulation, equilibrium indeterminacy results as the right-hand side of (28) will rise to validate the initial anticipated increase in the return on capital. It follows that raising the public-investment share to the corresponding critical level ϕ_I^{mim} ($= 0.2757$ when $\theta_{US} = 0.108$) or above is able to stabilize the economy against sunspot-driven cyclical fluctuations because of a denominating MPK effect.

Finally, when $\theta \geq 0.0774$ and rises, higher increasing returns-to-scale in investment induce households to move more productive resources out of the consumption sector, which in turn generate larger decreases in p_t and r_{t+1}, *i.e.* the price and MPK effects both become stronger. Per the same reasoning from the previous paragraph, Result 5 finds that ϕ_I^{mim} will increase as well $\left(\frac{\partial \phi_I^{mim}}{\partial \theta} > 0\right)$ such that the reduction in r_{t+1} dominates that in p_t to render the equilibrium unique and determinate. In sum, within the benchmark version of our two-sector RBC model with positive productive externalities in the investment sector, we show that the output fraction of public expenditures on investment goods needs to be sufficiently high in order to suppress endogenous business cycles driven by agents' animal spirits.

4 Alternative Specification on Government Spending

For sensitivity analysis, this section examines our two-sector RBC model under a different formulation of the government's spending policy rules. Specifically, public expenditures of consumption and investment goods are postulated as constant fractions of the economy's total output:

$$\frac{G_{ct}}{Y_t} = \varphi_c \quad \text{and} \quad \frac{p_t G_{It}}{Y_t} = \varphi_I, \tag{29}$$

where $0 \leq \varphi_c$, $\varphi_I < 1$ and $0 \leq \varphi_c + \varphi_I < 1$. In this case, it is straightforward to show that the steady-state proportion of capital and labor inputs allocated to producing consumption goods is given by

$$\mu = 1 - \varphi_I - \frac{\alpha\delta}{\frac{1}{\beta} - 1 + \delta}, \tag{30}$$

and that all other endogenous variables at the steady state can be derived accordingly. We then log-linearize the model's equilibrium conditions around this unique interior steady state to obtain the resulting Jacobian matrix $\mathbf{J}$ shown in Appendix B. In sharp contrast to Result 1 of our baseline specification, equations (B.1)-(B.7) show that the GDP fraction of government purchases in the consumption sector φ_c, as well as the public-investment share φ_I, now appears in the Jacobian matrix and thus affects the economy's local stability properties. To understand this finding, we note that the household's consumption spending within the alternative model can be expressed as

$$C_t = (\mu_t - \varphi_c)\, Y_t. \tag{31}$$

It follows that unlike in the benchmark counterpart (see footnote 8), the public-consumption share φ_c does not drop out from the log-linearization procedure and will play a role in the equilibrium dynamics. Intuitively, changes in φ_c not only yield a within-period trade-off between private and public consumption expenditures, they also influence the factor intensity across the two production sectors and the representative agent's intertemporal investment decision through substituting (31) into its consumption Euler equation (12) .

Under the same calibrated values of α, β, γ and δ as those in our baseline configuration, together with the investment externality fixed at $\theta_{US} = 0.108$, Figure 2 summarizes the model's local stability properties with public expenditures given by (29). Since the post Korean-war average of government spending to output ratio is around 0.2 in U.S., we set the individual upper bound of φ_c and φ_I to be 0.3 for the purpose of clear illustration. Following Azariadis (1993, p. 93), the economy displays saddle-path stability if and only if

$$-(Tr+1) < Det < Tr - 1 \quad \text{or} \quad Tr - 1 < Det < -(Tr+1); \tag{32}$$

our model possesses an indeterminate steady state if and only if

$$-1 < Det < 1 \quad \text{and} \quad -(Det+1) < Tr < Det + 1; \tag{33}$$

and the steady state is a source if and only if

$$Det > 1 \quad \text{and} \quad -(Det+1) < Tr < Det + 1, \tag{34}$$

where Tr and Det, as in (26)-(27), are the trace and determinant for the Jacobian matrix reported in Appendix B.

Given the above parameterization over the feasible $\varphi_I - \varphi_c$ space, Figure 2 shows that (i) the most-binding necessary and sufficient condition which separates the areas labeled "Saddle" versus "Sink" and "Source" turns out to be $Det + Tr = -1$; (ii) the downward-sloping curve that divides the regions of "Sink" and "Source" is characterized by $Det = 1$; (iii) when $\varphi_c = 0$, the minimum level for the GDP fraction of public spending in the investment sector that stabilizes the economy against belief-driven fluctuations is $\varphi_I^{mim} = 0.0783$, below which local indeterminacy results; and (iv) when $\varphi_I = 0$, the steady state is a sink under $0 \leq \varphi_c \leq 0.07283$, turns into a source as φ_c lies within the interval $[0.07284, 0.2755]$, and becomes a saddle point at $\varphi_c^{mim} = 0.2756$.

Figure 2 also depicts that similar to our benchmark specification, saddle-path stability is *ceteris paribus* more likely to occur under a "sufficiently high" value of φ_I or φ_c within the alternative model. For a given level of φ_c, the intuition for why higher public-investment shares make equilibrium determinacy easier to obtain is the same behind Result 4 of the preceding section: increases in φ_I will eventually generate a stronger MPK effect to invalidate the household's initial optimism. For a given level of φ_I, a higher φ_c shifts more productive resources into the consumption sector, which in turn raises the relative price of investment goods p_t along the convex social production possibility frontier and thus prevents agents' optimistic expectation from becoming self-fulfilling. Finally, combining these results on φ_I and φ_c implies that the dividing locus of $Det+Tr = -1$, above which the economy does not exhibit indeterminacy and sunspots, is negatively sloped. That is, there exists a trade-off between the public-investment versus the public-consumption shares for the design and implementation of stabilization policies in our model economy with government spending described by (29).

5 Conclusion

This paper has shown that in the context of a two-sector real business cycle model with positive productive externalities in investment, it is the sectoral composition, rather than the total amount, of government spending that governs the economy's macroeconomic (in)stability. Specifically in our benchmark specification whereby public expenditures from the consumption and investment sectors are set as constant fractions of their respective sectoral output, the public-consumption share plays no role in the model's local dynamics. We also find that the economy is more susceptible to indeterminacy and sunspots when the output fraction of government purchases of investment goods is relatively small; and that the threshold level for the public-investment share which yields saddle-path stability and equilibrium uniqueness is monotonically increasing with respect to the degree of investment externalities. When each type of public spending is postulated as a constant proportion of the economy's total output, we show that in sharp contrast to the baseline configuration, changes in the GDP fraction of government purchases in the consumption sector affect the local stability properties of competitive equilibria. In addition, there exists a trade-off between public consumption versus investment expenditures to stabilize the economy against endogenous belief-driven cyclical fluctuations within the alternative version of our model economy.

In terms of possible extensions, it would be worthwhile to study our model economy with useful government spending that contributes to the household's utility *à la* Guo and Harrision (2008, section 3.2) or the firm's productivity *à la* Barro (1990), or a generalized non-separable preference formulation *à la* Jaimovich and Rebelo (2009) that considers different degrees of income effect on labor supply, or a non-balanced budget with national debt *à la* Schmitt-Grohé and Uribe (1997, p. 990). Incorporating these features will allow us to examine the robustness of this paper's quantitative results and policy implications, as well as further enhance our understanding of the equilibrium dynamic effects of public expenditures in a representative-agent model with multiple production sectors. We plan to pursue these research projects in the future.

6 Appendix A

It can be shown that the elements which make up the benchmark model's Jacobian matrix $\mathbf{J}$ as in (25) are

$$\mathbf{J}_{11} = 1 - \delta + \frac{\alpha\delta\left(1+\theta\right)\left(1+\gamma\right)}{\left(\alpha+\gamma\right)\left(1-\mu\right)}, \tag{A.1}$$

$$\mathbf{J}_{12} = -\frac{\delta\left(1+\theta\right)\left[1-\alpha+\mu\left(\alpha+\gamma\right)\right]}{\left(\alpha+\gamma\right)\left(1-\mu\right)}, \tag{A.2}$$

$$\mathbf{J}_{21} = \frac{1}{\Pi}\left[\frac{\alpha\theta\left(1+\gamma\right)}{\left(\alpha+\gamma\right)\left(1-\mu\right)} + \Psi\mathbf{J}_{11}\right], \tag{A.3}$$

$$\mathbf{J}_{22} = \frac{1}{\Pi}\left\{\frac{\left(\alpha+\gamma\right)\left[1-\mu\left(1+\theta\right)\right]-\theta\left(1-\alpha\right)}{\left(\alpha+\gamma\right)\left(1-\mu\right)} + \Psi\mathbf{J}_{12}\right\}, \tag{A.4}$$

where μ is given by (24) together with

$$\Pi = 1 + \frac{\left(1-\alpha\right)\left[1-\beta\left(1-\delta\right)\left(1+\theta\right)\right] - \frac{\beta\theta\mu(1-\delta)(1+\gamma)}{1-\mu}}{\alpha+\gamma}, \tag{A.5}$$

and

$$\Psi = \beta\left(1-\delta\right)\left[1-\alpha\left(1+\theta\right)\right] - \left(1-\alpha\right) + \frac{\alpha\left\{\left(1-\alpha\right)\left[1-\beta\left(1-\delta\right)\left(1+\theta\right)\right] - \frac{\beta\theta\mu(1-\delta)(1+\gamma)}{1-\mu}\right\}}{\alpha+\gamma}. \tag{A.6}$$

7 Appendix B

It can be shown that the log-linearized dynamical system for the alternative specification of our model economy exhibits the same representation as in (25), and that the elements that make up the corresponding Jacobian matrix are

$$\mathbf{J}_{11} = 1 - \delta + \alpha\delta\,(1+\theta) + \frac{\alpha\,[\delta\mu\,(1-\alpha)\,(1+\theta) + \Omega\,(1+\gamma)\,(\mu-\varphi_c)]}{\mu\,(\alpha+\gamma)}, \tag{B.1}$$

$$\mathbf{J}_{12} = -\frac{\delta\mu\,(1-\alpha)\,(1+\theta) + \Omega\,(1+\gamma)\,(\mu-\varphi_c)}{\mu\,(\alpha+\gamma)}, \tag{B.2}$$

$$\mathbf{J}_{21} = \frac{1}{\Delta}\left\{\alpha\theta + \frac{\alpha\theta}{\alpha+\gamma}\left[\frac{(1+\gamma)\,(\mu-\varphi_c)}{1-\mu} + 1 - \alpha\right] + \Lambda\mathbf{J}_{11}\right\}, \tag{B.3}$$

$$\mathbf{J}_{22} = \frac{1}{\Delta}\left\{1 - \frac{\theta\,(1+\gamma)\,(\mu-\varphi_c)}{(\alpha+\gamma)\,(1-\mu)} - \frac{\theta\,(1-\alpha)}{\alpha+\gamma} + \Lambda\mathbf{J}_{12}\right\}, \tag{B.4}$$

where μ is given by (30) together with

$$\Omega = \frac{\delta\mu\,[(1-\mu)\,(1+\theta) - \theta\varphi_I]}{(1-\mu)\,(1-\mu-\varphi_I)}, \tag{B.5}$$

$$\Delta = 1 - \frac{(1+\gamma)\,(\mu-\varphi_c)\left[\frac{\beta\theta\mu(1-\delta)}{1-\mu}\right] - \mu\,(1-\alpha)\,[1-\beta\,(1-\delta)\,(1+\theta)]}{\mu\,(\alpha+\gamma)}, \tag{B.6}$$

and

$$\Lambda = \beta\,(1-\delta)\,[1-\alpha\,(1+\theta)] - (1-\alpha) - \frac{\alpha\left\{(1+\gamma)\,(\mu-\varphi_c)\left[\frac{\beta\theta\mu(1-\delta)}{1-\mu}\right] - \mu\,(1-\alpha)\,[1-\beta\,(1-\delta)\,(1+\theta)]\right\}}{\mu\,(\alpha+\gamma)}. \tag{B.7}$$

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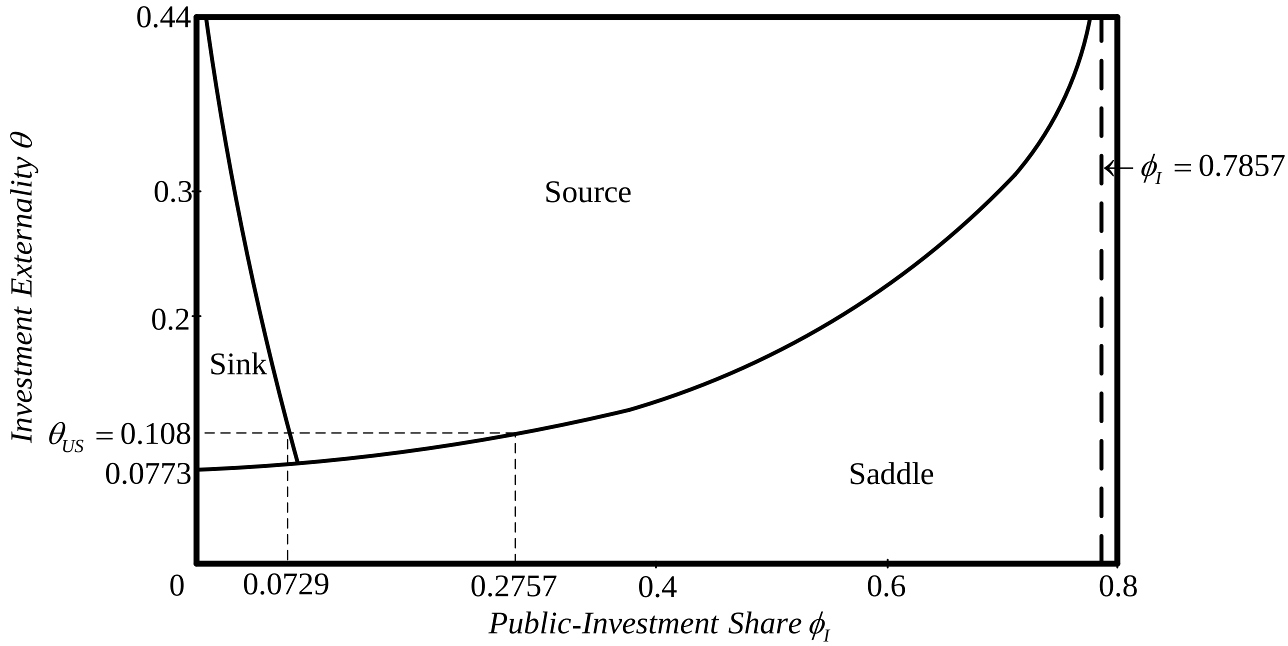

Figure 1. Benchmark Model with $\frac{\mathrm{G_{It}}}{\mathrm{Y_{It}}} = \phi_{\mathrm{I}}$

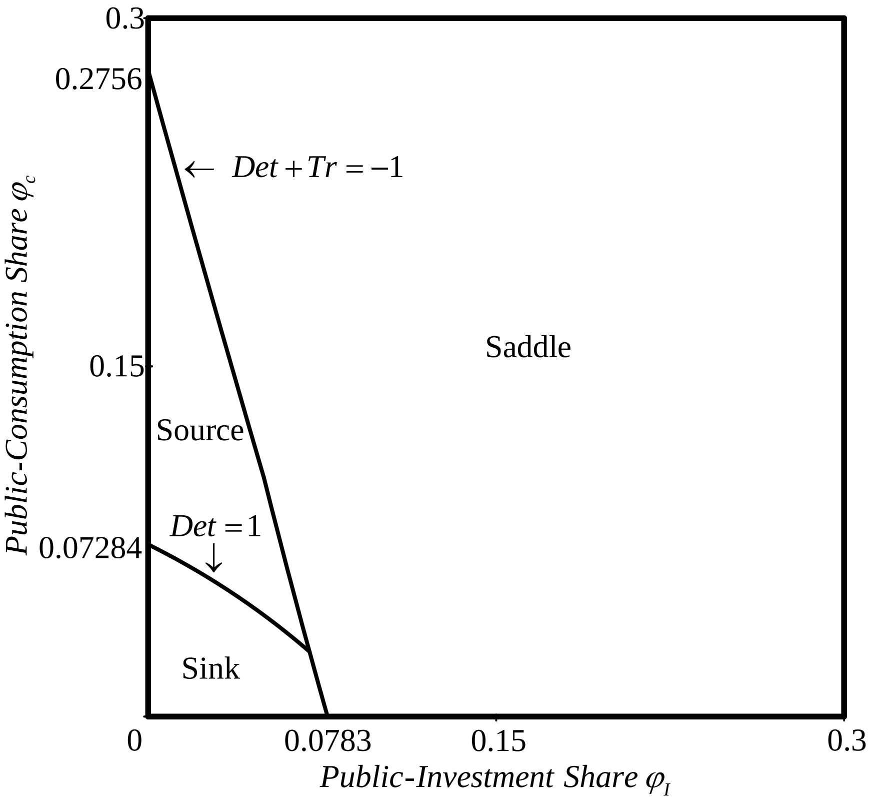

Figure 2. Alternative Model with $\frac{\mathrm{G_{ct}}}{\mathrm{Y_t}} = \varphi_{\mathrm{c}}$ and $\frac{\mathrm{p_t G_{It}}}{\mathrm{Y_t}} = \varphi_{\mathrm{I}}$